PREFACE

This book has been written to provide a textbook from which to teach the elements of Machine Design. The book is not intended to be a reference book in which may be found numerous applications of the same general idea. For each general problem which is discussed only a few applications have been analyzed, on the assumption that if the fundamental principles are well understood, these principles may be applied to a special problem without great difficulty.

The material has been arranged in an order which experience with classes has indicated would be *teachable*, and an attempt has been made to arrange the material for the convenience of the student and the instructor. It has been assumed that the student would consult freely such references as the various engineers' handbooks, but enough tables and diagrams have been included so that the ordinary data needed for design would be available.

Since students who are taking a course in Machine Design either have had or are studying Strength of Materials, it is assumed that the student will consult his textbooks on mechanics whenever necessary. For this reason the derivation of many of the formulas which may be found in any standard mechanics textbook have been omitted.

It is the opinion of the authors that the subject of Kinematics need not be taught as a separate subject, and that there are actual disadvantages in attempting to treat it separately. It is believed that enough material on Kinematics has been included in this book so that a separate text on this subject will be unnecessary.

The authors make no claim for originality of subject matter, but the arrangement, treatment, and choice of material are in some respects new. A number of useful tables have been added in the Appendix. References to Maurer and Withey's "Strength of Materials," John Wiley & Sons, Inc., New York, are to the first edition, 1925. References to Boyd's "Strength of Materials," McGraw-Hill Book Company, Inc., New York, are to the third edition, 1924.

To Professor Scott Mackay the authors are indebted for suggestions in arranging the chapter on Materials of Construction. The permission which has been granted by various manufacturers, authors, and publishers, to make use of material such as tables and figures, is hereby gratefully acknowledged. The authors will appreciate being notified regarding errors which may have been overlooked.

THE AUTHORS.

MADISON, WISCONSIN,
August, 1929.

ERRATA

Page 75, line 3 from top. Difference = 1.

Page 122, Formula (12) $d = \sqrt{\dfrac{6Px}{Sb}}$

Page 135, Formula (25) $\dfrac{S_{\max}}{S_{-1}} = \dfrac{3}{2 - r}$

Page 201, Equation preceding Formula (6)

$$I \text{ (not L)} = \frac{57.3 \times 2S \times 20d}{E_s d}$$

Page 235, Art. 236. The standard rolling circle which has been adopted, is one which will generate a hypocycloidal curve for a 12-tooth gear which is coincident with the diameter of the pitch circle, thus giving the gear radial flanks.

Page 239, Formula in footnote:

$$S_t = \left(\frac{150}{200 + V} + 0.25\right) \times 6{,}000$$

Page 255, Fig. 28. In the lower diagram of forces $F = W \tan \beta$ is the diagonal, and the angle α is the complement of the one indicated as α.

Page 312. Fig. 14(b). The force F should be the force P.

Page 329. Figs. 23(a) and (b) should be interchanged to check with the text.

HYLAND AND KOMMERS—MACHINE DESIGN

CONTENTS

MACHINE DESIGN

CHAPTER I

GENERAL CONSIDERATIONS AND PROCEDURE AFFECTING DESIGN

1. The inception of an idea in the mind of a design engineer may be looked upon as the first step in the design of a machine, but there are other matters which must be considered item by item, until all chances of the machine's failing to meet its requirements have been overcome. The design of a machine or structure implies full responsibility of the designer for complete drawings as a part of the specifications, for the building of models if necessary, for suggestions, advice, and inspection during erection, and finally, the release of the finished machine to the owner.

After a machine has been tested under working conditions, as a critical proof test of what may be expected in service, it is considered to be a marketable product. The arrangements for building similar machines in large numbers is an engineering problem of "production," and the important things involved are closely connected with the following considerations, although not all of them necessarily apply to each design problem.

1. The determination of the motion of parts, or the kinematics of the machine.

2. The selection of the materials from which the machine is to be constructed.

3. The determination of the form and size of the machine parts.

4. A study of the frictional resistance of moving parts and the means of lubrication.

5. A study of convenient and economical features in the operation and maintenance of the machine.

6. A consideration of the employment of standard parts.

7. A consideration of the safety of the operator of the machine.

8. A study of the facilities of the shop in which the machine or structure is to be fabricated.

9. A consideration of the number of articles to be manufactured.

10. A study of the cost of construction and the cost of operation.

11. A study of the assembling of parts for the finished machine.

12. A consideration of the transportation of the machine.

2. Kinematics. The successful operation of any machine depends largely upon the simplest arrangement of parts which will give the motion required. Combinations of machine elements which experience has proved to be adapted to the conditions of the problem should be adhered to. On the other hand, in the case of at least one manufacturer, new and original methods have been developed because the design engineers were instructed to discard all previous methods, and attack the problem from the very beginning. Such a plan may succeed with a brilliant engineering staff, and when the cost of the project does not have to be taken into account.

The introduction of original features should be avoided unless the underlying physical laws have been thoroughly studied and the "new idea" has proven to be sound. It may be stated that successful design engineers are those who, early in their career, mastered the fundamental laws of physical science.

Geometry, trigonometry, and the calculus are used to determine the location of centers of rotation and changes in the position of parts. The velocities and displacements should be thoroughly worked out, with no thought at this stage of the work as to materials, form, size, and strength of parts.

3. Selection of Materials. It is essential that a general knowledge of the properties of materials and their behavior under working conditions should serve as a guide to their proper selection. Chance must not be allowed to function, because the designer must predict the action of any material under stress or in contact with any other material.

Some of the important characteristics of materials are: strength; durability; flexibility; weight; resistance to heat and to corrosion; ability to be cast, welded, or hardened; machinability; electrical conductivity; insulating capacity; and cost. Brass, for instance, may be used in places where cast iron would be used

if it were more resistant to corrosion. At high speeds rawhide or composition gears are used to eliminate noises that accompany the contact of harder materials. Unlike metals are used to reduce friction at the rubbing surfaces, but there are exceptions to this, for cast-iron piston rings are used in cast-iron cylinders of steam and gas engines.

The relation of necessity to invention is illustrated by the fact that metallurgical research concerns itself with seeking materials which will fulfill the requirements that must be met in industrial applications. The research departments are constantly delving in this field to discover new combinations of metals which will meet the demands of competitive conditions. A notable example of this is the discovery of high-speed tool steel, which increased production as high as 500 per cent.

4. The Form and Size of Parts.—Some parts of a machine require little if any consideration of strength. The form and size are based upon judgement, the smallest practicable cross-section being used; but it is well to form the habit of checking the design to determine whether the stresses are reasonably safe.

To design any machine part for form and size it is necessary to know the forces which the part must sustain, and it is important to anticipate any suddenly applied or impact load, which might cause failure. Normal loads are often exceeded by conditions which are outside of the usual range, and while only momentarily applied, they dictate the working unit stress which should be used. An example of this is the starting of a machine under full load, in which case the forces acting are considerably greater than those required for normal running.

The necessity for reducing vibrations by absorption may require that machine frames be made heavier than would be required if only strength were considered. There is a truism among designers "that whatever is right, looks right," and to the experienced and critical eye faulty design is readily detected. Each part of a machine should be the simplest resistant member that will safely withstand the stresses imposed by the load, and the general shape should conform with what usage and tradition have prescribed for machine parts of a given classification.

5. Frictional Resistance and Lubrication.—There is always a loss of power due to frictional resistance, and it should be noted that the friction of starting is higher than running friction. Careful attention should be given to the matter of lubrication

of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings. Such an important matter should not be left to the shop, as is often done, but the problem of providing for lubrication should be studied as soon as possible during design. The selection of the proper provision for lubrication will often radically influence the design of a machine. The drawings should indicate the size and location of oil holes and oil grooves for all parts that require them. A machine may have certain parts which require lubrication but which are not within easy reach of the operator, and it is well to insure against neglect by providing oil cups to furnish the lubrication needed at such places.

6. Convenient and Economical Features. During the process of design a study should be made of the operating features of the machine. The starting, controlling, and stopping levers should be located on the basis of convenient handling, with little chance of mistakes being made by the operator.

Facility for adjustment for wear must be provided, employing the various take-up devices, and arranging them so that alignment of parts will be preserved. If parts are to be changed for different products, or replaced on account of wear or breakage, easy access should be provided, and the necessity of removing other parts to accomplish this should be avoided if possible.

The economical operation of a machine which is to be used for production, or for the manipulation of material, should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. The Use of Standard Parts. The use of standard parts is closely related to cost, and a study of this matter may avoid needless expense, since the cost of standard or stock parts is frequently only a fraction of the cost of similar parts made to order. Standard or stock parts should be used whenever possible: parts for which patterns are already in existance, such as gears, pulleys, and bearings; and parts which may be selected from regular shop stock, such as screws, nuts, and pins. The use of special bolts, studs, and pins should be avoided. Bolts and studs should be of as few sizes as possible to avoid the delay caused by changing drills, reamers, and taps, and to decrease the number of wrenches required.

When a series of machines or structures of different capacities or sizes is being designed, the design engineer should endeavor

to have as many parts as possible common to two or more consecutive members of the series.

8. Safety of Operation.—More attention is being given to the safety of the operator of a machine than was formerly the case. The design engineer does not always have a free hand in this matter, because safety devices add to the cost of the machine, but it is his duty to provide for the safety of the operator to the greatest possible extent. Some machines are inherently dangerous to operate, especially those which have been speeded up to insure production at a maximum rate. Such machines should have the driving and transmission mechanism guarded to remove all possible hazards. Many ingenious devices are now used to promote the safety of machine operators, and persistent and concerted efforts along this line of endeavor have opened up a new field which is called "safety engineering."

9. Shop Facilities.—The facilities of most shops are limited in making parts of machines which are not a regular product. For this reason the design engineer should be familiar with the limitations of his employer's shop, in order to avoid the necessity of having work done in some other shop. It is sometimes necessary to plan and supervise shop operations, and to draft methods for casting, handling, and machining special parts, because designing implies responsibility for the building of the machine designed. Special machines and temporary rigging are costly and should, in general, be avoided.

10. The Number of Articles to Be Manufactured.—The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or "overhead expense" are distributed over the number of articles which are manufactured, so that if only a few are to be made, extra expenses are not justified unless the machine is large or of special design. Usually an order calling for a small number of the product will not permit any undue expense in the shop processes, so that the designer should restrict his specification to standard parts as much as possible. A large machine or a special machine may be of sufficient importance to justify the expenditure of a considerable amount of money for its design and construction.

11. Cost of Construction and Operation.—Many of the matters already discussed have a direct bearing on cost, and this is one of the most important considerations involved in design. When

the question "Will it work?" has been answered satisfactorily, the next most important question is "What will it cost?" In some cases it is quite possible that the high cost of an article would immediately bar it from further consideration. If an article has been invented and tests of hand-made samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. In this case it is the monetary return on the article which the machine produces, and not the profit on the machine itself, which justifies the high cost of the machine.

Under all conditions the design engineer should use all of his skill in an endeavor to reduce the cost of the following items: design, material, shop processes, assembling, testing, transportation, and upkeep of the machine in the hands of the purchaser.

12. Assembling. Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested, and then taken apart to be transported to their place of service. The final location of any machine is important, and the design engineer must, in many instances, anticipate the exact location and the local facilities for erection. Erection changes in the shop, due to lack of foresight or accuracy, are irritating and expensive.

13. Transportation. The product of the shop must be loaded by the builder, transported, unloaded by the purchaser, and then moved to its final location. If railroad transportation is used the usual limits of size are about 9 ft. in width, 10 ft. in height, and 30 ft. in length. Restriction in size and weight of any part, or the extent to which a machine may be assembled for shipment, depends upon conditions which have been taken into consideration in the design specification.

Manufacturers often own and furnish special railroad cars for the shipment of extra large or very heavy units. The transportation department of the local railroad which is to handle the shipment should inspect and approve the consignment. A circuitous route of travel is sometimes necessary to insure a solid road bed, avoidance of sharp curves, and unsafe bridges. Large flywheels and rotors are made in sections, partly to meet restrictions of transportation.

If a machine must be shipped by sea, the maximum weight of any unit depends upon the capacity of the loading equipment at the ports where the machine must be loaded and unloaded. The best equipped ports can handle a complete medium-weight locomotive, and such locomotives completely assembled, have been loaded and unloaded at these ports. Light parts of locomotives, such as stacks, which might be broken during handling or transport, are usually boxed separately. Machinery is usually dismantled and boxed, for ease in handling, protection, and economy of space. If material is destined for the interior of an undeveloped country, rigid restrictions are placed upon the allowable size and weight of parts, because of transportation difficulties. Machinery and structural equipment used in the mining and oil industries must sometimes be carried by burros or camels. The motor car, however, is rapidly becoming available in all parts of the world.

14. General Procedure in Design.—The first consideration in the design of any machine is to understand the requirements of the problem. What is it that the machine is intended to do, or what is the structure to be used for? It often happens that the modification in the design of a known machine or structure will prove to be the most economical solution of the problem. Examination of an old wood cut of the Pearl Street Station in New York, which was first operated in 1882, shows that many of the elements of the original electric generators are incorporated in the design of the most modern electrical machines.

Design, in general, involves the application of known engineering principles to certain applications, the application, as a rule, being what is new. If there is any conflict between engineering and practice, it is due to an incomplete understanding of the problem involved. In his "Applied Mechanics" Rankine makes the following statement: "Sound theory in physical science consists simply of facts, and the deductions of common sense from them, reduced to a systematic form."

Science, in its application to engineering problems, is frequently referred to in an antagonistic sense as theory, the implication being that there is a difference between theory and fact, and that theory does not "work" in its applications. This is perhaps due to the popular misinterpretation of the words "theory" and "hypothesis." By hypothesis is meant an explanation, the truth of which remains to be proven. Webster confounds

theory with hypothesis, but the Standard Dictionary defines theory as "a scheme subsisting in the mind, verifiable by experiment and observation; a rational explanation that agrees with all the facts and disagrees with none." No engineering design ever failed where correct theory was used by the designer, because correct theory must agree throughout with correct practice.

The design engineer must be certain that his assumptions are correct and his data as complete as possible before proceeding, and to guard against errors in this direction he should investigate the status of the art that is under consideration.

Many problems involve the preliminary layout of the displacement of parts, and often the kinematics of such a problem may be solved by geometry. In some cases the problem may be of such elementary nature as to require no special solution. If a preliminary layout is employed, it should be made to as large a scale as is convenient (full size is advantageous); and the elements should be represented by points and lines only, no consideration being given at this stage to shape or strength.

The particular movement sought for is often made available by the use of several kinematic combinations, but it is important to remember that the simplest combination that will meet requirements is the one to use.

The form and size of component parts of any machine or structure are intimately associated with the kind of material to be used, but the selection of the material is generally the least difficult portion of the problem. The available materials for machine construction are few, although there are many varieties of some of them. In an electric motor, for example, there will be no question but what the bed plate and frame should be made of cast iron, the shaft of steel, and the bearings lined with brass; because it is well known from past experience that such metals are adaptable to such uses.

15. Rational Design.—When mathematics can be employed to determine the form and size of parts, the design is called rational design; but unfortunately rational design cannot be applied to the solution of all problems. Very often rational design is used to check a design based upon other considerations.

When rational design is employed, the sizes of parts should be calculated from the forces to which the part will be subjected, and on the basis of safe limits of stress intensity. This involves

skill in the methods of computation, and a knowledge of fundamental mechanics. As illustrations of machines and structures to which the rational method of design is applicable, the following may be noted: a bridge, a boiler shell, a crane, or an electric motor.

Machines in which rigidity is of prime importance often do not lend themselves to mathematical treatment, except as a check. In general they are designed to give satisfactory operating results, and considering strength only, some parts may have various degrees of oversize. The science of mechanics as applied to problems of this nature has not advanced sufficiently to be of value in originating designs. In such cases carefully conducted experiments in conjunction with the principles of mechanics might be employed to determine the design of similar machines. The principles of mechanics may also be employed in designing a machine similar to one already in successful operation, but which is on a larger or smaller scale.

16. Empirical Design.—For cases in which rational design cannot be applied, the design engineer should make use of the proportions of a similar machine which may have been developed by a process of evolution. Empirical design is the result of using data derived from machines and designs in actual use, and such information is usually tabulated in various handbooks for ready reference.

If the history of the modern lathe is studied, it will be found that its principle was known and utilized in ancient times. It was improved during the middle ages, and in the early part of the nineteenth century an English engineer, Henry Maudslay[1] (1771–1831) employed the then known slide rest, power-driven lead screw, and change gears, for generating and cutting fairly accurate screw threads. One of Maudslay's early lathes is now in the South Kensington Museum in London, and it incorporates the general principles upon which our modern machines are based. Chronologically, engineer *A* may have designed an engine lathe which proved to have a weakness either in design or construction; *B* followed along similar lines, strengthening the parts where necessary; *C*, adapting the machine to a new product, still further improved it; and finally, about 1900,

[1] Roe, J. W., "English and American Tool Builders," McGraw-Hill Book Company, Inc.

the discovery of high-speed tool steel for cutting edges, necessitated radical changes which resulted in the lathe as it is today.

In empirical design a survey of machines, similar to the one contemplated, leads to the incorporation of the good features and the avoidance of the bad features of existing machines. If an actual machine cannot be studied, recourse should be had to the technical press, catalogues, photographs, or verbal descriptions. It is at all times legitimate to take advantage of the experience which others have gained in the same field, and by this method costly errors may often be avoided. While the direct copy of a machine, without permission, must be classed as improper ethics, the systematic study of the state of the art is an imperative duty.

17. Combined Rational and Empirical Design. In many cases a combination of rational and empirical design methods is possible. For example, in the design of a punching machine a fair approximation of the total load may be obtained from the size of the opening to be punched and the shearing strength of the metal. From this value the frame, gears, and shaft sizes may be calculated. The sizes thus found should be compared with the parts of similar machines commonly used and therefore well proven in practice.

The advantage of this procedure over direct copying is that it is possible to apply the principles of mechanics in checking the design, and at the same time to avoid errors due to incomplete data. This method should be employed whenever possible.

Another method which has been employed is to combine rational design with actual test. This is especially useful in complicated designs in which it is difficult to determine the exact stresses by calculation. During the World War the Forest Products Laboratory at Madison, Wisconsin, improved the wing ribs of aeroplanes by this method. The object in view was to obtain the strongest wing rib per pound of material. A machine was devised which applied load to the wing rib approximately equivalent to conditions of service. By testing a rib, determining its weak point, and then strengthening it at the proper place, it was found possible to develop a rib which greatly exceeded in strength the original design.

18. Designing by Experience.—If psychologists had been allowed to name this kind of design they would perhaps have called it "behavioristic design," because it is copying from

memory, or is the result of a mental calculation combined with memory. A design engineer, working on a certain type of apparatus, may be employed by another manufacturer. The designer's mind is stored with information concerning the sizes and shapes of various parts, and when he wishes to use a certain machine element he recalls the form and dimensions from his former association, and adapts them to his new problem. This type of designing lends itself to deductive reasoning, and when successfully practiced, calls for the greatest skill. To employ this method the design engineer should possess a large fund of information derived from careful observation, and he must have the necessary resourcefulness to apply it.

In the past, before the principles of mechanics as applied to strength of materials had been developed, all design was necessarily based upon experience. Under such conditions certain individuals, families, or guilds might become very expert through the cumulative information gained from experience.

CHAPTER II

METAL WORKING AND SHOP PROCESSES

19. In considering the principal metal-working processes it is not possible in all cases to locate the several processes in a definite classification, because a given process may partake of one or several of the metal-working operations. This chapter however, includes a brief statement of the salient features of the different methods, each process considered as a separate operation.

In the *melting* process a metal is brought to a fluid condition by the application of heat, for the purpose of uniting it with a second metal having a higher fusing point, as in the soldering process, or to cause two pieces of like metal to unite, as in welding.

20. Soldering and Brazing. The processes of soldering and brazing are essentially the same, except that in the former the solder or joining metal usually contains a large proportion of lead, and melts below a red heat; while in the latter, known also as "hard soldering," the joining metal, of brass, bronze, silver or gold alloys, requires the application of a high temperature to effect the melting.

In any form of soldering, a flux is usually necessary to dissolve the metallic oxides, and thus allow the metallic surfaces to come in contact, without which the joining by metal is not possible. A capillary action appears to take place, particularly in brazing, as it is impossible to bring two pieces of iron or steel into such close contact as to prevent the entrance of the spelter into the joint.

All the ordinary metals may be easily soldered or brazed with the exception of aluminum and cast iron. The former requires special solders and fluxes, and even then the result is usually not satisfactory. Cast iron likewise requires special materials and a long heating at high temperature just below melting point, but if the operation is carefully performed the result is satisfactory. This method is now used in the repair of large sections,

such as bed plates and heavy machine frames. The melting processes involved in soldering, brazing, and welding do not require a mold, since only a thin film of metal adheres, and whatever excess of metal is applied is removed by a soldering or scraping tool.

21. Casting.—*Casting* involves a melting of the metal, but differs from melting in that the fluid metal is poured into a mold, the form of which the metal retains upon solidification. All of the ordinary metals may be cast with the exception of wrought iron, which cannot be brought to a fluid condition without changing the characteristic properties of the material.

Casting has the advantage over other forming processes because complicated forms can be produced at low cost, and also because the brittle non-forgeable metals, such as cast iron, become available in construction. Most of the steels are adapted to casting, so that for some purposes steel castings are substituted for expensive forgings, and in other cases shapes formerly made of cast iron are replaced by a tougher and more ductile material, at a cost comparable to that of cast iron.

Nearly all metals have a greater volume when liquid than when solid, so that after pouring, a casting will be smaller than the pattern from which it has been formed. This difference is called shrinkage, and is overcome by making the pattern a definite amount larger than the required casting. For cast iron this allowance is about 1/8 in. per lineal foot, but shrinkage is dependent on the form and size of the casting, and on the composition of the cast iron. Cast iron expands at the moment of solidification and then contracts rapidly, resulting in castings with sharper lines than if cast of a metal not having this property. Castings are usually made in sand molds, which give a rough appearance to the surface. For many purposes, when the number of castings to be made justifies the cost, permanent molds are made, and the resulting castings have smooth surfaces.

Steel is a more difficult metal to cast than iron because the shrinkage is greater, being about 1/4 in. per lineal foot, and because of the short period of fluidity greater amounts of gas are expelled. Molds for steel castings are made much the same as for other castings of similar shape and size, the principal differences being the quality of sand used and the fact that steel castings require more and larger shrink-heads and risers.

22. Die-casting.—Die-casting consists of forcing molten metal into steel dies, and after the metal has cooled, opening the dies and removing the castings. The die-casting process is adapted to alloys having low fusing points, and practice has narrowed down to the five following groups.

White-metal alloys are zinc-base alloys, also known as white brasses. The tensile strength is about 17,000 lb. per square inch, and the weight of the casting is generally not more than 8 lb. The minimum wall thickness is 0.1 in. for the larger castings and 1/16 in. for the smaller castings. The variations in dimensions per inch may be held to 0.001 in. for all directions. These alloys will corrode, but may be plated with nickel, copper, silver, or gold. They are used extensively for the parts of phonographs, automobile-body trimmings, stamp-affixing machines, and for a great many similar applications.

Tin-base alloys are babbitt metals which are used in die-cast bearings, and since these metals will resist corrosion, they are used extensively for the parts of milking machines and other apparatus used in the food industries. The maximum weight of parts made from these alloys is about 10 lb., the variation from exact dimensions per inch is 0.0005 in., but the tensile strength is not over 8,000 lb. per square inch.

Aluminum alloys are superior to pure aluminum because the addition of copper or magnesium improves the quality of the metal for die castings, resulting in light-weight parts of a tensile strength from 32,000 to 35,000 lb. per square inch. Copper hardens the metal, while magnesium produces a closer grain in the metal than can be obtained in an alloy of aluminum and copper. Nickel, in quantities from 1 to 3 per cent, will produce a still harder alloy and allow for a much brighter finish. The aluminum-magnesium alloy is suitable for intricate castings because it flows freely and fills all the cavities in the mold perfectly. The maximum weight of castings is about 5 lb. with a least wall thickness of 1/16 in., and a variation per inch from exact dimensions of 0.0025 in. in any direction. These castings are used widely in the manufacture of parts of magnetos, battery ignition and lighting systems, and vacuum sweepers.

Brass and *bronze* die-cast parts are now made with many compositions, but due to the action of the molten metal, it is difficult to maintain the dies in good condition. The result is

that production is limited, the number of castings being often 1,000 or less, resulting in high unit cost of product.

Lead alloys are composed of lead, antimony, and tin, and are used for die-cast parts when a low tensile strength and a good non-corrosive quality is desired. The variation per inch in dimensions is about 0.001 in. in all directions, the maximum weight of the cast part is about 15 lb., and the maximum wall thickness is about $\frac{1}{32}$ in. Lead-alloy castings are used extensively for the parts of fire extinguishers, low-pressure bearings, ornamental metalware, and for parts that come in contact with corrosive chemicals. Their relative cheapness recommends these castings for coffin trimmings, and during the World War they were used for grenades, trench-mortar fuse plugs, and other applications where resistance to corrosion was a requirement.

23. Hot Forging.—*Welding* may be classified as *forge welding* or *fusion welding*, according to the method employed in the operation. When ductile metals are worked or formed by hammer blows or pressure while in a plastic state, the operation is known as hot forging. Figure 1 shows a continuous semi-hot forging machine which will form heads on rivets, bolts, tappet valves, and similar work at the rate of 120 to 200 per minute. Forging is one of the oldest of the metal-working operations, and when done by hand it is so costly that, where possible, it has been replaced by the use of steel castings or drop forgings.

Drop forgings are made from ductile metals by forcing the metal while in a plastic condition into dies which will give the metal the required form. The metal is heated to a temperature below the melting point and placed on the lower die, the upper die is then dropped down, and the plastic metal is forced into the die and shaped to the form of the die. The surplus metal is squeezed out in a "fin" along the parting lines of the die, and is later sheared off in a cutting die. For deep forms more than one set of dies may be necessary, thus bringing the metal to final form by successive forging operations. Drop forgings of excellent quality are obtained, and are used largely for all high-grade tools and machine parts.

Large forgings, such as crankshafts, are preferably forged by slow deformation in a hydraulic press instead of by a drop hammer. The slowly applied pressure gives the metal time to flow and properly adjust itself, and the finished product is less

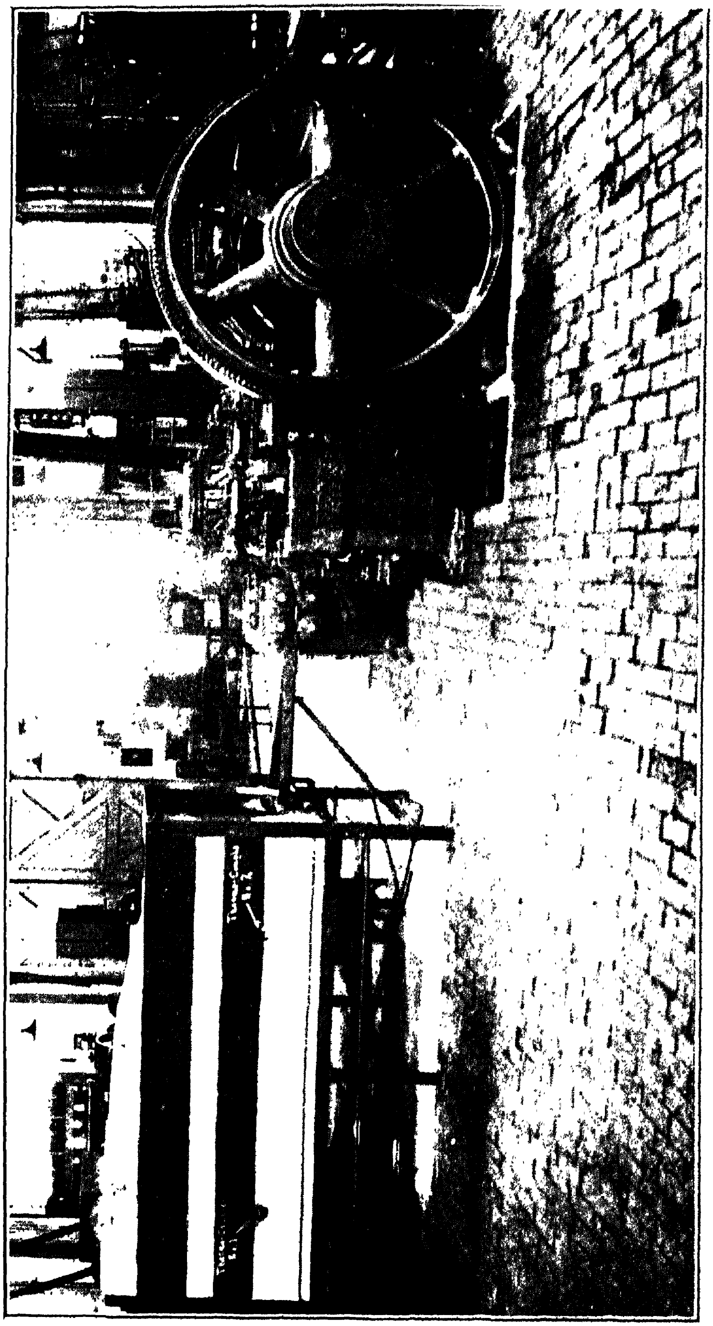


FIG. 1.—Continuous semi-hot forging machine. The National Machinery Co. Tiffin, Ohio.

likely to have cracks and hidden defects than when formed by sudden blows. Figure 2 shows a forging press for heavy work.

The method practiced in the hot rolling of structural shapes, rails, steel plate, and locomotive tires, may be classed as a continuous forging process, since the plastic metal passes between pairs of rolls of varying distance or section.

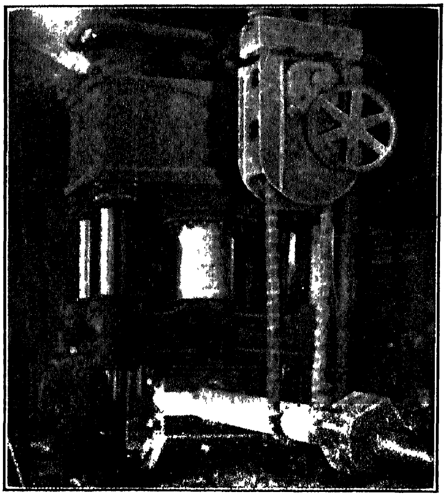


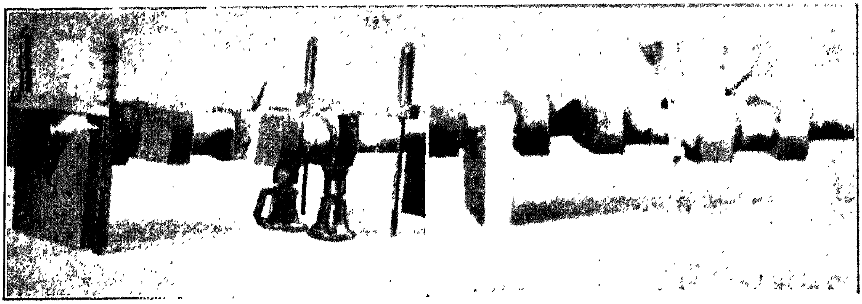
FIG. 2.—1,000-ton high-speed forging press. (*Southwark Foundry and Machine Co., Philadelphia, Pa.*)

24. Fusion Welding.—*Fusion welding* is to be distinguished from forging, in that the welding is done without the application of pressure or hammer blows and the metals are worked in the fluid and vapor states. The introduction of machinery provided means for speeding up the welding operations, and also increasing the size of the sections welded. Long seams are now welded by heating the metal to the fusion temperature by a gas flame, and then passing the plates between rollers which spread the metal while the weld is being made. After finishing, it is almost impossible to detect the weld. There are three main divisions of the fusion type of welding, and these are known as *thermit, gas,* and *arc welding.*

25. Thermit Welding.[1]—Thermit steel is formed by igniting a mixture of iron oxide and finely divided aluminum. The

[1] OWENS, JAMES W., "Fundamentals of Welding," 1st. Ed., The Penton Publishing Co., Cleveland, Ohio, 1923.

aluminum combines with the oxygen of the oxide, reducing it and forming aluminum oxide and iron. The pure iron produced by this reaction is too soft and ductile for commercial purposes, therefore the reaction is tempered by adding steel particles containing the proper proportion of carbon, silicon, manganese, and nickel, to match the steel in the section to be welded. The weld is made by tapping the steel of the thermit reaction into a mold surrounding the fractured parts. The thermit steel has a temperature of 5000° F., about double the temperature of ordinary molten steel, and as it flows between the fractured parts it melts the surfaces of the fracture and in cooling solidifies with the parts. Thermit welding is fundamentally a melting and casting process.



(a) (b)

Fig. 3.—(a) Thermit weld on a $6\frac{1}{2}$ in. crank shaft showing the fracture cut out ready for the weld; (b) thermit weld on a $6\frac{1}{2}$ in. crank shaft showing the weld with pouring gate and riser. (*Metal and Thermit Corp., New York City.*)

Figures 3(*a*) and 3(*b*) show the application of thermit welding to a broken crank shaft.

The thermit weld is used for heavy welding, such as rails, truck frames, locomotive frames, and other large sections used on steam and electric railroads; for stern frames, rudder frames, shafts, and anchors, in shipyard work; and in the steel mills the process is employed to replace broken gear teeth, to weld new necks on rolls and pinions, and to repair broken shears.

26. Gas Welding.—A *gas weld* is made by applying the flame of a welding torch upon the surfaces of a prepared joint. The torch consists of two tubes, usually independent of each other, one for oxygen and the other for acetylene or other combustible gas. The two tubes are connected to a mixing head, shown by Fig. 4, in which the gases are mixed in the proper proportion,

the burning jet coming from the mixing head being controlled by valves in the handle of the torch. The intense heat at the

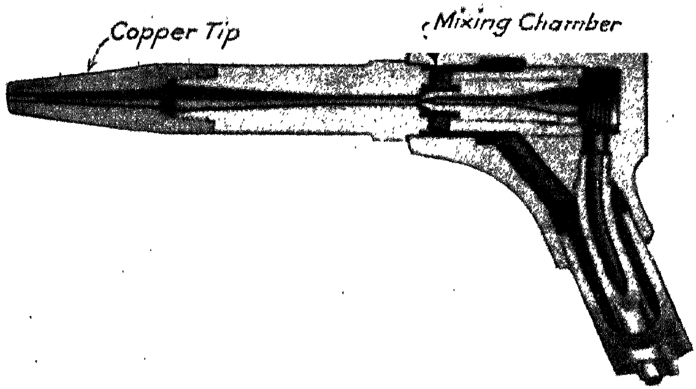


FIG. 4.—Cross-section of blowpipe head. Oxweld welding nozzle. (*Oxweld Acetylene Co., Long Island City, N. Y.*)

white cone of the flame heats up the local surfaces to the fusion point, while the operator manipulates a welding rod to supply the metal for the weld, a flux being used to remove the slag. Figure 5 shows a gas-welding apparatus.

The technique of gas welding varies with the kind of metal to be welded. The quality of the welds is largely dependent upon the skill of the operators, and these men have developed their own methods from successful practice. Cast iron is one of the metals which may be welded successfully by this method.

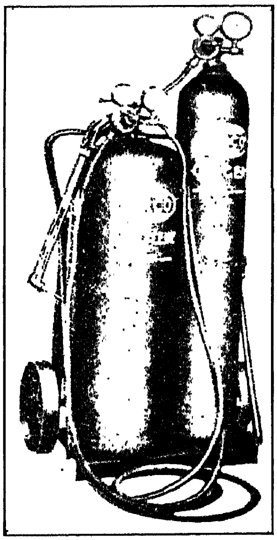


FIG. 5.—Portable welding outfit. (*Air Reduction Sales Co., New York City.*)

27. Arc Welding.—For *arc welding* the work is prepared in the same fashion as for gas welding. The operator, with his eyes and face protected, strikes an arc by touching the work with the electrode, then withdraws it about 1/8 in. to allow the arc to form, which instantly fuses the tip of the electrode and an adjacent spot on the work. Arc welding does not require the metal to be preheated, and since the temperature of the arc is about 6000° F.,

the fusion of the metal is almost instantaneous. There are two kinds of electrodes used for arc welding, carbon and metal, the carbon electrode being used chiefly for thin plates and where the joint is such that it will supply its own fused metal filling. The substitution of a metal rod for the carbon electrode has the advantage that the metal electrode as well as the work fuses and serves as a filling metal for the weld. The usual voltage for arc welding is about 20 volts, although at the start a voltage of about 60 is required to break down the resistance of the air gap between the electrode and the work. Both direct and alternating current are used at the present time for electric welding, but it is more difficult to maintain the arc with alternating current than

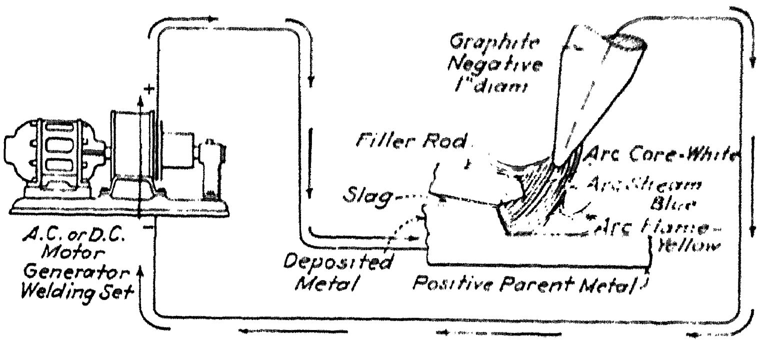


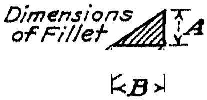
FIG. 6. Arc-welding circuit.

with direct current. Figure 6 shows a diagram illustrating the application of the arc-welding process.

28. Application of Arc Welding.[1] Well-made fabricated parts joined by arc welding are much stronger in proportion to their weight than castings, and may be readily produced at low cost. This process is comparatively new, and its use is impeded to some extent by the lack of design data and engineering information. Table I gives design data for the strength of flush, reinforced, and concave fillets which are employed in making joints for a variety of work. Figures 7(a) to 7(n) show the application of arc welds to the joints of a variety of steel shapes. The form of the fillet and its length, together with the allowable strength, having been decided upon, the strength of the joint may be calculated.

[1] Westinghouse Electric and Manufacturing Co., East Pittsburgh, Pa.

TABLE I.—DESIGN ENGINEERING DATA FOR ARC-WELDED JOINTS



Allowable strength of flush-welded fillets per inch of length for various fiber stresses

Dimensions in inches			Pounds per square inch					
A	B		4,000	5,000	9,000	12,000	30,000	
1/8	1/8	0.0884	345	442	769	1,060	2,650	3,540
1/8	3/16	0.104	416	520	936	1,250	3,120	4,160
3/16	3/16	0.133	532	670	1,200	1,600	3,990	5,320
3/16	1/4	0.150	600	750	1,350	1,800	4,500	6,000
1/4	1/4	0.177	708	885	1,590	2,120	5,310	7,080
1/4	3/8	0.208	832	1,040	1,870	2,500	6,240	8,320
3/8	3/8	0.266	1,060	1,330	2,390	3,190	7,980	10,600
3/8	1/2	0.300	1,200	1,500	2,700	3,600	9,000	12,000
1/2	1/2	0.354	1,420	1,770	3,190	4,260	10,600	14,200
1/2	5/8	0.391	1,560	1,960	3,520	4,690	11,700	15,600
1/2	3/4	0.416	1,660	2,080	3,740	4,990	12,500	16,600
5/8	5/8	0.442	1,770	2,210	3,980	5,300	13,300	17,700
5/8	3/4	0.480	1,920	2,400	4,320	5,760	14,400	19,200
5/8	7/8	0.509	2,040	2,550	4,580	6,110	15,300	20,400
3/4	3/4	0.531	2,120	2,660	4,780	6,370	15,900	21,200
3/4	1	0.600	2,400	3,000	5,400	7,200	18,000	24,000
	1	0.707	2,830	3,540	6,360	8,480	21,200	28,300

Reinforced fillets, , add 10 per cent to above strengths.

Concave fillets, , subtract 50 per cent from above strengths.

Strength of welding material
Maximum fiber stress in pounds per square inch

Kind of stress	Ultimate strength	Dynamic or vibration load	Lifting load	Static load
Bending	40,000	5,000	5,000	12,000
Tension	40,000	5,000	5,000	12,000
Shear	30,000	4,000	4,000	9,000
Tension and shear	30,000	4,000	4,000	9,000
Compression	40,000	5,000	5,000	12,000

The relative strengths of the various welded joints may be taken from Table II.

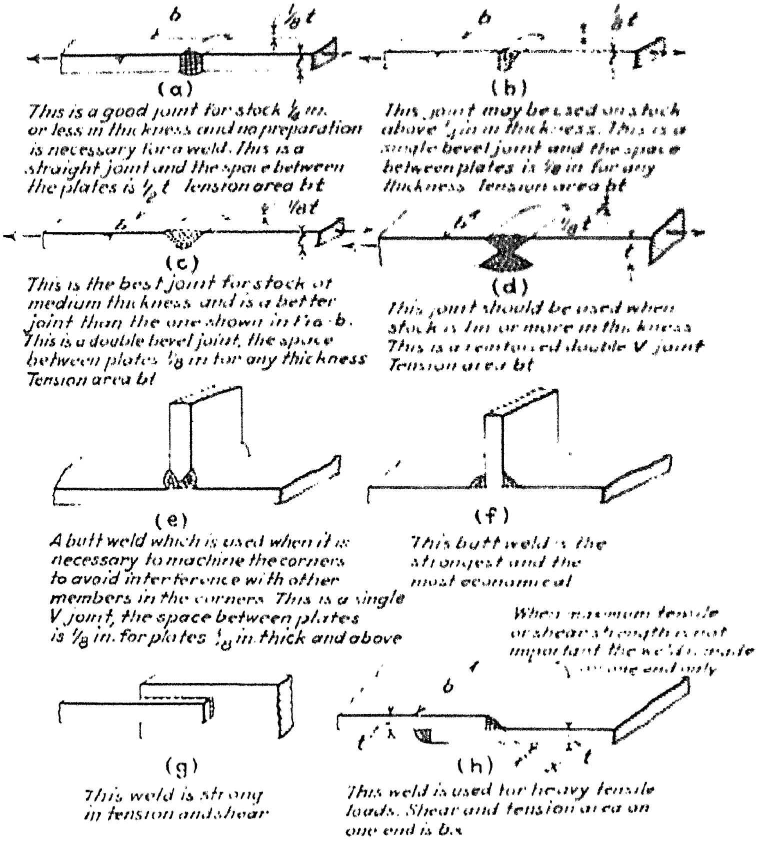


Fig. 7.

Table II. Approximate Relative Strength of the Weld to the Strength of the Welded Parts

Class of joint	Reinforced	Flush	Concave	Oil tight*
1. Weld in shear	0.90	0.75	0.25	0.00
2. Weld in shear and tension	0.75	0.40	0.10	0.00
3. Weld in tension	0.50	0.30	0.00	0.00

* "Oil tight" is a term applied to a weld in which the density of the metal used to close up the seam is such that no leakage is visible when the weld is subjected to water, oil, or air pressure of 25 lb. per square inch.

Figure 8 makes clear the basis upon which the values given in Table I were calculated. In Class 1 the joint is in shear, in Class 2 simultaneously in shear and in tension, and in Class 3 it is in tension. The three classes of joints under the action of static loads with the permissible unit stresses given in the lower portion of Table I will be considered. Suppose in the Class 1 joint the value of A is $\frac{1}{2}$ in., the value of B is $\frac{5}{8}$ in., and the length

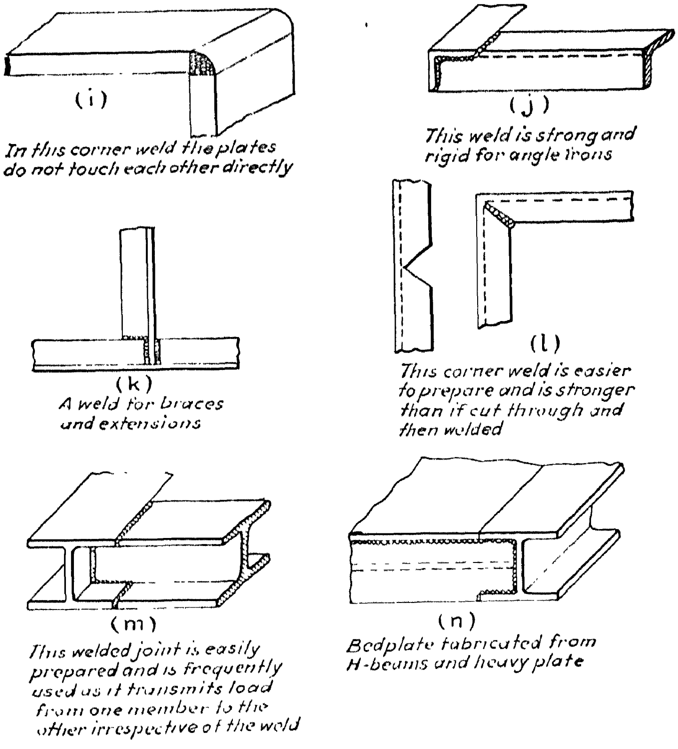


FIG. 7. (*Continued.*)

of the joint is 3 in. How much total load can the joint carry in shear if two plates are welded to a third as shown in Fig. 8? With a permissible unit stress in shear of 9,000 lb. per square inch, the load per inch which may be carried, according to Table I, is 3,520 lb. The total load is:

$$\text{Load} = 4 \times 3{,}520 \times 3 = 42{,}200 \text{ lb.}$$

Suppose the Class 2 joint in Fig. 8 has the values of A and B given above and the length of the joint is 3 in. The total load

which can be carried in simultaneous shear and tension will be based upon a permissible unit stress of 9,000 lb. per square inch. From Table I the load is 3,520 lb. per inch, and the total load is:

$$\text{Load} = 2 \times 3 \times 3{,}520 = 21{,}100 \text{ lb.}$$

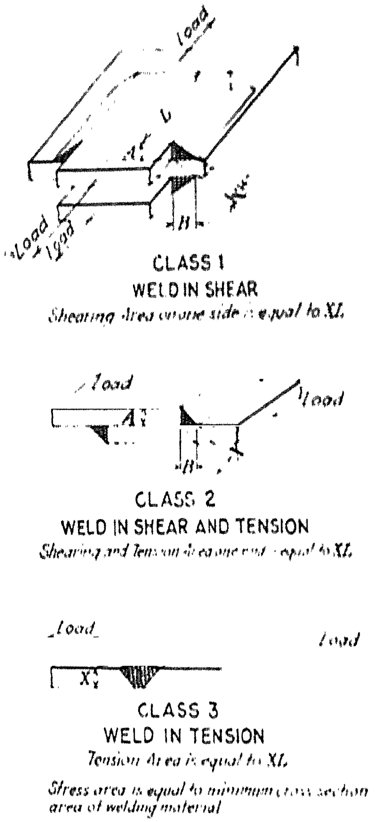


FIG. 8.

The same plate 3 in. wide and ½ in. thick, welded as shown for Class 3 in Fig. 8, could carry a load in tension based upon an allowable unit stress of 12,000 lb. per square inch, as follows:

$$\text{Load} = \tfrac{1}{2} \times 3 \times 12{,}000 = 18{,}000 \text{ lb.}$$

29. Rolling.—*Rolling* is the same in principle whether forming is done on hot or cold metal. The art of rolling in order to shape metals was known in 1553, for it is recorded that a French smith used rolls to produce gold and silver plate of uniform thickness

for the coinage of money. The modern process of rolling steel shapes from the cast material was used by Henry Cort, an Englishman, in 1783, in his iron works at Gosport, England; and he obtained a patent on the use of grooved rolls for rolling

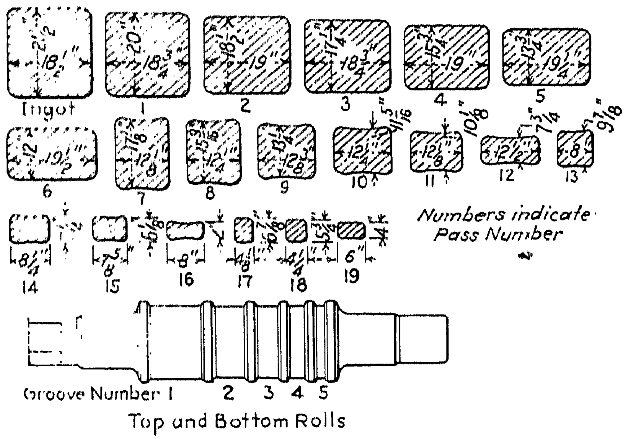


FIG. 9.—Drawing of rolls and sections of bloom as it is reduced in section.

TABLE III[1]

Number of groove on rolls	Size of groove on rolls, inches	Number of passes and manipulation		18-by 21-in. ingot reduced to
1	24	2	Bloom turned 90 deg.	$18\frac{1}{2}$- by 19-in. bloom
1	24	4	Bloom turned 90 deg.	12- by $19\frac{1}{2}$-in. bloom
2	12	4	Bloom turned 90 deg.	$11\frac{3}{4}$- by $12\frac{1}{2}$-in. bloom
2	12	2	Bloom turned 90 deg.	$7\frac{3}{4}$- by $12\frac{1}{2}$-in. bloom
3	8	2	Bloom turned 90 deg.	$7\frac{1}{2}$- by $8\frac{1}{4}$-in. bloom
3	8	2	Bloom turned 90 deg.	4- by 8-in. bloom
5	4	1	Bloom turned 90 deg.	$6\frac{7}{8}$- by $4\frac{1}{8}$-in. bloom
4	6	1	Bloom turned 90 deg.	$5\frac{3}{4}$- by $4\frac{1}{4}$-in. bloom
4	6	1	Finish	4- by 6-in. bloom

Total: 19 passes and 8 turns of 90 deg.

[1] Figure and Table by permission of Carnegie Steel Company, Pittsburgh, Pennsylvania.

iron into bars. Cort is credited with being the father of the rolling-mill industry.

The shaping of steel by the rolling process consists of passing the material between two rolls revolving at the same peripheral

speed and in opposite directions, the distance between the rolls being slightly less in height than the thickness of section entering them. The steel in passing between the rolls is reduced in cross-section and increased in length in proportion to the reduction, except for a slight lateral spreading, which is negligible in most sections. Figure 9 shows the shape of the rolls used in reducing the cross-section of a bloom, and also the reduction which takes place during each pass through the rolls. Table III supplements the information given in Fig. 9. Many shapes, for example crank shafts, are so intricate in design that rolling them is impossible, so that they must be forged under a hammer or with a press. Material formed by the forging process is under better control than if formed by the rolling process, but forging is a slower and more costly process.

Steel products which are manufactured by the rolling process include rails, plates, bars of various sections, armor plate, and such structural sections as channels, I-beams, and angles.

30. Seamless Tubing. Brass and copper have been the materials commonly used for tube making, because they are readily worked, but their relatively high cost excluded their use for most commercial purposes. In 1837, a process was patented in England, by which a short, thick cylinder of cast steel was squeezed, while hot, through an orifice and around a punch, to form a *seamless tube*. Several other patented methods were developed later, and in 1885, Rheinhard and Max Mannesmann patented what is known as the Mannesmann process, the principle of which was discovered accidentally as follows: The Mannesmann firm was a manufacturer of tool steel, and had built a cross-rolling machine with the rolls inclined at an angle, for the purpose of producing round bars from hot billets. After much experimentation with the rolls placed at different angles, it was found that with a high rotative speed of the billet and a slow forward feed, a small axial hole was produced through the length of the bar, which led to the application of this discovery to the manufacture of seamless tubing.

The Mannesmann method of piercing round billets for making seamless tubing can be applied in several ways. The modern piercing machine has heavy steel rolls about 40 in. in diameter and 24 in. long, set side by side at an angle of 6 to 12 deg. in opposite directions, so that the rolls cross at the center as shown in Fig. 10.

A round billet of the proper size for the desired tube is centered at one end, heated, and then pushed between the rolls so that the point of the mandrel will enter the center hole of the billet. The rolls grasp the billet, rotate it, and at the same time pull it slowly forward, forcing the metal over the mandrel. The mandrel serves to increase the size of the hole which is formed at the center of the billet as it passes through the rolls.

The method outlined is known as a piercing process, but little is known about the action of the rolls on the billet. The angle of the roll axes causes the forward movement of the billet, and the high rotative speed seems to cause the metal to be drawn away from the center. The mandrel is not forced through the metal, but the metal seemingly flows around it while in a plastic state.

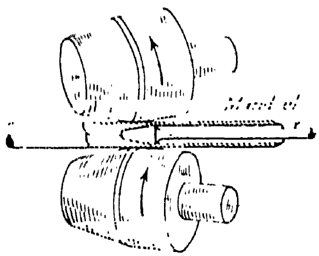


FIG. 10. Diagonal rolls and mandrel for rolling seamless tubes by the Mannesman process. (*National Tube Co.*)

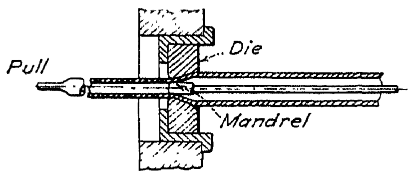


FIG. 11.—Sketch showing relative position of die and mandrel in the cold drawing of tubes. (*National Tube Co.*)

Many improvements were made in the application of the principle of piercing round billets, and equipment has been rapidly developed to produce hot-rolled seamless tubing of fine quality and excellent finish.

31. Cold-drawn Tubes.—*Cold drawing* is done after the tube has been formed by the hot process: to secure a smooth finish, to reduce the wall thickness, to change the shape to other than round, to finish to more exact sizes, or to increase the strength and stiffness of the tube. Figure 11 shows the relative position of the die and the mandrel during cold drawing. In the preparation for drawing, tubes are subjected to the same inspection and treatment as described in Sec. 36 for wire.

32. Cupping Process.—Seamless tubes are made in the large sizes by *cupping* a hot circular plate of steel in dies by means of a

hydraulic press. The process is repeated from four to five times, each time reducing the diameter and increasing the length of the cup. If further reduction is required, it is done on the draw bench in a manner similar to that of drawing tubes formed by the piercing method. The closed end of the tube is cut off after the finishing process has been completed.

There are other methods of producing seamless tubing, but they partake of the principle of piercing or cupping as described above. The inquisitive student may find his interest stimulated by reading the discussion as outlined in "The Making, Shaping and Treating of Steel," Carnegie Steel Company, Bureau of Instruction, 4th Ed. pp. 1,084 to 1,116, inclusive.

33. Stamping. The term *stamping* is usually applied to the forming of shallow patterns in sheet metal, in which the sheet is stamped or pressed rather than drawn to form between dies. In pressing deep forms there is a tendency for the metal to form radial corrugations, which can be kept to a minimum by having the pressure a maximum at the end of the stroke, which renders the toggle-joint press one of the most satisfactory mechanisims for this kind of work. Hollow ware is readily formed by enclosing the article to be shaped in a close-fitting die, and admitting water under heavy pressure. The water pushes the metal against the sides of the die so that it will take on the exact form of the die, suitable packing rings being provided to prevent water leakage.

34. Spinning. *Spinning* is the formation of sheet-metal parts into circular shapes by pressing the metal against revolving forms by means of hard tools. The ductile metals are used, and the softer the stock the more readily it lends itself to the spinning process. Spinning is usually resorted to when the production of parts is too small to justify the expense of making dies for the drawing process. Other advantages of spinning over the drawing process are: a cheaper grade of steel may be spun than can be drawn with dies, beads may be rolled at the edges of shells at little expense, experimental work may be done quickly, and difficult forms may be readily spun which could not possibly be made with the pressing process. Spun work is readily detected because the working marks are circular instead of radial. The art of spinning was known to the ancients, and many examples of their handiwork compare favorably with the products of today, using modern shop and tool facilities.

35. Swaging. *Cold swaging* is a method of reducing the size of metals or of forming metals while cold, by striking the metal a large number of times with a pair of hammers or dies. It is applied principally to reducing wires, rods, and tubes, for tapering or pointing the ends, or reducing the diameter at one or more places. It is the only known method for working plated metal without destroying the plated surface, and for this reason it is largely employed in the manufacture of jewelry and plate. Making the points of steel needles and pins, reducing wire so that it will enter the drawing die, reducing the middle portion of bicycle spokes and similar articles are examples of the use of the swaging process.

Hand swaging is a very old process, and one of the first machines for replacing this method was developed about 1865 at the works of Wallace and Sons, of Ansonia, Connecticut. Swaging machines operate at a high speed, work being turned out very rapidly. An automatic machine, designed for automobile-wheel spokes, takes the wire from the coil, straightens it, swages the spokes between butts, and cuts them off to length, at a speed of three spokes per minute.

36. Drawing. Processes known as *drawing* differ widely in character, but may be generally defined as those in which the metal in cylindrical or prismatic section is pulled through dies which reduce or change its section, thus implying that the metal is drawn over a die surface, in distinction from pressing, rolling, or stamping. In steel-wire drawing the material consists of $\frac{1}{4}$- to $\frac{1}{2}$-in. rods which have been obtained in the steel mill by hot rolling. The rod should be as nearly round as possible, and free from such defects as cracks and flaws. The first procedure in wire drawing is the cleaning of the rod by "pickling," to remove the scale or oxide. This is done by immersing the rod coil in a bath of acid, after which the rod coils must be washed with water to clean off the acid. The rod is annealed sometimes before it is cleaned, but this depends upon the class of the material. One end of the rod is reduced in section by swaging or by passing it through a pointing roll so that it will pass through the die, the projecting end is then caught by a grip, and the wire is pulled through by a power reel. This process is repeated by using holes of decreasing size, until the required size has been produced. In the drawing of hard wire it is necessary to coat the surface of the wire with a lubricant, usually

some hydrated iron oxide, oil, tallow, or soap, to cushion the drawing action of the wire as it passes through the die. Figure 12 shows the cross-section of a die hole for wire drawing.

The first draw through the die, or "pass" as it is called, reduces the section the least, the reduction at subsequent passes being fairly heavy. In terms of cross-section this reduction is about 17 per cent for the first pass, and as much as 33 per cent for the others. Drawing speeds vary according to the nature of the material, and may be as low as 10 ft. per minute for large diameters and hard material, to 900 ft. per minute for fine copper wire in continuous machines.

Chilled-iron dies are used in drawing the heavier sizes and the cheaper grades of wire, or the dies may be made of a high-grade tungsten steel. Holes are drilled and reamed in the die, tapering to the correct size for the wire to be drawn, several holes of different size often being finished in the same draw plate. For the smaller sizes of wire the so-called "diamond die" is used, consisting of a diamond inserted in a brass body. On account of the extreme hardness of the diamond, large amounts of wire of constant diameter may be drawn by its use. There is a record of a diamond die that was used for drawing 500,000 lb. of copper wire, 0.004 in. diameter, with no measurable change in diameter. When diamond dies are worn too much they are redrilled for a larger gage number. The average time for enlarging a hole 0.001 in. in diameter is 1½ hr., the operation is performed on small automatic lathes, and the cutting medium is an abrasive of diamond dust and oil. Diamonds as large as 3½ carats are used for the larger sizes of wire. In the production of spring brass wire, annealing is regulated so as to produce wire of the required temper.

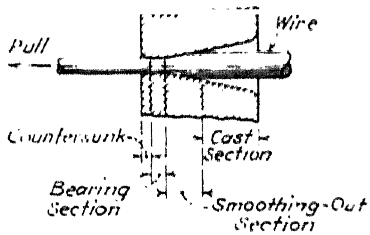


FIG. 12. Wire drawing, cross-section of die hole. (*American Steel and Wire Co.*)

37. Extrusion.—*Extrusion* is a method by which shapes of plastic metal are produced by forcing the metal under high pressure through an aperture of the required shape. The metal is usually heated, lead and tin being extruded at a temperature as low as 250° F., while copper requires a temperature of 1750° F. On account of the high pressure under which metal is extruded,

its structure becomes more compact, its strength is increased, and the surfaces are smooth and free from cracks and flaws. Extrusion permits the forming of unusual sections at low cost, and the size of extruded shapes can be gaged so that little if any additional finishing is required. There are many shapes that are readily extruded, a good example being the lead covering of telephone cable, which is produced by this process. Extrusion was found possible as early as 1797, but it was in the late eighties, with the development of the hydraulic press for high pressures, that the extrusion process became a commercial success. Figure 13 shows a number of sections which are formed by the extrusion process.

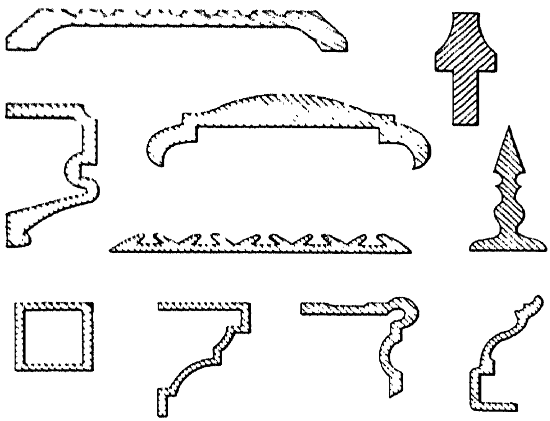


FIG. 13. Extruded shapes in bronze, brass and copper.

38. Thread Rolling. The formation of threads by rolling is done by means of hardened rolls or dies having threads which roll grooves into the blank, the squeezed metal being raised above the surface, forming a thread.

Threads may be formed by one of the following methods: (*a*) Using an automatic screw machine, the thread-forming tool, in the form of a disk, is pressed against the rotating blank, and as the tool is fed along the axis it forms the thread. (*b*) The blank is passed between two flat dies which have V-shaped ridges to correspond with the pitch and helix angle of the thread. One of these die plates reciprocates while the second one is stationary. (*c*) A very rapid thread-rolling process, called the rotary method, is carried out on a machine in which the blank is rolled between a cylindrical die, rotating on a vertical axis,

and provided with thread grooves on the outside, and a hollow stationary cylindrical die surrounding the first, which has thread grooves cut on the inside.

Thread rolling increases the outside diameter, and since the metal is not cut away to form the groove, there is no waste of metal. The strength of the rolled threads is about equal to that of cut threads, and while the rolled threads are not as accurately formed, the advantage lies in the rapidity of rolling and its consequent low cost. A blank $1\frac{1}{2}$ in. in diameter can be threaded in 3 sec., while a smaller $\frac{3}{16}$-in. blank can be threaded in $\frac{1}{5}$ sec. Nearly all classes of screws may be rolled.

FIG. 14. Airco-Davis-Bournonville Radiagraph. Showing the applicati of the oxy-acetylene cutting process to heavy work. (*Air Reduction Sales C New York City.*)

39. Cutting.—The *cutting* processes are more numerous and varied than any of the other metal-working methods discussed thus far, and include cutting by hand-power as in sawing, chiseling, scraping, and filing, as well as cutting by all forms of machine-tool operations on the drill press, lathe, planer, and milling machine.

Grinding is a cutting process and the use of grinding wheels for the finishing cut is now common practice. Heat-treated products are ground to remove any distortion that might arise due to heating and cooling, and to finish and polish the surfaces. Grinding wheels are made of artificial abrasives because of their uniformity. Alundum, an aluminum oxide product, is used for

abrasives for high-tensile strength metals such as steel; while carborundum, a carbide of silicon product, is used for abrasives for cast iron, brass, and low-tensile-strength materials. Alundum and carborundum are trade names, and other names are given to similar abrasives. Emery is an impure form of alumina, and is popular on account of its low cost. Abrasive wheels rotate at relatively high speeds depending upon their diameter, a wheel 1 in. in diameter rotates at 15,000 to 23,000 revolutions per minute, and a wheel 36 in. in diameter rotates at 425 to 640 revolutions per minute. Ordinarily a peripheral speed of 5,000 ft. per minute is employed to insure a margin of safety against the bursting of the wheel.

The gas torch, described in Sec. 26, is extensively used for making rough cuts, and especially in the salvaging of scrap materials. Figure 14 shows the oxy-acetylene process applied to the rough cutting of heavy work.

40. Galvanizing. Iron and steel are often coated with zinc as a protection against corrosion. Zinc is low in cost, has an agreeable color, and a hard smooth coating is readily obtained. The surfaces which are to be *galvanized* should be given an acid bath to remove the scale, washed to remove the acid, and thoroughly dried. If the surfaces are hard, due to cold working, the hardness is removed by annealing. The zinc coating is applied by one of the following methods:

The *hot-bath* process is best adapted for large work such as plate, wire, pipe, and castings, the article to be coated being drawn through a bath of molten zinc.

In the *vapor-bath* process small parts are placed in a metal drum with zinc dust and heated to a temperature of about 600° F., the fumes causing a zinc coating to form on the surfaces. This method is known as the *sherardizing* process.

In the *electrolytic* process the articles to be coated are placed in a bath of soluble zinc salts, the articles acting as cathodes and the metallic zinc as the anode.

41. Nickel Plating. Coating finished surfaces with nickel prevents corrosion and gives the surface a pleasing lustre. The color may be blue, grey, yellow, white, or silver-white. Nickel plating does not oxidize or tarnish readily, and its principal service is decorative. The nickel is applied to surfaces by the electrolytic process, the bath consisting of a solution of nickel salts.

42. Tin Plating. The principal application of tin-coating is on thin iron and steel sheets, and the coating is applied by the hot-bath process. The electrolytic method of tin plating has been used, but not as much as the hot-bath process.

43. Copper Plating. A thin coating of copper may be applied to surfaces by the electrolytic process, and copper plating is of importance to the engineer because of its high thermal and electrical conductivity, as well as its protection against corrosion.

44. Chromium Plating. Chromium plating has an extreme resistance to corrosion in air, moisture, and many chemicals, and its use is supplanting nickel plating for decorative purposes. It has a high wearing resistance, and is applied for this purpose to machine bearings, shafting, rollers, pistons, and many other parts The life of hardened steel dies is greatly extended when chromium plated. Chromium plating is applied by the electrolytic process

45. Wire Gages and Sheet Plate Gages. In the early days of the metal industries gage numbers were used to designate the different sizes of wires and plates that could be most readily produced by the different steps in the process of manufacture The practice of the several manufacturers varied, even in the same line of work, and soon a great number of gages appeared on the market. This led to confusion, and it soon became apparent that for commercial reasons a more uniform system of wire and plate measurement should be adopted.

Wire gages have never been legally standardized in the United States, the steel wire manufacturers in this country having adopted gages that were practically alike, and upon the suggestion of the Bureau of Standards the so-called United States Steel Wire Gage, column 4, Table IV, is taken as a standard for all steel wire other than music wire. For wire other than iron and steel, the American Wire Gage, column 2, Table IV, is taken as a standard in the United States.

The gage for Sheet and Plate Steel, known as the United States Standard Gage, and shown by Table V, was established by an act of Congress, approved Mar. 3, 1893. It is based upon weights per square foot in pounds avoirdupois, and the thickness for each gage can readily be expressed both as a common fraction and a decimal fraction of 1 in. To avoid confusion, it is customary for the designing engineer to specify the diameter of wire or the thickness of plate in decimals of an inch in addition to the gage size, thereby making the specification more complete than if the gage number alone were given.

TABLE IV. DIFFERENT STANDARDS FOR WIRE GAGES IN USE IN THE UNITED STATES

(Dimensions of sizes in decimal parts of an inch)

1	2	3	4	5	6	7	8	1
Number of wire gage	American or Brown & Sharpe	Birmingham, or Stubs' iron wire	Washburn & Moen, or steel wire gage	American S. & W. Co.'s music wire	Imperial wire	Stubs' steel wire	U. S. Standard gage for sheet and plate iron and steel	Number of wire gage
00000000			.....	.....	......	.....	....	00000000
0000000			0.4900	.....	......	.....	....	0000000
000000			0.4615	0.004	0.464	.....	0.46875	000000
00000			0.4305	0.005	0.432	.....	0.4375	00000
0000	0.460	0.454	0.3938	0.006	0.400	.....	0.40625	0000
000	0.40964	0.425	0.3625	0.007	0.372	.....	0.375	000
00	0.3648	0.380	0.3310	0.008	0.348	.....	0.34375	00
0	0.32486	0.340	0.3065	0.009	0.324	.....	0.3125	0
1	0.2893	0.300	0.2830	0.010	0.300	0.227	0.28125	1
2	0.25763	0.284	0.2625	0.011	0.276	0.219	0.265625	2
3	0.22942	0.259	0.2437	0.012	0.252	0.212	0.250	3
4	0.20431	0.238	0.2253	0.013	0.232	0.207	0.234375	4
5	0.18194	0.220	0.2070	0.014	0.212	0.204	0.21875	5
6	0.16202	0.203	0.1920	0.016	0.192	0.201	0.203125	6
7	0.14428	0.180	0.1770	0.018	0.176	0.199	0.1875	7
8	0.12849	0.165	0.1620	0.020	0.160	0.197	0.171875	8
9	0.11443	0.148	0.1483	0.022	0.144	0.194	0.15625	9
10	0.10189	0.134	0.1350	0.024	0.128	0.191	0.140625	10
11	0.090742	0.120	0.1205	0.026	0.116	0.188	0.125	11
12	0.080808	0.109	0.1055	0.029	0.104	0.185	0.109375	12
13	0.071961	0.095	0.0915	0.031	0.092	0.182	0.09375	13
14	0.064084	0.083	0.0800	0.033	0.080	0.180	0.078125	14
15	0.057068	0.072	0.0720	0.035	0.072	0.178	0.0703125	15
16	0.05082	0.065	0.0625	0.037	0.064	0.175	0.0625	16
17	0.045257	0.058	0.0540	0.039	0.056	0.172	0.05625	17
18	0.040303	0.049	0.0475	0.041	0.048	0.168	0.050	18
19	0.03589	0.042	0.0410	0.043	0.040	0.164	0.04375	19
20	0.031961	0.035	0.0348	0.045	0.036	0.161	0.0375	20
21	0.028462	0.032	0.0317	0.047	0.032	0.157	0.034375	21
22	0.025347	0.028	0.0286	0.049	0.028	0.155	0.03125	22
23	0.022571	0.025	0.0258	0.051	0.024	0.153	0.028125	23
24	0.0201	0.022	0.0230	0.055	0.022	0.151	0.025	24
25	0.0179	0.020	0.0204	0.059	0.020	0.148	0.021875	25
26	0.01594	0.018	0.0181	0.063	0.018	0.146	0.01875	26
27	0.014195	0.016	0.0173	0.067	0.0164	0.143	0.0171875	27
28	0.012641	0.014	0.0162	0.071	0.0149	0.139	0.015625	28
29	0.011257	0.013	0.0150	0.075	0.0136	0.134	0.0140625	29
30	0.010025	0.012	0.0140	0.080	0.0124	0.127	0.0125	30
31	0.008928	0.010	0.0132	0.085	0.0116	0.120	0.0109375	31
32	0.00795	0.009	0.0128	0.090	0.0108	0.115	0.01015625	32
33	0.00708	0.008	0.0118	0.095	0.0100	0.112	0.009375	33
34	0.006304	0.007	0.0104	.....	0.0092	0.110	0.00859375	34
35	0.005614	0.005	0.0095	.....	0.0084	0.108	0.0078125	35
36	0.005	0.004	0.0090	.....	0.0076	0.106	0.00703125	36
37	0.004453	.....	0.0085	.....	0.0068	0.103	0.006640625	37
38	0.003965	.....	0.0080	.....	0.0060	0.101	0.00625	38
39	0.003531	.....	0.0075	.....	0.0052	0.099	....	39
40	0.003144	.....	0.0070	.....	0.0048	0.097	....	40

TABLE V. SIZES OF NUMBERS OF THE UNITED STATES STANDARD GAGE FOR SHEET AND PLATE IRON AND STEEL

Number of gage	Approximate thickness in fractions of an inch	Approximate thickness in decimal parts of an inch	Weight per square foot in ounces avoirdupois	Weight per square foot in pounds avoirdupois
0000000	1/2	0.5	320	20.00
000000	15/32	0.46875	300	18.75
00000	7/16	0.4375	280	17.50
0000	13/32	0.40625	260	16.25
000	3/8	0.375	240	15.00
00	11/32	0.34375	220	13.75
0	5/16	0.3125	200	12.50
1	9/32	0.28125	180	11.25
2	17/64	0.265625	170	10.625
3	1/4	0.25	160	10.00
4	15/64	0.234375	150	9.375
5	7/32	0.21875	140	8.75
6	13/64	0.203125	130	8.125
7	3/16	0.1875	120	7.5
8	11/64	0.171875	110	6.875
9	5/32	0.15625	100	6.25
10	9/64	0.140625	90	5.625
11	1/8	0.125	80	5.00
12	7/64	0.109375	70	4.375
13	3/32	0.09375	60	3.75
14	5/64	0.078125	50	3.125
15	9/128	0.0703125	45	2.8125
16	1/16	0.0625	40	2.5
17	9/160	0.05625	36	2.25
18	1/20	0.05	32	2
19	7/160	0.04375	28	1.75
20	3/80	0.0375	24	1.50
21	11/320	0.034375	22	1.375
22	1/32	0.03125	20	1.25
23	9/320	0.028125	18	1.125
24	1/40	0.025	16	1
25	7/320	0.021875	14	0.875
26	3/160	0.01875	12	0.75
27	11/640	0.0171875	11	0.6875
28	1/64	0.015625	10	0.625
29	9/640	0.0140625	9	0.5625
30	1/80	0.0125	8	0.5
31	7/640	0.0109375	7	0.4375
32	13/1280	0.01015625	6 1/2	0.40625
33	3/320	0.009375	6	0.375
34	11/1280	0.00859375	5 1/2	0.34375
35	5/640	0.0078125	5	0.3125
36	9/1280	0.00703125	4 1/2	0.28125
37	17/2560	0.006640625	4 1/4	0.265625
38	1/160	0.00625	4	0.25

Problems

Instructions for Problem Work.—Drawings are to be made on 15- by 22-in. paper. The paper should be of a quality which

will take ink, and any one of the better quality yellow detail papers are satisfactory. The border lines are to be 1½ in. from the left edge and ½ in. from the other edges. A 2- by 4-in. title block should be placed in the lower right-hand corner. Any bill of materials should be placed directly above the title block. The dimensions given in the problems for locating the drawing on the paper are full size.

Problem calculations and sketches are to be made on 8½- by 11 in. cross-section paper of good quality, with 8 lines to the inch. Leave a generous margin at the left edge of the paper. Place the statement of the problem at the top of the paper, and number the problems in the order of their assignment. Place answers near the right edge, and underscore with two lines. Give reference and page number for formulas. Place your name and the date on each sheet of paper.

1. What allowance for shrinkage should be made in the pattern for a cast-iron pulley which has the following dimensions when finished: diameter of pulley is 18 in., diameter of hub is 3¾ in., length of hub is 4¼ in., face of pulley is 5 in., and bore is 2 in. Make a skecth of the pulley pattern, showing the necessary dimensions.

2. The statement of the problem and the data are the same as for problem 1, but the pulley is to be made of cast steel. Make a sketch of the pulley pattern, showing the necessary dimensions.

3. (*a*) Measure the flywheel on the. . . .engine in the mechanical laboratory, and estimate its weight; (*b*) If the finished casting costs. . . .cents per pound, estimate the cost of the flywheel.

For estimating purposes, when the pattern is available, the weights of castings may be determined by the use of Table VI.

TABLE VI. WEIGHT OF CASTINGS DETERMINED FROM THE WEIGHT OF PATTERN

Pattern weighing 1 lb. made of	Will weigh when cast of			
	Cast iron, pounds	Steel, pounds	Yellow brass, pounds	Gun metal, pounds
Mahogany (Nassau)...	10.7	11.5	12.2	12.5
Mahogany (Honduras)	12.9	13.8	14.6	15.0
Mahogany (Spanish)..	8.5	9.2	9.7	9.9
Pine (red)............	12.5	13.4	14.2	14.6
Pine (white)..........	16.7	17.8	19.0	19.5
Pine (yellow).........	14.1	15.1	16.0	16.5

4. An arc-welded joint similar to the one shown in Fig. 7(*g*) is made with a flush fillet. The smaller bar is 2 by $\frac{1}{2}$ in. and the members are lapped 4 in. (*a*) Using a $\frac{1}{4}$-by $\frac{3}{8}$-in. fillet, determine the strength of the joint for withstanding a horizontal pull for an allowable value of 9,000 lb. per square inch (Table I). (*b*) How does the strength of the weld compare with the strength of the members?
5. An arc-welded joint like that shown in Fig. 7(*b*) is made with a reinforced fillet. The dimensions are $h = 6$ in. and $t = 1$ in. The allowable strength of the weld is 9,000 lb. per square inch. Determine the strength of the joint in tension for a static load.
6. A dead load of 4,500 lb. is placed on the horizontal member of Fig. 7(*a*), inducing a shear stress in the weld. What should be the width of the members if they are $\frac{3}{8}$ in. thick and if a $\frac{3}{8}$- $\times$ $\frac{1}{8}$-in. reinforced fillet is used?
7. Two plates 6 in. wide are to be welded as shown in Fig. 7(*j*). With an allowable unit stress of 9,000 lb. per square inch for a $\frac{5}{8}$ $\times$ $\frac{5}{8}$ in. fillet, determine the horizontal shear load which could be transmitted from one plate to the other.
8. Plates 1 in. thick are to be welded according to each of the three classes of welds shown in Fig. 8. If $A = 1$ in., $B = 1$ in., and $L = 6$ in., determine for each class the total maximum load which could be carried.

CHAPTER III

PIPES AND PIPE FITTINGS

46.—Pipes are used to carry fluids or gases from the source of supply to the place where they are used, and the pipes are subjected principally to an internal pressure. Pipes may be made of cast iron, wrought iron, steel, copper, and copper alloys. The terms steel pipe and wrought iron pipe are sometimes used indiscriminately, and since steel has largely superseded wrought iron in the manufacture of pipe, steel pipe is usually supplied unless it is clearly stated that wrought iron pipe is wanted.

47. Cast-iron Pipe.—Cast-iron pipe was first made in Coolbrooksdale, England, about 1780. The pipe, with the bell end

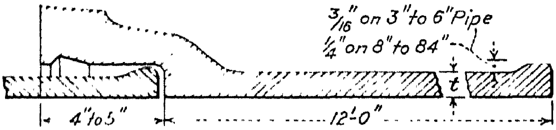


FIG. 1. Section of cast-iron pipe, bell and spigot joint.

down, is usually cast in a vertical position to insure a sound casting. A section through the wall of a cast-iron pipe is shown in Fig. 1. The large end is called the bell, and the small end the spigot, and pipes are joined together by placing the spigot end of one pipe into the bell of the next. The joint is packed with some fibrous material and made tight by caulking with lead. Such a joint will have some flexibility and will also allow for expansion and contraction.

The thickness of the pipe wall is determined by Brackett's formula:

$$t = 0.25 + \frac{(P + P')r}{3{,}300}, \tag{1}$$

in which t denotes the thickness of the pipe wall, in inches.

P denotes the maximum static pressure, in pounds per square inch.

P' denotes the allowance made for water hammer, in pounds per square inch.

r denotes the inside radius of the pipe, in inches.

The value of P' in formula (1) is given in Table I.

TABLE I

Inside diameter of pipe, in inches......	3 to 10	12	14	16	18	20	24	30	36
Value of P', in pounds per square inch.....	120	110	105	100	95	90	85	80	75

Cast-iron pipe is classified according to the pressure which it will carry safely, as shown in Table II

TABLE II

Class	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Head, feet	100	200	300	400	500	600	700	800
Pressure, pounds per square inch	43	86	130	173	217	260	304	347
Smallest pipe, diameter in inches	3	3	3	3	6	6	6	6
Largest pipe, diameter in inches	84	84	84	84	36	36	36	36
Wall thickness of 12-in. pipe in inches	0.54	0.62	0.68	0.75	0.82	0.89	0.97	1.04

All cast-iron pipe and pipe fittings that are used for water supply should conform to the rules and specifications of the American Water Works Association.

Cast-iron pipe is used principally for underground pipe lines because of its ability to resist corrosion. The pipe is made of a soft and tough quality of iron. It is not made with a machine-finished surface, but is coated by dipping in hot coal tar.

48. Flexible Joint Pipe.—Water pipes are sometimes submerged in water beds so that the pipe must follow the contour of the water bottom and also allow for slight movement caused by shifting. There are various forms of flexible joint pipe made for this purpose, and Fig. 2 shows a ball-and-socket type that is in common use. For this type the ball and socket are machined, a lead gasket and retaining ring are used, and a maximum movement of 20 deg. is possible. This joint is also made with bell and spigot ends.

49. Steel and Wrought-iron Pipe.—Steel pipe is used for ordinary engineering work; it costs less than wrought-iron pipe, and for most purposes is as satisfactory. Steel pipe is made

from soft steel that is homogeneous in composition. Both steel and wrought-iron pipe are formed from flat stock, and the seam is either butt welded or lap welded.

In the *butt welding process* the pipe material, called "skelp," is cut to the correct length and width for the desired size, and

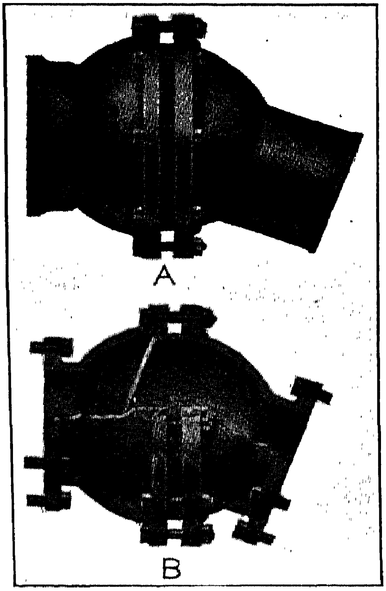


FIG. 2.—Flexible joint for cast-iron pipe; A, bell and spigot ends; B, flanged ends

the edges are beveled so that when the pipe is formed the edges will come together squarely. The flat strips are heated to a welding temperature and then drawn through a bell-shaped die

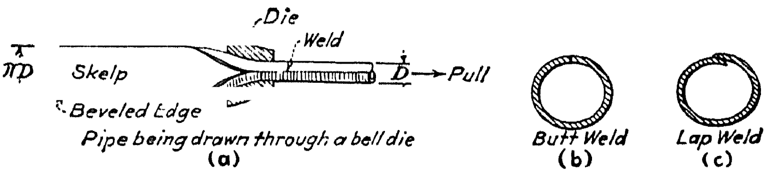


FIG. 3.

as shown by Fig. 3(*a*), which forms the plate so that the edges come together and are welded. The pipe is correctly sized and finished by passing through cross-rolling and sizing mills.

In the *lap-welding process* the skelp is prepared as described in the butt-welding process, except that the edges are scarfed by

hammering. The plate is then drawn through a die in which the edges are overlapped and welded, the large welding surface producing a very strong joint. Figures 3(*b*) and 3(*c*) show sections of a butt and lap-welded pipe.

50. Classification of Steel and Wrought-iron Pipe. Steel and wrought-iron pipe is classified according to the working pressure which the pipe will carry, as follows:

Standard weight or ordinary (standard full weight) for pressures up to 125 lb. per square inch.

Extra heavy for pressures up to 250 lb. per square inch.

Double extra heavy for pressures above 250 lb. per square inch, or when used under severe service conditions.

When very high pressures are used a special grade of pipe is made to order by boring a hole in a round ingot, and this pipe will safely carry pressures up to 10,000 lb. per square inch.

Pipe is designated by the nominal inside diameter of the standard weight grade. For convenience and interchangeability all grades of pipe of a given size have the same outside diameter. For example, the standard weight, extra heavy, and double extra heavy ¾-in. pipe have an outside diameter of 1.050 in., but the internal diameters are 0.824, 0.742, and 0.434 in., respectively. Hence ¾-in. double extra heavy pipe has only about one fourth the capacity as a standard weight pipe of the same size. These nominal pipe sizes orginated in England about 100 years ago. Pipe as it comes from the mill is known as black or mill-finished pipe, and is furnished in random lengths which vary from 12 to 24 ft.

51. Pipe Threads. Pipe connections are usually made by cutting a thread on the pipe ends, and then screwing the pipe together with a coupling or fitting. For many years the American Briggs Standard of pipe threads was used, which had been formulated by Robert Briggs, one time engineering editor of the *Journal of the Franklin Institute*. The pipe thread which is standard in the United States at the present time is known as the American Standard, or National Standard, and is identical with the older Briggs system, but is designated in different terms.

Figure 4 shows the form of the American Standard thread, and Table III gives the thread and pipe information for standard, extra heavy, and double extra heavy pipe. The fine-thread system for pipe has the following advantages: a tight joint is insured by the engagement of at least five threads, the small lead

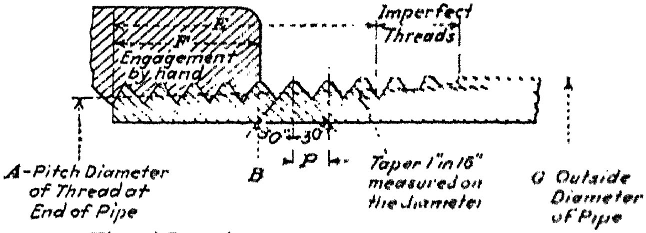
TABLE III.—DIAMETERS AND AREAS OF STANDARD, EXTRA-HEAVY AND DOUBLE EXTRA-HEAVY PIPE
(American Standard pipe threads)

Diameter (nominal) inches	Outside diameter, inches	Threads per inch	Internal diameter: Standard weight, inches	Internal diameter: Extra heavy, inches	Internal diameter: Double extra heavy, inches	Transverse area: Standard weight, square inches	Transverse area: Extra heavy, square inches	Transverse area: Double extra heavy, square inches	Drill size for pipe tap, inches
1/8	0.405	27	0.269	0.215	....	0.057	0.036	....	21/64
1/4	0.540	18	0.364	0.302	....	0.104	0.072	....	29/64
3/8	0.675	18	0.493	0.423	....	0.191	0.141	....	19/32
1/2	0.840	14	0.622	0.546	0.252	0.304	0.234	0.050	23/32
3/4	1.050	14	0.824	0.742	0.434	0.533	0.433	0.148	15/16
1	1.315	11 1/2	1.049	0.957	0.599	0.864	0.719	0.282	1 3/16
1 1/4	1.660	11 1/2	1.380	1.278	0.896	1.495	1.283	0.630	1 15/32
1 1/2	1.900	11 1/2	1.610	1.500	1.100	2.036	1.767	0.950	1 23/32
2	2.375	11 1/2	2.067	1.939	1.503	3.355	2.953	1.774	2 3/16
2 1/2	2.875	8	2.469	2.323	1.771	4.788	4.238	2.464	2 11/16
3	3.500	8	3.068	2.900	2.300	7.393	6.605	4.155	3 5/16
3 1/2	4.000	8	3.548	3.364	2.728	9.886	8.888	5.845	3 13/16
4	4.500	8	4.026	3.826	3.152	12.730	11.497	7.803	4 3/16
4 1/2	5.000	8	4.506	4.290	3.580	15.947	14.455	10.066	4 11/16
5	5.563	8	5.047	4.813	4.063	20.006	18.194	12.966	5 1/4
6	6.625	8	6.065	5.761	4.897	28.891	26.067	18.835	6 5/16
7	7.625	8	7.023	6.625	5.875	38.738	34.472	27.109	
8	8.625	8	8.071	7.625	6.875	51.161	45.663	37.122	
9	9.625	8	8.941	8.625	....	62.786	58.426		
10	10.750	8	10.192	9.750	....	81.589	74.662		
11	11.750	8	11.000	10.750	....	95.033	90.763		
12	12.750	8	12.000	11.750	....	114.800	108.434		
14 (O.D.)	14.000	8	13.250	13.000	....	137.890	132.730		
15 (O.D.)	15.000	8	14.250	14.000	....	159.480	153.940		
16 (O.D.)	16.000	8	15.250	15.000	....	182.650	176.710		
17 (O.D.)	17.000	8	16.214						
18 (O.D.)	18.000	8	17.182						
20 (O.D.)	20.000	8	19.182						
22 (O.D.)	22.000	8							
24 (O.D.)	24.000	8							
26 (O.D.)	26.000	8							
28 (O.D.)	28.000	8							
30 (O.D.)	30.000	8							

allows for close lateral adjustment, and enough metal is left at the thread root to give adequate strength. The thread on the pipe is given a taper, with the result that erection is facilitated and the joint between the threads becomes tighter as the parts are screwed together.

52. Built-up Pipe.—Large diameter pipe is sometimes built up of steel plates riveted or welded across the joints. Figure 5 shows

a form of pipe used for exhaust steam, paper pulp, suction, and for general low-pressure work. It is a light weight but strong pipe



Thread Formula:
$A = G - (0.05G + 1.1)P$
B = Pitch of thread at gaging notch $= A + 0.0625F$
E = Length of effective thread $= P(0.8G + 6.8)$
Depth of thread $= 0.8P$
F = Engagement by hand between male and female $= 5P$
The crest of the thread is truncated $= 0.1P$

FIG. 4.

made of a single strip of steel rolled in the form of a helix. The edges are overlapped and riveted, and steel flanges are riveted

FIG. 5.—Spiral riveted pipe and flanged joint.

on the ends so that connections can be made with through bolts. This pipe is called "spiral riveted" pipe, and is manufactured

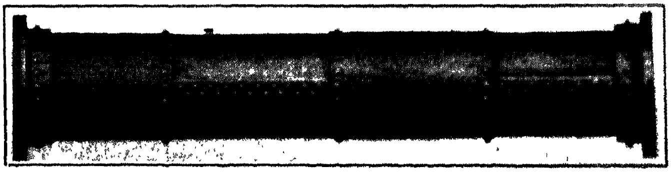


FIG. 6.—Straight seam riveted steel pipe.

in sizes from 3 to 16 in. inside diameter, increasing by 1-in. increments; and from 18 to 40 in., increasing by 2-in. increments; the maximum length of the pipe being 20 ft.

Figure 6 shows a steel-riveted pipe using a straight seam, which is used for all purposes and is made in the larger sizes.

53. Copper and Brass Pipe and Tubing.—The processes used in making copper and brass tubing is similar to those described in Sec. 30 for the making of steel tubing. Copper and brass pipe is made by a process similar to that described for making steel pipe; sheet material is used, and the seam is closed by brazing instead of welding. This pipe is made in the same outside diameters as steel pipe so that ordinary steel fittings may be applied, but brass fittings are generally used.

54. Lead Pipe.—Lead pipe was used by the Romans for water conduits before the Christian era. The pipe was made from sheet lead and the seams were soldered. At the present time lead pipe is made in many diameters and wall thicknesses, and its length is limited only by transportation facilities. Lead pipe is made by the extrusion process, and on account of the ease with which it can be bent, it is useful as a covering for telephone cables and as a water conduit.

55. Pipe Bends.—Lap-welded pipe is used for bends because the seam at the weld is less liable to open during the bending process than the seam of a butt weld. The seam should be at the side of the bend to avoid maximum stresses at the seam. The radius of the bend is usually about 5 diameters to prevent wall buckling on the compression side. At each end of the bend there should be a length of straight pipe equal to 1 diameter, because there is a tendency for the ends to flatten, which would prevent the forming of a tight screwed fitting. The threads should be cut and the flanges fitted on before the pipe is bent. Pipe bends of large radius are used to allow for expansion and contraction in pipe lines, and are standardized in form and dimensions.

56. Pipe Fittings.—All parts of a pipe line except the pipe and the valves are called *pipe fittings*. The form and wall thickness of pipe fittings are made according to the American Standard Codes of the American Engineering Standards Commitee.[1] The following paragraphs discuss briefly some of the fittings in common use.

[1] This committee is sponsored by the Heating and Piping Contractors' National Association, the Manufacturers' Standardization Society of Valve and Fittings Industry, and the American Society of Mechanical Engineers.

Cast-iron Screwed Fittings.—Cast-iron screwed fittings are used for maximum saturated steam working pressures of 125 and 250 lb. per square inch. The 125-lb. pressure fittings are made to be used for work of the standard-weight class, the 250-lb. pressure fittings are for work of the extra heavy class, and both classes of fittings are made in all sizes up to 16 in.

Malleable-iron Screwed Fittings.—Malleable-iron screwed fittings are used for a maximum working pressure of 150 lb. per square inch, and are made in all sizes up to 8 in.

Cast-iron Flanged Fittings.—Cast-iron flanged fittings are made of standard weight in all sizes for a maximum saturated steam pressure of 125 lb. per square inch.

Cast-steel Flanged Fittings.—Cast-steel flanged fittings are made in the several weights for maximum working steam pressures of 250, 400, 600, 900, and 1,350 lb. per square inch, at a temperature of 750° F.

All the fittings classified above have marks cast on them indicating the manufacturer, and the maximum working steam pressure or service for which they are intended. The manufacturers of valves and fittings give detailed information in their catalogues for the application of their products, and also all dimensions for making engineering piping drawings to scale.

Stock Fittings.—Stock fittings, shown in Fig. 7, are available in the sizes classified above, and are made in the following forms. Elbow fittings, commonly called *ells*, are made for 90- and 45-deg. turns. When the turn is 180 deg. the fitting is called a *U* or return bend. *Tees* are made to permit branching out of a pipe run, and are specified by giving the run size and the outlet size. For example, a 2 by 2 by $1\frac{1}{2}$ tee is used to connect a $1\frac{1}{2}$-in. pipe to a 2-in. pipe. A *cross*-fitting is one with four openings 90 deg. apart. *Bushings* and *reducers* are used for connecting pipes of different sizes, *caps* and *plugs* are used for closing off the ends of pipes or fittings, and *Y bends* or lateral fittings are branch pipe connections used when the angle between the pipe line and branch pipe is other than 90 deg. *Union* fittings are used to make a joint in a pipe line so that a section of pipe with its fittings may be readily assembled or taken apart. *Long sweep* fittings for turns and bends are used to avoid the abrupt changing of direction of flow within the pipe line, and their use results in a smaller friction loss.

Special Fittings.—Special fittings are made in many forms and types, but their employment in design should be discouraged for cases in which the more common forms would be satisfactory. *Hand-rail* fittings are special fittings for joining pipe for guard

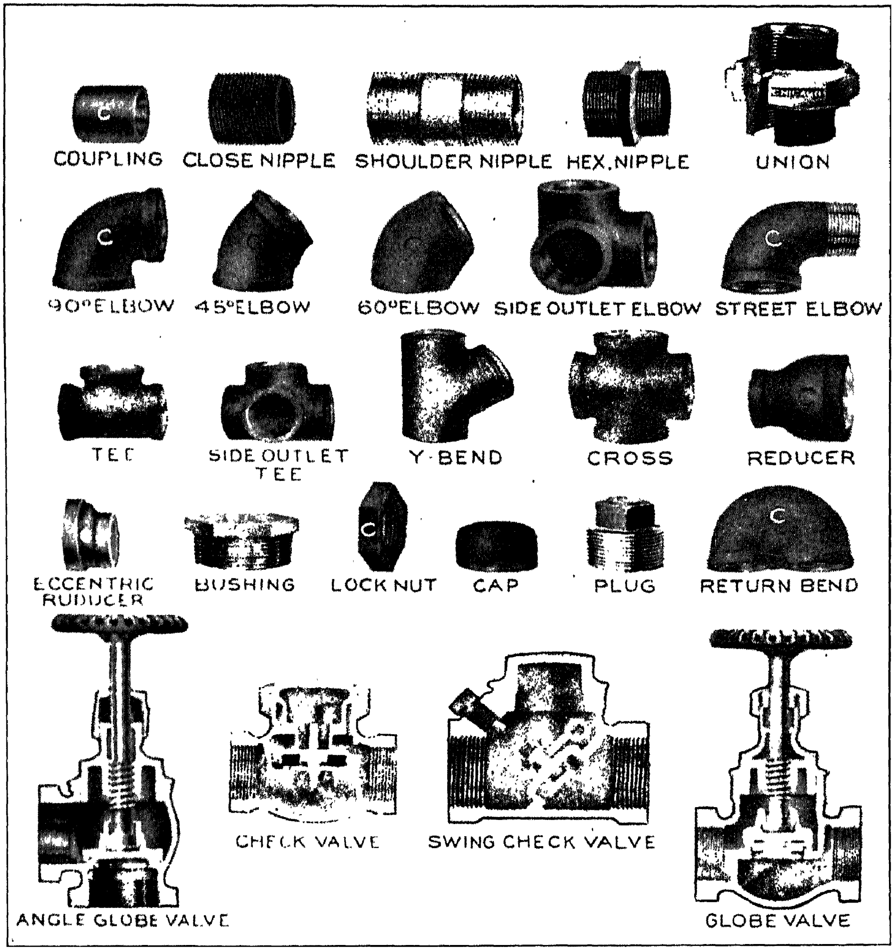


FIG. 7.—Screwed fittings. (*Crane Co., Chicago, Ill.*)

rails which enclose machinery, for exhibit spaces, and for protection against location hazards.

Bronze Hydraulic Fittings.—Bronze hydraulic fittings are made in the smaller sizes for working pressures up to 2,500 lb. per square inch, and the passages in the fittings are usually drilled.

Flanged Fittings.—Practically all forms and sizes of threaded fittings are obtainable in flanged fittings. In general, flanged fittings are used for the larger pipe sizes, and the choice of either one usually lies with the design engineer. Some of the common flanged fittings are shown in Fig. 8.

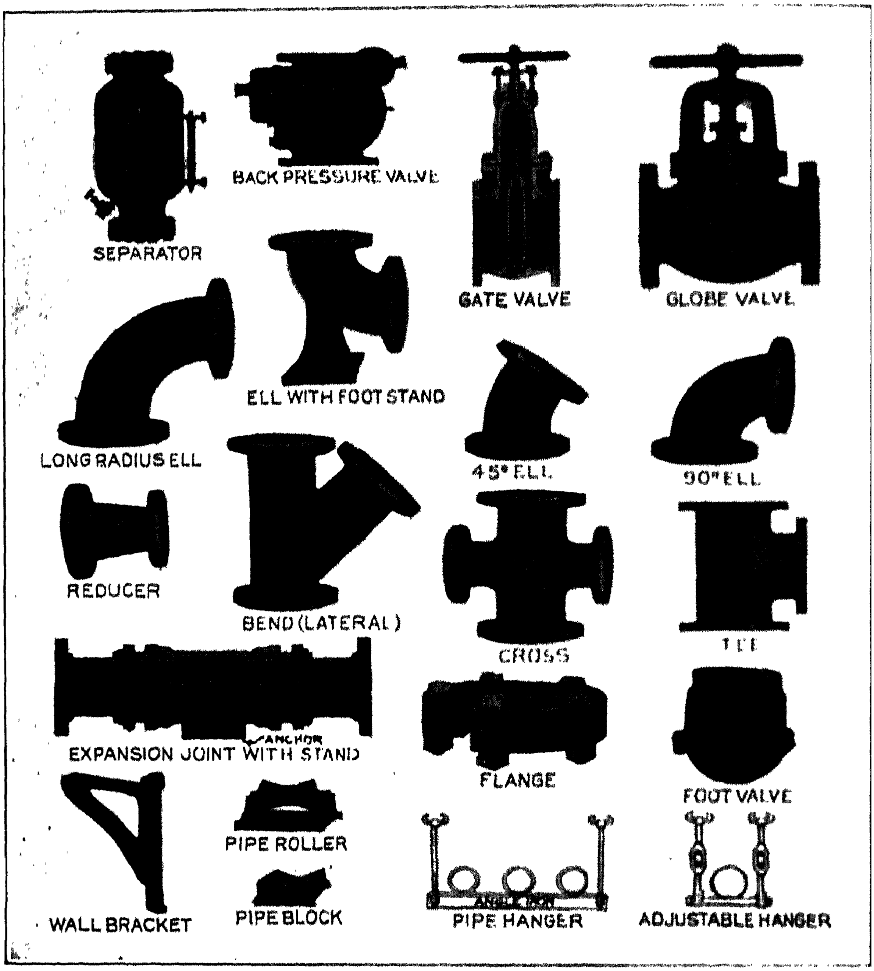


FIG. 8.—Flanged fittings. (*Crane Co., Chicago, Ill.*)

57. Pipe Flanges.—The screwed flange shown in Fig. 9 is the common form for joining two lengths of pipe, or for the connection to a flanged fitting or valve. The thread is cut on the pipe ends, the flange is screwed on, and for true alignment the flange

face should be trued in a lathe by a light finish cut, to insure that its surface will be normal to the axis of the pipe. The flange is

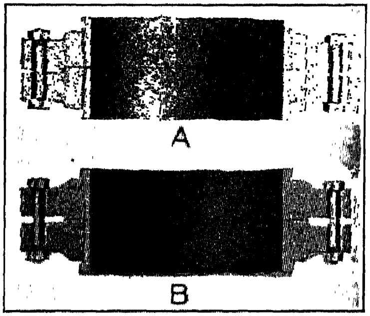


FIG. 9.—Screwed flanges; A, raised face; B, male and female face.

sometimes shrunk on the pipe end, and the edge of the pipe is spread as shown in Fig. 10. The joint shown in Fig. 11 is made

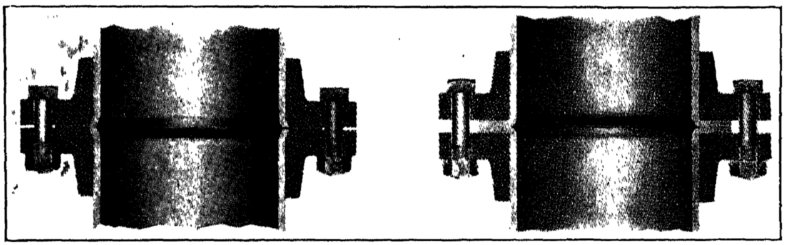


FIG. 10. FIG. 11.

FIG. 10. Shrunk and peened joint. The flange is bored slightly smaller than the pipe and shrunk on. The end of the pipe is spread by hand, hammered or by an expanding tool.

FIG. 11.—Van Stone joint. The flange is loose and can be tuned to any desired position for the alignment of the bolt holes.

by rolling the pipe end, and then facing it off in a lathe, so that when two flanges are bolted and pulled together the pressure will

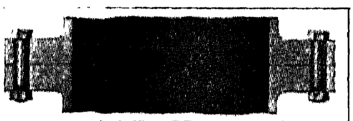


FIG. 12.—Welded flange.

be between the pipe ends at the joint and not on the flanges. This joint is common for high pressures, and is known as the

Van Stone joint, patented in 1897 by G. J. Rockwood; but it is now manufactured under various trade names by different manufacturers.

The welded flange joint shown in Fig. 12 is made by welding a rolled-steel flange to the pipe end, forming a solid connection. The face is machined in a lathe, and often grooved in various ways for retaining a gasket. For high pressures, flange bolts of high tensile strength should be used for the flanged joints described above.

58. Metal Conduit.—Steel pipe made in gas pipe sizes is used for electric wire conduit. The pipe is lined with a soft insulating material which will not crack when the pipe is bent to a comparatively small radius. Conduit fittings include junction boxes, elbows, tees, and others, with outside threads cut on them; and the joints are connected by long right- and left-hand threaded couplings.

59. Pipe Covering.—To prevent undue radiation losses in pipe lines conveying hot liquids or high temperature gases such as steam, the pipes should be covered with a material which is a poor conductor of heat. There are a number of commercial materials available for this purpose in which asbestos is the principal ingredient. A material of felt composition is used for covering pipes which carry a gas or liquid of low temperature, to insulate them and also to absorb the moisture which collects on the outside of the pipe.

60. Pipe-line Design.—The design of pipe lines involves a knowledge of the kind and quantity of gas or liquid to be delivered, the pressure and temperature, the distance over which the gas or liquid is to be transmitted, and the location of the line above or below the ground surface. The losses in pressure due to friction are indefinite, but they may be approximated by empirical formulas. The expansion of the pipe line due to changes of temperature should be provided for by long-sweep bends or loops, the slip type of joint being employed only when the pressure is below 25 lb. per square inch. The direction of expansion is controlled by using anchors at given points, so as to permit the movement of the pipe away from such fixed points. The vibration in pipes, due to the flow of gas or liquid, should be minimized by well-designed supports properly located. All low points of steam and gas lines should be drained to prevent the accumulation of condensate or liquid, the usual

practice being to incline the pipe line in the direction of flow towards drip pockets, which empty automatically when full.

61. Pipe Drawings.—Pipe-line drawings should be made large enough to show the size of the pipe to scale, the usual scale being $\frac{1}{8}$ or $\frac{1}{4}$ in. to the foot. Large pipe is cut, threaded, flanged, and faced in the shop, and should fit into place when erected in the field. Small pipe is usually cut on the job. Drawings should show the exact dimensions from face to face of flanged pipe lengths, valves, and fittings. Conventions as shown by Fig. 13 are used to represent the position of valves and other fittings.

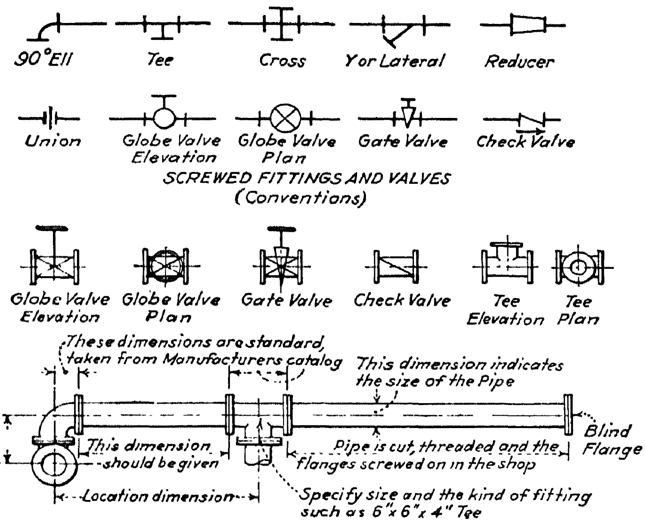


FIG. 13. Screwed and flanged fittings and valves. (Conventions.)

Drawings for small pipe are usually made by representing the pipe by a single line, and when water, gas, steam, and other pipe lines are shown on the same drawing, the different pipes are represented by full, dotted, and dot-dash lines, for clearness.

The design engineer should have constant access to manufacturers' catalogues, so that all dimensions shown on the drawings will agree with the measurements of the valves and fittings.

Problems

1. Using Brackett's formula determine the thickness of a 12-in. cast-iron water pipe to withstand a maximum static pressure of 85 lb. per square inch.

2. A 16-in. cast-iron pipe has a wall thickness of 0.64 in. (*a*) What static water pressure will the pipe carry? (*b*) What will be the equivalent head in feet?

3. Determine the capacity of a 1-ft. section of pipe of the following: standard weight, extra heavy, and double extra heavy, and also determine the reduction in cross-section in percentage of the section of the standard weight pipe.

4. A 2½-in. standard weight pipe is to be replaced by an extra heavy pipe. If the carrying capacity of the pipe is to be the same, and the velocity of the substance in the pipe may not be increased, what size of extra heavy pipe should be used?

5. Using the same data as given for Problem 4 what size of double extra heavy pipe is required?

6. Using Barlow's formula (p. 364) determine the theoretical bursting pressure for a 1-in. pipe: (*a*) a standard-weight steel pipe; (*b*) an extra-heavy steel pipe; (*c*) a double extra-heavy steel pipe.

NOTE: The ultimate tensile strength of butt- and lap-welded pipe has been determined experimentally as 40,000 and 50,000 lb. per square inch, respectively. The common factor of safety used for determining the safe working pressure is 8.

7. Determine the safe working pressure for a 2-in. pipe: (*a*) a standard-weight steel pipe; (*b*) an extra-heavy steel pipe; (*c*) a double extra-heavy steel pipe.

8. The linear coefficient of thermal expansion for steel is 0.0000065 in. per degree Fahrenheit. What expansion must be provided for in a 10-in. steam pipe which is 130 ft. long, if the temperature of the steam is 225° F., and the temperature of the surrounding air is 68° F.?

CHAPTER IV

LINKWORK, INSTANT CENTERS, AND VELOCITY DIAGRAMS

62. A machine or structure is an assemblage of unit parts called links, which either transmit some form of motion from one part to another, or lock one part to another so that no relative motion can take place.

A link with two pin joints which connect the link to two adjacent members is a *simple link.* A link with more than two pin joints, and connected by them to more than two other members is a *compound link.*

A series of links connected to form a closed system is called a *kinematic chain.* A kinematic chain which allows no relative motion between its members is called a *locked chain.* Figure 1(a) shows a simple locked chain, as used in structures such as bridges. A kinematic chain which transmits definite relative motion to all of its members when one member is displaced with respect to a fixed member, is called a *mechanism.* A machine is dependent upon one or more mechanisms to convert energy into work or to transmit forces. If the relation of the movable links to the fixed link of a mechanism is such that all points in the moving members follow fixed paths, the moving links are said to have *constrained* motion. If the links of a kinematic chain are free to assume other than a definite position with respect to a fixed link, the motion of the links is *unconstrained.* The latter form of kinematic chain has little if any application in design engineering.

A link may be other than a rigid body. A leather belt is a link which compels constrained motion from the driving pulleys to the driven pulley, but it functions as a link only when subjected to an unbalanced tensile load. Gases and liquids may function as a link when acting as compression members. A link which will impart motion to a second link under all conditions of loading without apparent deformation is called a *rigid* link.

Kinematics of machines is the study of the motion of various parts, and in the solution of problems the links are represented by lines and the pin joints by points.

63. Four-link Mechanisms.—The simplest mechanism is a four-link chain, shown in Fig. 1(*b*). Any one of the four links may be fixed, and each change in the fixed link will result in a different mechanism. The links of the chain shown in Fig. 1(*c*) are unconstrained, but by the insertion of the link *f*, the chain becomes a mechanism.

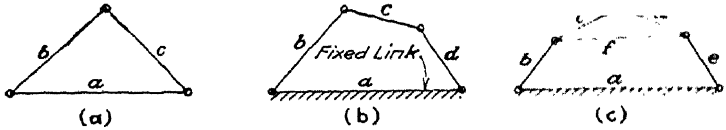


FIG. 1.

The four-link mechanism shown in Fig. 2(*a*) is an *inversion* of the four-link mechanism shown in Fig. 1(*b*). The replacement of the guide and sliding block *b* by the link *b′* does not affect the relative movement of the other parts. If the link *b′* of Fig. 2(*a*) is made infinite in length, the mechanism will be like that shown in Fig. 2(*b*), which is the well-known crank, connecting rod, cross-head, and guide mechanism of the gas and steam engine. This is called the *slider-crank mechanism*. Linkwork is commonly employed in design to convert rotation to linear translation or conversely.

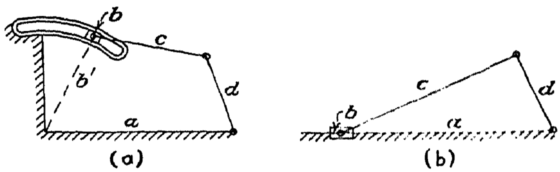


FIG. 2.

64. Instant Centers.[1]—An instant center is a point coincident in two bodies about which either body may tend to rotate with respect to the other, and being a common point it will have the same linear velocity in each body with respect to a third body.

For the solution of velocity problems it is convenient to assume that the relative positions of the links of a mechanism are fixed positions for any instant, and that at any other instant the links have a new relation to each other. When the velocity is a con-

[1] "Instant center" is an abbreviated form for instantaneous center. Some authors use the word "centro" in the same sense. For conciseness this book will employ the abbreviation.

stant quantity, any cycle position may be taken as a true velocity relation of the links. When the velocity is a variable quantity the velocity relation is true only for the relative position of the links for a particular instant. Instant centers are useful for the solution of velocity problems, when employed in connection with a theorem which is credited to C. E. Kennedy.[1]

65. Kennedy's Theorem.—"If any three bodies, a, b, and c, have plane motion, their virtual centers ab, bc, and ac, are three points upon a straight line."

Proof.—In Fig. 3 let a, b, and c, be any three bodies having relative motion with respect to each other. Then ab is the common center of the body b with respect to a, and ac is the common center of the body c with respect to a. Assume that the third common center is at the point k. As the body b rotates about its center, the point k will move along the line xx' which is normal to kab, the instant radius of k with respect to the center ab. Also as the body c rotates about its center, the point k will move along the line yy', which is normal to kac, the instant radius of k with respect to the center ac. The point k cannot have motion along both lines xx' and yy' unless they are coincident, and they will be coincident only when k lies on the line ab-ac.

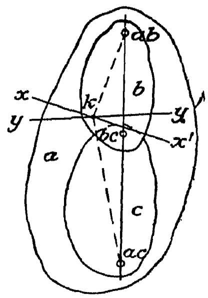


FIG. 3.

This proof is true whether the point bc lies within or outside of the mechanism, and the student will find the solution of velocity problems greatly simplified if this theorem is kept in mind.

66. Number and Location of Instant Centers.—In any mechanism a number of the instant centers are readily located at the joints of the links, others lie outside the links, and may be readily found. The number of instant centers is:

$$N = \frac{n(n-1)}{2}, \tag{1}$$

in which N denotes the number of instant centers.

n denotes the number of links in the mechanism.

To locate the instant centers of the mechanism shown in Fig. 4(a), the number of centers is determined first.

$$N = \frac{4(4-1)}{2} = 6.$$

[1] KENNEDY, C. E., University College, London. Translator of "Kinematics of Machinery," by Franz Reuleaux.

Tabulating the links in alphabetical order, and taking each link in combination with every other link, the instant centers are:

Links:	*a*	*b*	*c*	*d*
Centers:	*ab*	*bc*	*cd*	
	ac	*bd*		
	ad			

The instant centers *ab*, *bc*, *cd*, and *ad*, Fig. 4(*b*), are found by inspection at the joints of the links, leaving *ac* and *bd* to be located by means of Kennedy's theorem. To find *ac* the links

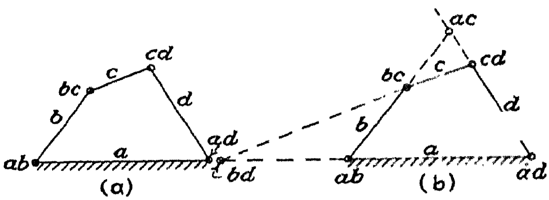


FIG. 4.

a and *c* must be taken in combination with any other link such as *b*:

Links:	*a*	*b*	*c*
Centers:	*ab*	*bc*	
	ac		

If two of these are already located, the third lies in line with them. The tabulation above shows that *ab* and *bc* are already located, so that *ac* will be found on a line connecting them. By a similar procedure *ac* is also found to lie on a line connecting *cd* and *ad*, so that *ac* must lie at the intersection of the lines drawn through *ab* and *bc*, and through *ad* and *cd*.

67. Instant Center at Infinity.—When one link is guided to move in a straight line, the link may be considered to be rotating about the guide, the center of rotation being located on a line normal to the guide at infinity. The slider-crank mechanism shown in Fig. 5 is an example of this.

Links:	*a*	*b*	*c*
Centers:	*ab*	*bc*	*cd*
	ac	*bd*	
	ad		

The fixed instant centers at the link joints *bc*, *cd*, and *ad* are readily located. The slider *b* is rotating about the guide *a*, and the instant center *ab* must lie on a normal drawn through *bc* perpendicular to *a*. The center *ab* must also lie on a normal through *ad* perpendicular to *a*, and since it lies at the intersection of two parallel lines it must be at infinity.

The instant center *ac* must lie on a line connecting *ab* and *bc*, and therefore on a perpendicular through *bc*. The center *ac* must also lie on a line connecting *ad* and *cd*, and therefore lies on a line through *ad* and *cd* and a perpendicular at *bc*.

The student should note that the correct position of instant centers cannot be found unless the proper combination of letters is used. A check may be applied to prove that the correct combination of letters has been used in naming an instant center. To locate the center *ac* of Fig. 5, any two instant centers involving *a* and *c* which have a common letter, such as *cd* and *ad* may be used. The common letter, in this case *d*, is cancelled out leaving the combination *ac*, and this indicates that the instant center *ac* lies on the line connecting *cd* and *ad*. In the same way *ac* must lie on the line connecting *ab* and *bc*.

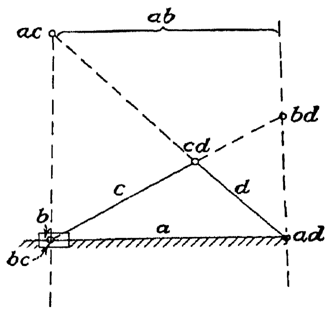


FIG. 5.

68. Relative Linear Velocity Problems.—Relative linear velocity problems are solved by using geometry, and one of the following methods may be employed:

(*a*) The direct instant center method.

(*b*) The link-to-link method.

(*c*) The velocity component method.

69. The Direct Instant Center Method.—The direct instant center method is based upon the following: The velocity of any point in link *a* and the velocity of any point in link *b* may be compared through the velocity of the common point *ab*.

Figure 6 shows any three links of a mechanism, in one of which the point *x* is located, whose velocity is known, in another of which the point *y* is located, whose velocity is unknown, and in which the third link is the fixed link. The instant center *ab*, about which every point in the link *b* rotates with respect to the fixed link *a*, is first located. Next the instant center *ac*,

about which every point in the link c rotates with respect to a, is located. The instant center bc, a common point for links b and c, is located. The point x is then rotated about its fixed center ab into the line of centers, and the velocity of x is laid off to any scale, such as V^{-x}. The velocity[1] of bc, which is V^{-bc}, may be found by completing the similar triangles 1 and 2 as indicated. The point y is now rotated about its fixed center ac into the line of centers, and by completing the similar triangles 3 and 4 as shown, the velocity V^{-y} is determined.

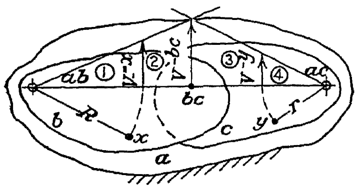


FIG. 6.

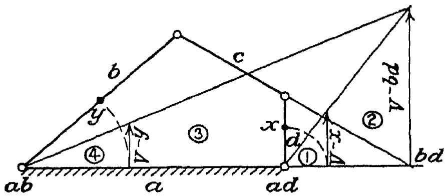


FIG. 7.

Example.—For the mechanism shown in Fig. 7 the velocity of the point x is given, and the velocity of the point y is to be determined. As stated before, the links which are to be considered are the links containing the points and the fixed link.

Links:	a (fixed)	b	d
Centers:	ab (fixed)	bd	
	ad (fixed)		

These three instant centers are located, and the line of centers is found to be along the base of the figure as shown. The point x is rotated into the line of centers, the vector V^{-x} is laid off, and the velocity of the common point bd is found by drawing the similar triangles 1 and 2 as shown. This is true because the instant center bd may be looked upon as the instant center of d with respect to b. Therefore both bd and x may be looked upon as points in the link d, whose linear velocities will be proportional to their radii with respect to the fixed point ad. The point y is then rotated into the line of centers, and the velocity V^{-y} is determined from the similar triangles 3 and 4.

The student will note that in Fig. 6 the triangles are on opposite sides of the vector representing the common-point velocity, and in Fig. 7 the triangles are on the same side of the common-point vector. In both cases, however, there are two sets of triangles, and the common point vector is a common side to the larger pair of triangles.

The same reasoning may be followed in solving the velocity problem shown in Fig. 8. The links are a (fixed), b, and d, and the instant centers are ab (fixed), ad (fixed), and bd. The line of centers in this case is of infinite length and passes through ad and bd as shown, bd being loated in the usual way. The point x is now rotated about the center ad into the line of centers,

[1] The symbol V^{-bc} is to be read: the linear velocity of bc.

and its velocity is indicated by the vector V^{-x}. The similar triangles 1 and 2 are drawn to determine the velocity of the common point V^{-bd}. The point of the triangles 3 and 4 is at infinity, hence the vertical line through the end of the vector V^{-bd} is the hypotenuse of the triangles 3 and 4. The point y is rotated into the line of centers as before, and its velocity vector will be V^{-y}.

The direct instant center method of solving linear velocity problems may be applied to any mechanism. When the mechanism is a compound chain, a number of instant centers may have to be located to determine the line of centers, but the solution of the velocity problem is just as simple as that involved in Figs. 7 and 8.

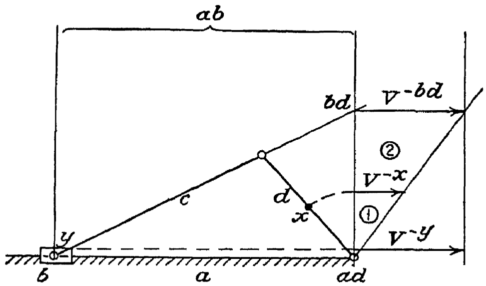


FIG. 8.

70. The Link-to-link Method.—The link-to-link method is readily applied to simple mechanisms. It is dependent upon geometry and the location of one instant center. The method depends upon the fact that if the velocity of any point in a link is known, the velocity of all other points in that link may be found by similar triangles, since the velocities of all points in any link are proportional to the distances from the points to the center of rotation. In Fig. 9 the velocity of the point x in the link d is given, and it is desired to determine the velocity of the point y in the link b.

The vector V^{-x} is first laid off normal to the link d and to any convenient scale. By drawing the similar triangles 1 and 2 the velocity of the end of the link d, V^{-cd}, is determined, and this is also the velocity of the end of link c. The link c rotates about the instant center ac with respect to the link a, and the velocity of all points in link c are proportional to their radii to the instant center. The points n and y are both points on the link c, therefore their velocities will be proportional to their radii from n to ac and from y to ac. Figure 9 shows that the velocity V^{-cd} has been swung into the line drawn from n to ac. The line mp is drawn parallel to the link c. Then in the similar triangles 3 and 4, if nm represents

the vector V^{-cd}, yp will represent the vector V^{-y}. That this is true may be seen by drawing the line mo parallel to acy. In the similar triangles nmo and $nacy$, the radii from ac to n and to y are proportional to mn and mo, and therefore to V^{-cd} and to V^{-y}.

The velocity of any intermediate point, such as k, may be found by joining the point k to the instant center ac, and V^{-k} may be found as shown. The direction of V^{-k} is, of course, along a line through k normal to the instant radius.

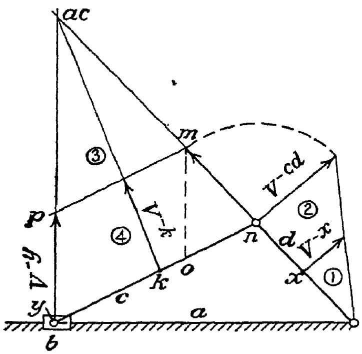


Fig. 9.

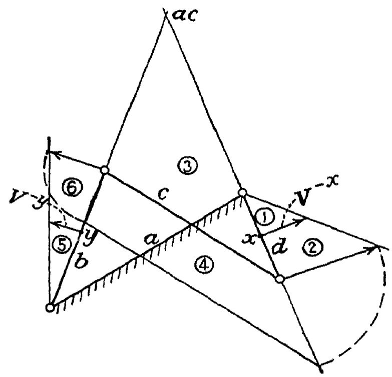


Fig. 10.

Example.—An example is shown in Fig. 10, in which the velocity of a point x in the link d is known, and the velocity of a point y in the link b is to be determined. The vector V^{-x} is laid off to any convenient scale, and the velocity of the end of the link d is determined as shown, this being also the velocity of the end of link c. This vector is swung into the line of centers as shown, and then projected over to the other line of centers from ac, parallel to the link c, thus determining the velocity of the other end of link c. Since this is also the velocity of the outer end of link b, the vector may be swung out as indicated, and the velocity V^{-y} determined from the similar triangles 5 and 6.

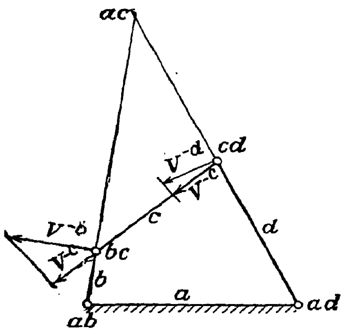


Fig. 11.

71. Velocity Component Method.—The velocity component method takes into consideration the direction as well as the magnitude of a velocity. In Fig. 11, the point cd in the link d has a velocity shown by the vector V^{-d}, with respect to the center ad, and its direction will be normal to the link d. The point cd being also a point in the link

c, will have motion along the vector V^{-c}, which is along the link. The magnitude of this vector is found by drawing the two rectangular components of V^{-d}, one along the link and the other normal to the link. Use is then made of the fact that for any rigid link the velocities of two points will be the same along the line connecting the points, although the velocities may be different in any other direction. The vector V^{-c} is transferred to the b end of link c, and a normal to the link b is drawn through the point bc. The vector V^{-b} is drawn as the hypotenuse of a right-angled triangle of which V^{-c} is one leg.

72. Velocity Diagrams.—The velocity of any point in a mechanism may be found for a series of successive positions of the

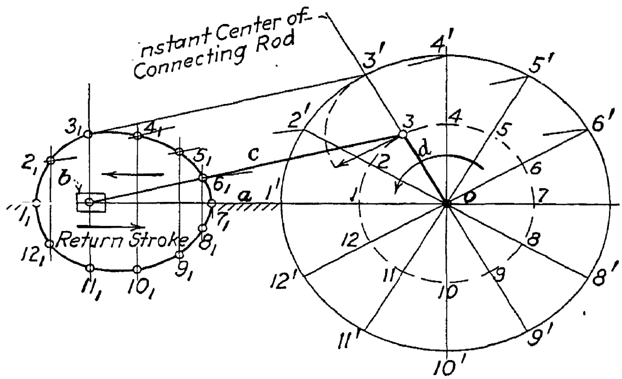


FIG. 12.

links as they move in a cycle, and a curve may be drawn through the ends of the velocity vectors. Such a curve forms a *velocity diagram.* A velocity diagram is a convenient method of getting a picture of the variation in velocity of a point or a part of a mechanism, and is a means for determining the velocity at intermediate positions of the links. The velocity diagram of a point or a link which is moving with translation, is called a *linear velocity diagram.* The velocity diagram of a point or link that is rotating is called a *polar velocity diagram.*

The method of drawing a velocity diagram will be illustrated by constructing the polar and linear velocity diagrams for the slider-crank mechanism shown in Fig. 12. The crank d is turning about the center O at a uniform rate, or approximately so, and when the length of the connecting rod c is as shown, the cross-head b

travels back and forth between the points 1_1 and 7_7 with a variable linear velocity.

The crank-pin circle is first divided into any number of increments (equal increments are most convenient) such as 12. The end of the crank has a linear velocity indicated by the vector which is laid off normal to *30* at the point 3. With the point 3 as a center, and with a radius equal to the length of the vector, the latter is swung into the radial line *30*, locating the point 3′. Point 3′ is one point in the polar diagram, and since the angular velocity of the crank *30* is constant, the other points of the polar diagram will fall on the circle 3′7′11′.

On the line 1_17_1 the positions of the crosshead are located by points to correspond with the positions of the crank *10*, *20*, . . . *120*. Through these points vertical lines 2_1, 12_1, 3_1, 11_1, . . . 6_18_1 are drawn.

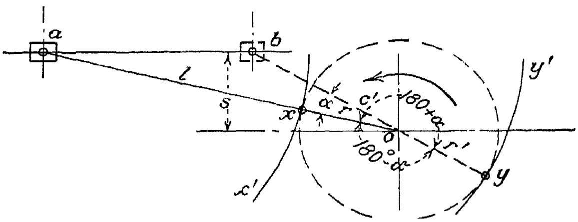


FIG. 13.

The position of the connecting rod c for each position of the crank is then determined, and through the points 2′, 3′ . . . 12′ of the polar diagram lines are drawn parallel to the connecting rod, thus locating the points 2_1, 3_1, . . . 6_1 above the line 1_17_1. Since the center line of the crosshead 1_17_1 is on a line with the center of the crank *O*, the linear velocity diagram of the crosshead will be symmetrical about the line 1_17_1.

The above method is based upon the fact that the instant center of the connecting rod c with respect to the fixed link a of the machine is at the intersection of the line *O3′* and the vertical line through the crosshead. Therefore, from similar triangles, if 33′ is the linear velocity of one end of the connecting rod, $b3_1$ will be the linear velocity of the crosshead.

In Fig. 12 the velocities measured above the line 1_17_1 would be the velocities to the left, and those below the line would be the velocities on the return stroke to the right.

The slider-crank mechanism shown in Fig. 13 has the center line of the crosshead offset a distance s from the center of the crank. With the length of the stroke equal to ab, the connecting rod of length l, and the crank arm of radius r, the construction of the velocity diagram is as follows. With a radius equal to l, and with a and b as centers, the arcs xx' and yy' are drawn. The center o of the crank is located by trial so that the crank-pin circle will be tangent to the arcs xx' and yy'.

When the crosshead is at a, the connecting rod and crank will be in line and the distance oa is $l + r$. When the crosshead is at b, the connecting rod and crank will be in line and the distance ob is $l - r$. Assuming that the angular velocity of the crank is constant, or nearly so, it will turn through $(180 - \alpha)$ deg. while the crosshead travels from a to b. While the crank turns through $(180 + \alpha)$ deg., the crosshead will travel from b to a. Hence the crosshead travels faster while going from a to b than it does in returning from b to a, and the linear velocity diagram of the crosshead, if constructed, will indicate the difference in velocities of the forward and return strokes.

The distance s is limited, because if it is made too large the vertical thrust of the crosshead against the guides becomes excessive. Within the practical limits of the offset distance s, the time ratio of the forward to the return stroke of the crosshead is close to unity, and therefore has little practical application as a quick return mechanism.

When it is desirable that the sliding links have a quick motion in one direction, this may be accomplished by applying an inversion of the slider-crank mechanism. One of the most widely used quick-return motion mechanisms is credited to Sir Joseph Whitworth, who applied it to machine tools in his shop at Manchester, England. It has also been applied to shapers and slotters by American machine-tool builders.

73. Whitworth Quick-return Mechanism.—The elements of the Whitworth quick-return mechanism are shown in Fig. 14. The constant radius arm r, with its center at o, is provided at its outer end with a sliding block b. The lower circle is the path of the block b as the driving arm r turns with a constant angular velocity. The block b pushes the variable radius arm v around its center c, and at the same time is constrained to slide along on the arm v. The ram s is attached to the variable radius arm by the connecting rod l.

This mechanism will give a quick return to the ram, the time ratio being as high as two to one. That is, the arm v will sweep through an angle of 240 deg. while the ram is moving through a working stroke, and through an angle of 120 deg. while the ram

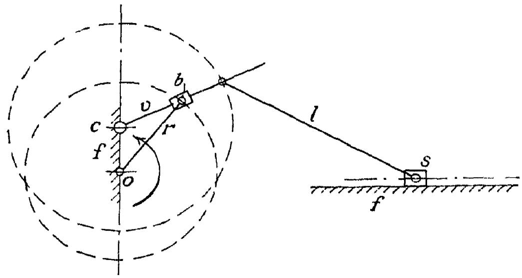


FIG. 14.

is idling back to get into position for another working stroke. In other words, this mechanism permits a machine to do work two-thirds of the time that it is operating.

The velocity diagram of the Whitworth mechanism is constructed as shown in Fig. 15. The stroke of the ram ab and the

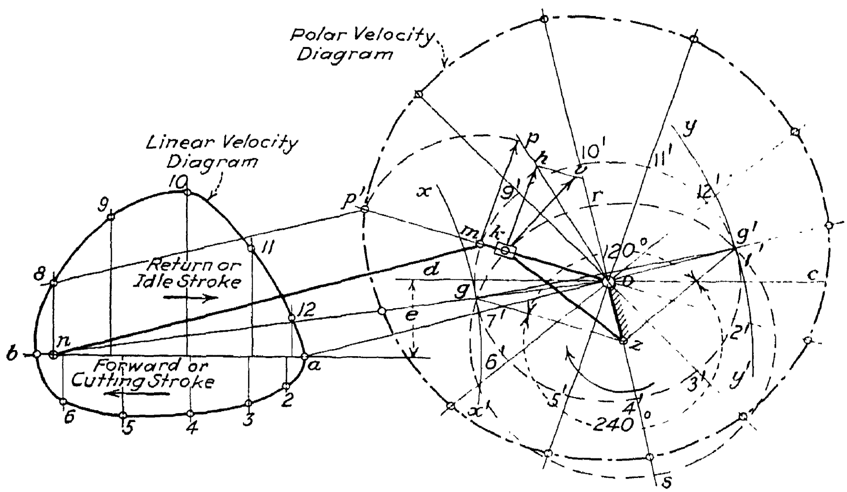


FIG. 15.

length of the connecting rod mn are given, and the time ratio of the working stroke to the return stroke is to be 2 to 1. The center line of o is a distance e above the center line ab. The line ab with the center line a distance e above it is first laid off. With

a radius equal to the length of the connecting rod, and with centers at b and at a, the arcs xx' and yy' are drawn. The center o is determined by trial, and a circle is drawn tangent to xx' and yy'. This circle will be the path of the point m in the variable radius arm mo.

The positions of the connecting rod mn and the crank mo are drawn in for the extreme positions of the ram, locating the points g and g'. The points g and g' are connected with a line, and normal to it through the point o the line rs is drawn. The variable radius arm om is in line with og when the ram is at one extreme position (at b), and in line with og' when the ram is at the other extreme position (at a). A line is drawn through g so that the angle gzr is 60 deg., locating the point z. The line $g'z$ is drawn making the angle $g'zs'$ equal to 120 deg., and the angle gsg' equal to 240 deg. This arrangement will make the time ratio of the working stroke to the return stroke 2 to 1.

The arcs $g10'g'$ and $g'4'g$ are divided into any number of equal increments, such as $1'o2'$, $2'o3'$ . . . $12'o1'$. The angle increments for $g10'g'$ and $g'4'g$ will of course be of different size. The corresponding positions of the ram are determined for the twelve positions of the arm mo, and numbered 1, 2, . . . 12.

The velocity vector kv, the linear velocity of the driving block k, which is turning at a uniform rate, is laid off to any convenient scale. The point k is a point in the link kz and the link mo. The vector kv is resolved into two rectangular components parallel and perpendicular to the link mo, determining kh as the linear velocity of k about the center o, to the same scale that kv is the linear velocity of k about the center z. Knowing the linear velocity of the point k, the linear velocity of the point m in the same link may be found from similar triangles, determining the vector mp.

The point p is now swung into the radial line om, locating the point p', which is one point on the polar velocity diagram. Parallel to the position of the connecting rod mn and through the point p' a line is drawn, locating the point 8 on the vertical line $n8$, and this point is one point in the linear velocity diagram.

The other points in the polar and linear velocity diagrams may be located in a similar manner. Smooth curves drawn through the points so located determine the diagrams as required by the statement of the problem.

The student should note the relative flatness of the lower portion of the linear velocity diagram, between the points 2

and 6 especially, which indicates that the cutting speed is fairly uniform for the cutting portion of the stroke.

74. Swinging-arm Quick-return Mechanism.—The swinging-arm mechanism is an inversion of the slider-crank mechanism. It is similar to the Whitworth mechanism in form and operation, the essential difference being in the length of the fixed arm *oz*. If the fixed arm *oz* of Fig. 15 is made longer, so that the point *z* falls outside of the circular path of *k*, the arm *zk* will be constrained to rock back and forth with a variable angular velocity, and it cannot sweep through an angle of 360 deg. The arm *ok* would be made the constant radius arm, and the arm *kz* the

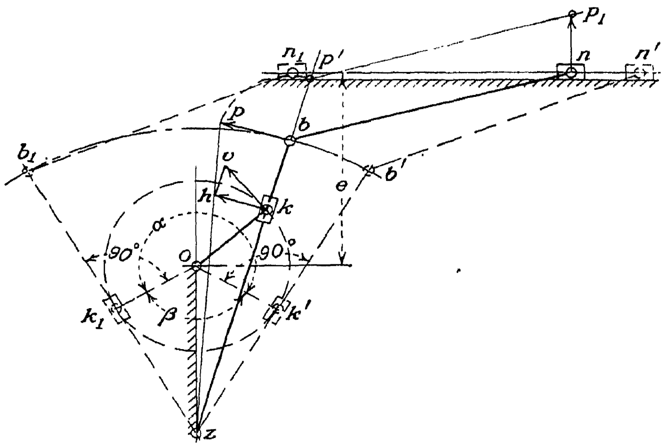


Fig. 16.

variable radius arm. The elements of the swinging-arm mechanism are shown in Fig. 16.

The construction of the swinging-arm mechanism will now be discussed. The following values are given: the constant radius arm *ok*, the length of the swingng arm *zb*, the length of the connecting link *bn*, and the distance *e* between the center *o* and the center line of the ram. The time ratio of the working stroke to the idle stroke will be the ratio of the angle α to the angle β.

The center *o* and the line n_1n' are first located. The circular path of the sliding block *k* is drawn with a radius *ok*. When the swinging arm is in its extreme position, the constant radius arm *ok* is normal to it. The position of the arm *ok* is drawn so that the angles β and α divide the circle k_1kk' into angles which

are proportional to the desired time ratio of the idle to the working stroke. The positions of the swinging arm $b'z$ and b_1z are drawn so that they are normal to the lines ok' and ok_1, locating the center z on the center line oz. The arc $b'b_1$, the path of the end of the swinging arm, is then drawn. With the connecting link bn as a radius, and with centers at b' and b_1, the extreme positions of the ram may be located at n' and n_1.

The velocity diagrams for the swinging-arm mechanism may be developed as follows: The vector kv, representing the linear velocity of k about the center o, is laid off to any convenient scale, normal to the arm ok. The point k is a point in the link zb and also in the link ok, and has the velocity kh in the swinging arm about the center z. Knowing the linear velocity of k in the swinging arm, the linear velocity of the point b in the same arm may be found by similar triangles, determining the vector bp. The vector bp is now swung into the radial line zp', locating the point p' in the polar diagram. Through the point p' and parallel to the connecting link bn the line $p'p_1$ is drawn, locating the point p_1 in the linear velocity diagram of the ram n. Proceeding in a similar manner, other points in the polar and linear velocity diagrams may be located. In doing this the circle $kk'k$, is divided into any number of angle increments to represent the different positions of the link ok in its cycle. If the angle α is divided up into the same number of angle increments as the angle β, then the α increment will be to the β increment, as the time ratio of the working to the idling stroke

Problems

1. State Kennedy's theorem and prove it.
2. Make a sketch of the kinematic chain formed by the connecting rod and crank arms of a pair of locomotive driver wheels, and locate all the instant centers.
3. Make a sketch of an open belt which transmits power from one pulley to the other, and, neglecting the stretch in the belt, locate all the instant centers.
4. Same as Problem 3, but for a crossed belt.
5. Locate all the instant centers in the mechanism shown by Figs. 1(*b*) and 1(*c*).
6. Locate all the instant centers in the mechanism shown by Figs. 2(*a*) and 2(*b*).
7. In Fig. 1(*b*), with the link *b* fixed, assume the linear velocity of a point in the link *a*, and find the velocity of a point in the link *c*: (*a*) by the direct method; (*b*) by the link-to-link method; (*c*) by the component method

8. The links shown in the mechanism of Fig. 5 have the following lengths: the crank is 8 in., and the connecting rod is 24 in. Find the linear velocity of the crosshead when the crank is turning at 200 r.p.m., when the angle *cd-ad-bc* is 60 deg. Use any method and choose any convenient scale for the velocity vector.

9. For the mechanism shown in Fig. 17: (*a*) locate all the instant centers; (*b*) given the linear velocity of the point x, find the linear velocity of the point y, each point being located at the middle of the link.

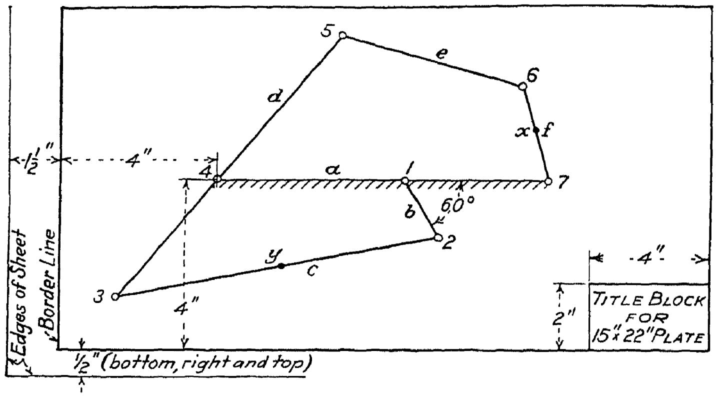


FIG. 17.—Figure for Problem 9.

Data.—Link 14 = 4 in.
12 = 1½ in.
23 = 7 in.
34 = 3½ in.
45 = 4¼ in.
56 = 4 in.
67 = 2¼ in.
17 = 3 in.
The angle 712 = 60 deg.

The length of the velocity vector for the point x is to be laid off 2 in. long. If the linear velocity of x is 6 ft. per second, determine the linear velocity of y in feet per minute. For the solution of this problem use the direct instant center method. Scale 12 in. = 1 ft. 0 in.

10. For the mechanism shown in Fig. 18: (*a*) locate all of the instant centers; (*b*) given the linear velocity of the point x find the linear velocity of the instant center *ef*.

Data.—Link 12 = 3⅞ in.
23 = 3$^{13}/_{16}$ in.
34 = 2⅝ in.
45 = 1¾ in.
56 = 6 in.
14 = 7 in.
The angle 341 = 60 deg.

In laying off the velocity vector of the point x use the largest scale consistent with the limits of the problem sheet. In solving this problem use any of the three methods discussed in the preceding chapter. Scale 12 in. = 1 ft. 0 in.

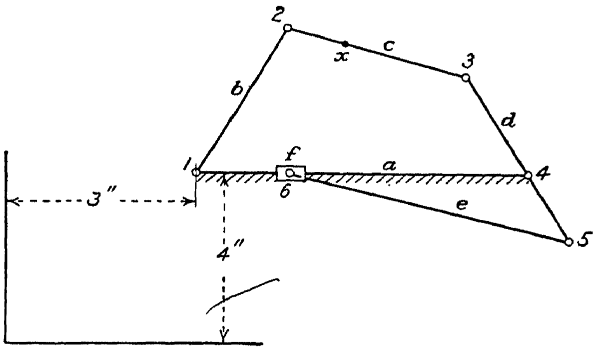


FIG. 18.—Figure for Problem 10.

11. For the mechanism shown in Fig. 19: (*a*) locate all of the instant centers; (*b*) if the crank arm 12 is turning at 30 r.p.m., what is the linear velocity of the end of the link *e*?

Lay off the vector representing the linear velocity of the point *bc* to the scale 1 in. equals 15.7 ft. per second.

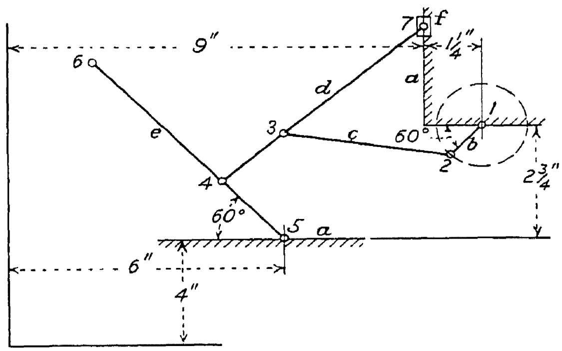


FIG. 19.—Figure for Problem 11.

Data.—Link 12 = 1 in.
34 = $1\frac{5}{8}$ in.
45 = 2 in.
46 = 4 in.
47 = $5\frac{5}{8}$ in.

The angles are as shown in the figure. Scale 12 in. = 1 ft. 0 in.

12. (*a*) Develop the polar and linear velocity diagrams for the slider-crank mechanism shown by Fig. 9. Locate 16 points by dividing the crank pin circle into 16 equal angles. After the diagrams have been developed

draw in the links in any one position, showing the shape and the proportions of the parts, using your judgment for sizes.

Data.—The length of the crank arm is 2½ in. and the length of the connecting rod is 12 in. Locate the center of the crank on a line 5 in. up from the lower border line, and place the drawing central with respect to the right and left border lines. The velocity vector representing the linear velocity of the crank pin is to be 2 in. long.

(*b*) What is the linear velocity of the crosshead at 25 per cent of the crank end and head end stroke?

(*c*) Why is the linear velocity diagram of the crosshead unsymmetrical with respect to its vertical center line?

(*d*) What would be the effect upon the linear velocity diagram of the crosshead if the connecting rod were infinite in length? Scale 12 in. = 1 ft. 0 in.

13. Develop the polar and linear velocity diagrams for the Scotch crosshead shown in Fig. 20.

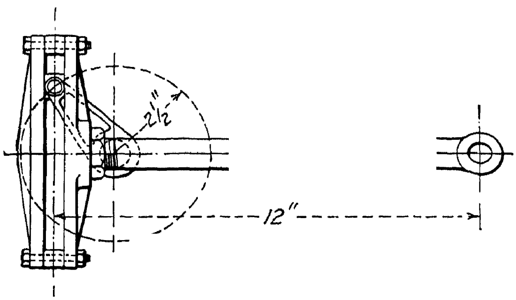


FIG. 20.—Figure for Problem 13.

Data.—The length of the crank arm is 2½ in. and the length from the center of the guide in the crosshead to the connection at the end of the link is 12 in. The length of the velocity vector representing the linear velocity of the crank pin is 2 in.

Locate the center of the crank shaft 5 in. up from the bottom border line, and place the drawing central on the sheet with respect to the right and left border lines.

NOTE: The data for this problem are similar to the data of Problem 12. The velocity diagrams of the two problems may be developed from the same centers, superimposing the linear velocity diagrams upon each other, thereby obtaining a direct comparison of them. Scale 12 in. = 1 ft. 0 in.

14. (*a*) Lay out the polar and linear velocity diagrams for the Whitworth quick-return mechanism shown in Fig. 15.

Data.—Maximum stroke of ram, b to a, is 20 in.

$$e = 7\frac{1}{2} \text{ in.}$$
$$mn = 36 \text{ in.}$$

The time ratio of the cutting stroke to the idle stroke is 2 to 1.

The length of the vector representing the velocity of the point k in the constant radius arm is 2 in.

Locate the line *ba* 4½ in. up from the bottom line, and on it locate the point *b* 1¼ in. from the right border line.

NOTE: The radius *kz* is adjustable to allow the changing of the stroke of the ram. The maximum stroke of the machine for this problem is as given, the minimum stroke is zero inches.

(*b*) When the arm *kz* is turning at 25 r.p.m., what is the maximum velocity in feet per minute of the cutting stroke and idle stroke?

Scale 6 in. = 1 ft. 0 in. Dimensions which locate the drawing on the paper are full size.

15. Lay out the polar and linear velocity diagrams for the rocking-arm shaper mechanism shown in Fig. 16.

Data.—The maximum length of the driving arm *ok* is 10½ in., the length of the rocking arm is 36 in., and the length of the connecting rod is 24 in. The time ratio of the cutting stroke to the idle stroke is 2 to 1, and the offset distance *e* is 18½ in.

NOTE: The center of the driving arm is 7¼ in. from the left border line, and 6 in. up from the bottom border line. The driving arm radius *ok* is adjustable. For the data given the block is at its greatest distance from the center *o*. The velocity vector representing the linear velocity of the point *k* is 2 in. long.

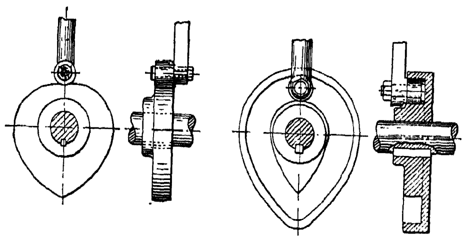
Scale 6 in. = 1 ft. 0 in. Dimensions which locate the drawing on the paper are full size.

CHAPTER V

CAMS

75. A *cam* is a machine part which, when displaced, transmits motion by means of a curved edge to a second part called a *follower.* A *single-edge* cam depends upon some external force produced by a spring or by gravity to return the follower to its starting point. A *two-edge* cam constrains the follower for the complete cam cycle by means of a groove, and is known as a *positive-motion* cam.

Cams are classified according to their form as *plate cams* or *cylinder cams.* A plate cam, sometimes called an *edge cam* or a *disk cam,* consists of a flat plate of variable thickness, which



(a) (b)

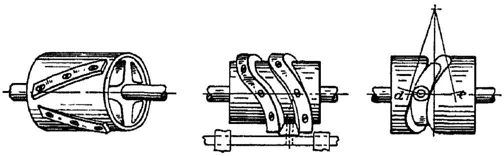
FIG. 1.

drives a follower in a plane normal to the axis of the cam. Figure (1*a*) shows a single-edge plate cam, and Fig. 1(*b*) shows a two-edge or grooved-plate cam which is called a *face cam.* When a plate cam pushes the follower away from the center of rotation the follower is said to *rise,* when the follower remains at a fixed position it is said to *rest,* and when the follower returns to its starting point it is said to *fall.*

Figure 2 shows several types of cylinder cams. Figure 2(*a*) is a drum-type cylinder cam having adjustable edges which may produce various movements of the follower. The cam of Fig. 2(*b*) produces straight-line motion in the follower. Figure 2(*c*) shows a cam with a follower which swings in the arc of a circle.

Figure 3 shows a single-edge cylinder cam which produces reciprocating motion in the follower.

A simple plate cam, known as a *sliding cam,* is shown in Fig. 4. The crank gives the sliding cam a reciprocating motion, and the follower rises and falls according to the contour of the edge *cd.* The movement which the follower is to have determines the slope



(*a*) (*b*) (*c*)
FIG. 2.

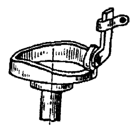


FIG. 3.

and curvature of the cam edge. Ordinarily cams have a constant angular motion, and the follower moves according to any one or a combination of all of the motions discussed in the following sections.

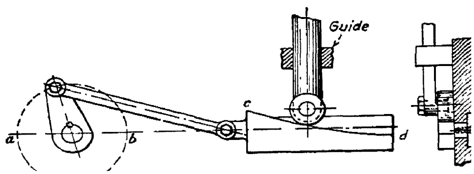


FIG. 4.

76. Uniform Motion.—If motion is defined as a change in position, and velocity as a measure of the motion, then a body has *uniform* motion when it has a constant velocity. In uniform motion the change in position takes place at a constant rate with respect to time, and equal distances are traveled in equal intervals of time. This may be expressed in a formula:

$$s = vt, \tag{1}$$

in which s denotes distance, in feet or inches.

v denotes velocity, usually in feet per second, or inches per second.

t denotes time, in seconds.

In engineering, distance is usually measured in inches, feet, or miles. Velocity is measured in inches per second or per minute, feet per second or per minute, or miles per hour. Angular

displacement is measured in degrees, radians, or revolutions; and angular velocity in degrees, radians, or revolutions per second or per minute.

77. Harmonic Motion.—When a point moves in a circle at constant speed, the motion of the point projected upon a diameter is called a *harmonic* motion. This motion may be applied to a cam follower as shown in Fig. 5. The semi-circle 04′8 is divided into any number of equal divisions, say eight, and the division points 1′, 2′ . . . 7′ are projected on the diameter 08, dividing it into eight divisions representing harmonic motion. In Fig. 5 the diameter 08 is the path of the follower.

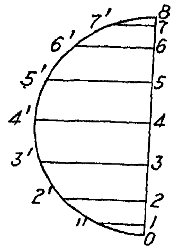


FIG. 5.

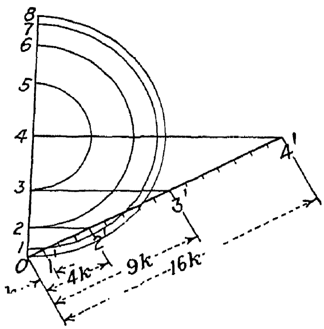


FIG. 6.

78. Motion with Uniform Acceleration.—A body which changes its position with a velocity which increases at a uniform rate with respect to time, is moving with uniform acceleration. This is sometimes called natural motion because it is the motion of a freely falling body. It may be expressed in a formula:

$$s = \tfrac{1}{2}at^2, \tag{2}$$

in which s denotes distance, usually in feet.

a denotes acceleration, usually in feet per second per second.

t denotes time, in seconds.

For a freely falling body experiment has shown that a is equal to 32.186 ft. per second per second. (Latitude 49° at sea level.)

If the angular displacement of a cam is substituted in formula (2) for t, and if the cam is turning at a uniform rate, then the angular displacement will be directly proportional to the time.

Hence:

during the first unit of angular displacement $s = a/2 = k$, difference = 1

during the second unit of angular displacement $s = 2a = 4k$, difference = 3

during the third unit of angular displacement $s = 9a/2 = 9k$, difference = 5

during the fourth unit of angular displacement $s = 8a = 16k$, difference = 7

To divide the follower path into divisions representing uniform acceleration, the line 08 in Fig. 6 is to be divided into divisions representing uniform acceleration up to the point 4. The line 04′ is drawn from 0 at any convenient angle, and with any unit length such as 01′ the divisions 01′ = k, 02′ = $4k$, 03′ = $9k$, and 04′ = $16k$, are stepped off. From 4′ the line 4′4 is drawn, and 3′3, 2′2, and 1′1 parallel to 4′4 through the points 3′, 2′, and 1′, locating 3, 2, and 1. The points 0, 1, 2, 3, and 4 are divisions representing uniform acceleration on the follower path.

The motion opposite to uniform acceleration is uniform retardation, and the divisions for it are stepped off from the point 4 to 8 as indicated in Fig. 6. The velocity of the cam follower slows up at the same rate at which it speeds up, hence uniform acceleration and uniform retardation are each an equal part of the follower's motion.

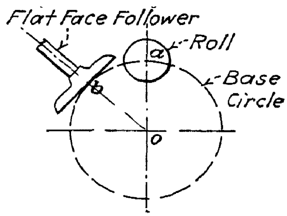


FIG. 7.

79. Base Circle.—Plate cams have their edges developed from a base circle, the radius of which is the distance from the center of the camshaft to the nearest position of the follower. In Fig. 7 the camshaft center is at o, and for a flat-face follower the radius of the base circle is ob, and for a roll follower the radius is oa.

The size of the base circle fixes the size of the cam, a large base circle producing a large cam, and a small base circle producing a small cam. There is always a practical limit for the size of any cam, so that a base circle is chosen which will result in a cam large enough to give the follower the desired displacement, and at the same time strong enough to transmit the power which is required to do the work.

80. Pressure Angle.—The pressure between the surface of the cam and the follower is along a line normal to the cam curve at

the point of contact, and the angle which this normal makes with the center line of the follower is the pressure angle. Figure 8 shows the pressure angle α for a cam which has been laid out for a 30-deg. slope, to raise and lower the follower through a distance h with a uniform motion. If the base of the triangles is lengthened, the slope of the curve will be less and the pressure angle will be smaller. Hence, if the base circle of a cam is made larger, the follower will have an easier motion, and there will be less bending action on the follower. For practical reasons it is desirable to give the follower an easy start and stop when uniform motion is employed, and the dotted lines in Fig. 8 show that to do this the base circle should be enlarged. Figure 9 shows the

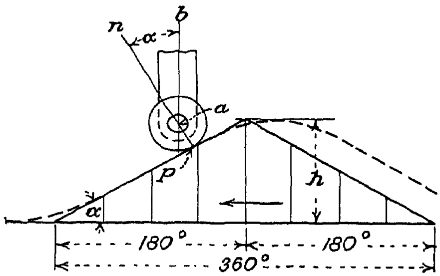


Fig. 8.

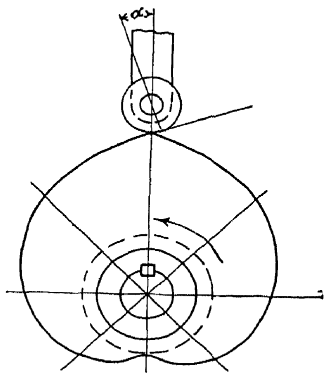


Fig. 9.

diagram of Fig. 8 bent around the base circle, resulting in what is commonly called a *heart cam.*

81. Roll and Flat-face Follower.—Contact between the curved surface of the cam and the follower may be made by a flat-surface follower, which slides over the cam surface as the cam turns, or the follower may be provided with a roll which rolls against the cam surface as turning takes place. Either type of follower may be made to reciprocate or to swing about a center. Figures 10(a) and 10(b) show a reciprocating flat-face and a reciprocating roll follower, while Figs. 10(c) and 10(d) show the swinging flat-face and swinging roll follower.

The roll used with a roll follower is limited in size by the curvature of the cam. The radius of the roll should in no case be larger than the least radius of curvature of the cam curve.

82. Theoretical Curve.—If the follower made contact with the cam by means of a pointed or knife-edged bearing, then the cam curve which would drive the follower with the desired motion would be called the *theoretical curve.* The theoretical curve must be modified to suit the requirements of a roll or flat-face follower, and the modified curve is called the *working curve.*

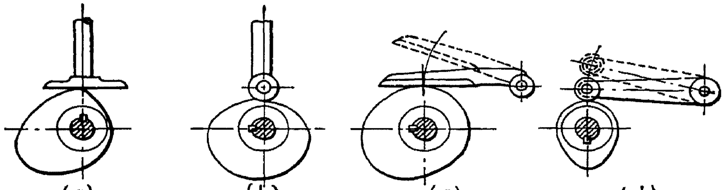


FIG. 10.

83. Cam Charts.—A cam chart is used to help the designer visualize just what takes place at any instant when the cam displaces the follower. The cam turns at a uniform rate while the displacement of the follower may be according to one or several motions. For convenience, a cam chart or diagram is drawn as shown in Fig. 11, in which the cam displacements are the abscissas and the follower displacements are the ordinates.

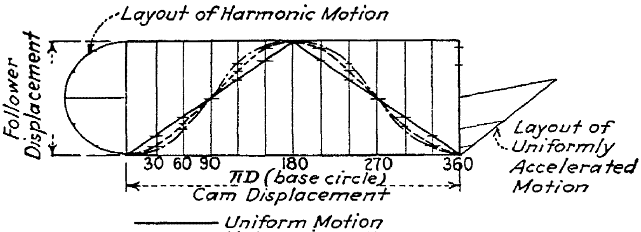


FIG. 11.

The plotted curves show that with uniform motion for one cycle the follower starts from rest suddenly with a uniform velocity, changes its direction abruptly at its highest point, and comes to rest suddenly. This condition should be avoided by modifying the curve as indicated by the dotted lines in Fig. 8, so as to round out the peak and valley of the curve. The harmonic curve shows that the sudden change in direction is avoided, while the curve plotted for uniformly accelerated and retarded motion

allows the follower to start slowly, pick up speed, and come to rest gradually.

If the base line in Fig. 11 were bent to form a circle, the base line would then become the base circle, the vertical ordinates would become radial lines, and the curves shown would be the profile curves of a plate cam.

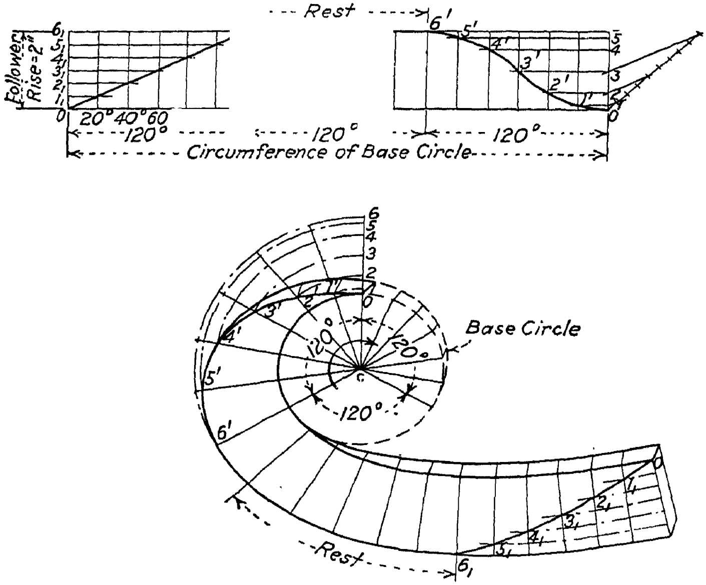


Fig. 12.

In Fig. 12 the cam chart shows the theoretical curve for a plate cam which turns clockwise to give the follower displacements according to the following schedule:

(*a*) A rise of 2 in. with uniform acceleration and retardation in 120 deg. of angular displacement of the cam.

(*b*) Rest for 120 deg. of angular displacement.

(*c*) A 2-in. drop with uniform motion in 120 deg. of angular displacement.

The lower portion of Fig. 12 shows the cam profile partly wrapped about the base circle. Because the cam's rotation is clockwise the profile must be wrapped to the left around the base circle, as shown in the figure.

84. Plate Cam with a Reciprocating Roll Follower.—To find the working curve which will give its follower the displacements

required by the data in the preceding section, the base circle is first drawn with a radius $c0$ as shown in Fig. 12. The profile curve is transferred to the base circle by stepping off on the radial lines distances $c0$, $c1'$ . . . $c6'$, which are equal to the radius of the base circle, plus the rise of the follower. A smooth curve drawn through these points will be the theoretical curve.

A geometrical layout which does not require the actual transfer of follower displacement is shown in Fig. 13. The base circle with a radius $c0$ is drawn first. The center line of the follower is measured off 2 in. along 06, and divided into any convenient number of divisions to represent uniformly accelerated motion, by the method described in Sec. 78. The first 120 deg. of the base circle is divided up into as many units of arc as there are divisions

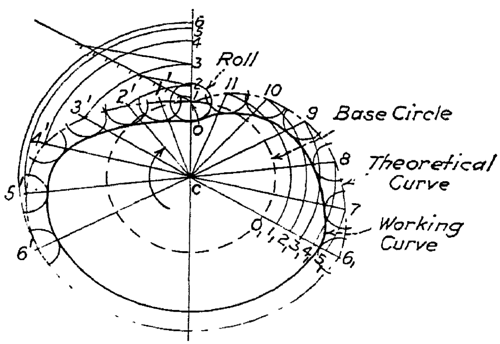


FIG. 13.

on the line 06. With c as a center, point 1 is swung into line $c1'$ locating point $1'$, and the points $2'$ . . . $6'$ are located in like manner. The points so located are points on the theoretical curve.

A rest indicates that the follower remains in a fixed position while the cam turns through a given angle. In this case the angle is 120 deg. so that the theoretical curve for the next 120 deg. will be the arc which has $c6'$ as its radius.

The third portion of the problem requires a 2-in. drop for the follower with uniform motion, as the cam is completing its cycle. On the line $c6_1$ six equal divisions are laid off, and with c as a center, point 5_1 is swung into line 7, point 4_1 into line 8, thus locating successively points 7, 8 . . . 11, which determine the theoretical curve for the third requirement.

The roll has its center on the base circle at point 0 on the theoretical curve. With the points 1′, 2′ . . . 11 as centers, and a radius equal to the radius of the roll, arcs are drawn as shown. The working curve for the cam is then determined by drawing a smooth curve tangent to the roll arcs.

85. Plate Cam with Reciprocating Flat-face Follower.—The kind of follower used with a cam determines the working curve only, and if the data for a cam with a flat-face follower are the same as for the problem in the preceding section, the theoretical curve will be the same. To locate the working curve for a flat-face follower the procedure is as shown in Fig. 14. The base of the follower is shown by the line *ab*, making an angle α with the

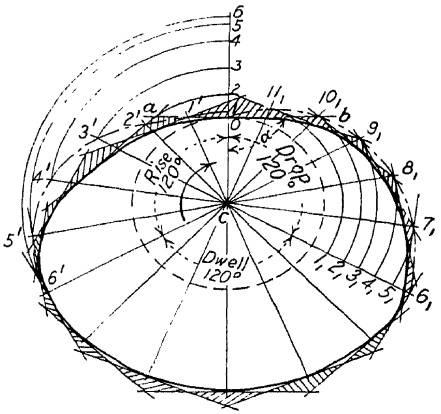


Fig. 14.

center line of the follower, and this center line must pass through the point 0 on the base circle when the follower is in its lowest position. Through the points 1′, 2′ . . . 11_1 on the theoretical curve, lines are drawn making an angle α (in this case a right angle) with the radial lines. These lines will form a series of triangles, which are shown by the cross-hatched lines in the figure. The bases of these triangles are formed by the follower at that particular angular displacement of the cam. A smooth curve, drawn tangent to the middle point of the base of each triangle, will determine the working curve for the cam.

The base line of a flat-face follower usually makes an angle of 90 deg. with the center line of the follower (α = 90 deg.), but it may have some other value and still result in a good working curve.

86. Plate Cams with Offset Follower.—The discussion thus far has pertained to cams with the follower center line passing through the axis of the cam. The center line of the follower may be to the right or left of the cam axis as shown in Fig. 15. The center line in this case is a distance *cb* to the left of the center of the cam. The large outside circle is drawn with *c* as a center using any convenient radius, and is divided into as many unit arcs as there are divisions in the follower path. Point 1 on this circle is located where the center line of the follower cuts the large circle. A circle with *cb* as a radius and *c* as a center is drawn, and the cam elements will be tangent to this circle. The theoretical curve is located as described in Sec. 84, keeping in mind,

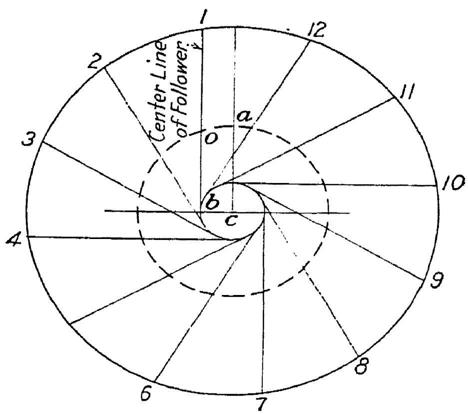


FIG. 15.

however, that the points in the follower path are always rotated about the *center c* into their corresponding cam elements. The working curve is found as described in the preceding sections. The base circle has a radius *ca* with its center at *c*.

87. Cam with Swinging Follower.—A follower may swing about a center as shown in Figs 16(*a*) and 16(*b*). Figure 16(*a*) shows a cam with a roll follower which swings through the angle 0*a*3 about the center *a* while the cam turns about the center *c*. The displacement of the follower roll must be along the arc 03, and the positions of the roll are shown by the points 1, 2, and 3. These points are located according to the motion of the follower, which in this case rises and falls with harmonic motion. The points 1, 2, and 3 are swung about the center *c* into the cam

elements $c1'$, $c2'$, $c3'$, respectively, and the theoretical curve is thus located. The working curve is found as before.

Harmonic motion cannot be laid out on the arc 03, and the usual scheme is to lay off the harmonic divisions on a straight line of length equal to the length of the arc, and then transfer the division points to the arc. In many cases, if the cord of the arc is divided into harmonic divisions, the error is small and may be neglected.

The follower arm shown in Fig. 16(*a*) may be a bell crank, indicated by the center lines 0*a* and *ab*, and the motion may then

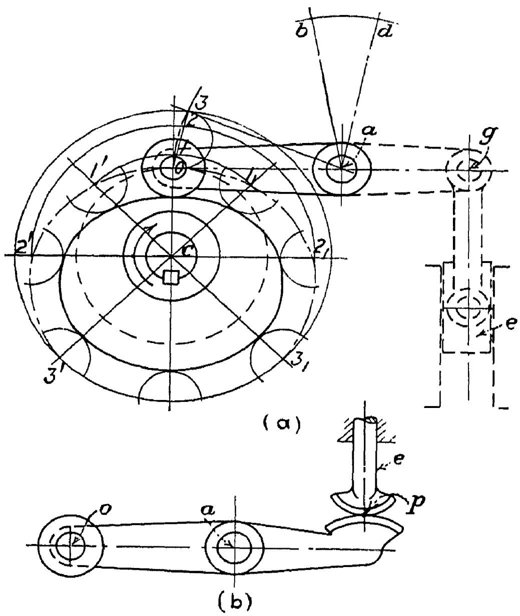


FIG. 16.

be laid off on the arc *bd*. In other cases the follower arm may be extended to form the lever 0*ag*, connected to the sliding head *e* by the link *ge*. In this case if the motion of *e* is to be harmonic the harmonic divisions must be laid off on the center line of the sliding head *e*.

Figure 16(*b*) shows a follower lever connection to the guided rod *e*, with sliding contact between the surfaces at *p*.

The layout for a cam with an oscillating flat-face follower is shown by Fig. 17. The line 0*a* is the base line of the follower, which rises and falls about the center *a* as the cam turns about

the center c as indicated by the arrow. The theoretical curve, as before, will pass through the points 1, 2, 3, . . . 0. If a circle is drawn through a with its center at c, it is possible to locate the points a, b, d, . . . i, which are the relative locations of the fixed end of the follower for the corresponding displacements of the cam. Starting at a, the circumference of the circle is divided into as many parts as were used for the base circle. Through the points b, d, e, . . . i the lines $b1$, $d2$, $e3$, . . . $i7$, are drawn, which are the positions of the follower base line which will give the follower the required motion. If these lines are extended they will form a series of triangles which will enclose the cam, and the working curve of the cam will be

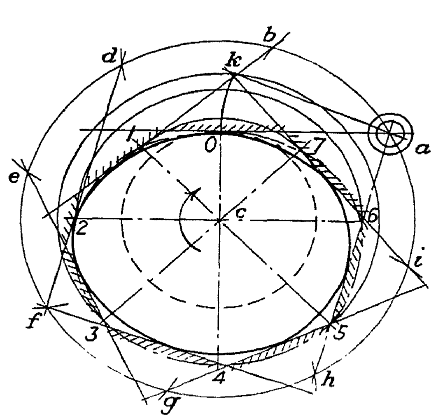


FIG. 17.

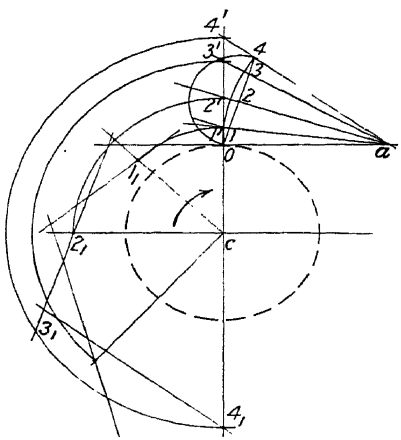


FIG. 18.

tangent at the midpoint of the base of each of these triangles. In Fig. 17 the bases of the triangles are shown cross-hatched for clearness.

A second method of drawing the lines $b1$, $d2$, $e3$, . . . $i7$ is shown in Fig. 18. The path of the follower is divided into divisions according to the motion of the follower, which in this case is harmonic. The lines $a1$, $a2$, $a3$, and $a4$ are extended until they cut the line $04'$, at the points $1'$, $2'$, $3'$, and $4'$. The points $1'$, $2'$, $3'$, and $4'$ are swung into the lines $c1_1$, $c2_1$, $c3_1$, and $c4_1$, locating the points 1_1, 2_1, 3_1, and 4_1. Through 1_1 a line is drawn making the angle $01'a$ with $c1_1$, and through 2_1 a line is drawn making the angle $02'a$ with $c2_1$, and so on. These lines are the base lines of the flat-face follower as it turns about

the fixed point a, and a smooth curve drawn tangent to them as before will be the working curve for the cam.

A simple method for transferring the follower base lines to the new positions may be employed by the use of a protractor made of a piece of tough transparent paper or tracing cloth pinned at the center c. On it the lines $c4'$ and the lines $0a$, $1'a$, $2'a$, $3'a$, and $4'a$ (Fig. 18) are drawn. The protractor is swung about the center c and the line $1'a$ drawn through the point $1'$, then the template is swung so as to draw the line $2'a$ through the point $2'$, and so on.

88. Other Types of Plate Cams.—Names given to plate cams are sometimes derived from the kind or form of the follower, or from the profile curve of the cam edge. A cam which has its edge shaped like the involute curve is known as an involute cam, and this type is used to raise the follower to a fixed height and then allow the follower to fall due to its own weight. The principal use of involute cams is to raise the hammer of stamp mill machinery used in the mining industry.

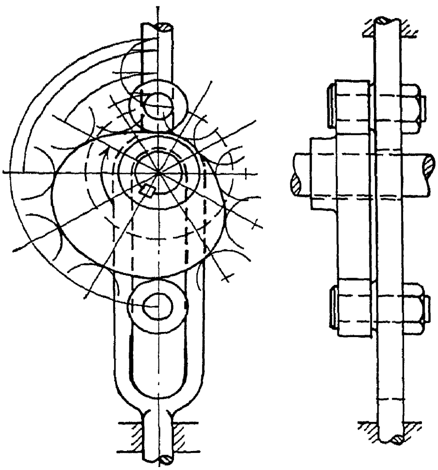


Fig. 19.

89. The Constant-diameter Cam.—The constant-diameter cam is one which has two rolls, as shown in Fig. 19, one to drive the follower according to any given law of motion, and the other roll located on the opposite side of the center to drive the follower back to its starting position. If the lower follower roll were omitted in Fig. 19, then, after the upper follower had reached its highest position, it would require gravity, or some external force to push it down to maintain contact with the cam. When both follower rolls are used the motion is positive.

The first half of the cam curve is developed according to the methods described in preceding sections, and the second half of the curve is the complement of the first. Figure 19 shows this type of cam which has been laid out for a rise with harmonic motion for 120 deg. of clockwise revolution, and a rest or dwell

for the remaining part of the 180 deg. With the centers of the two rolls fixed, the second half of the cam contour was formed by taking centers on the first half of the theoretical curve, and with dividers set equal to the distance between roll centers, the centers of the second roll were located by measuring this distance on each diameter across the center of the cam.

90. The Constant-breadth Cam.—The constant-breadth cam functions like the constant-diameter cam, the follower, however, being flat faced. If, in Fig. 19, two lines were drawn through the center lines of the rolls to represent the edges of the follower, the working curve could be found as described in the preceding section by keeping the distance between the edges of the followers constant. This cam is also a positive motion cam and may be laid out for 180 deg. of cam motion.

91. Cylinder Cams.—If the chart shown in the upper portion of Fig. 12 were wrapped around a base cylinder so that the lines 06 and $0'6_1$ coincided on an element of the cylinder, then the curve shown would be the theoretical curve for a cylinder cam. There are a number of machine elements which are cylinder cams by virtue of their form and function, although they may be classified otherwise. A screw thread is cut on a cylinder in the form of a helix, and is identical with a cylinder cam which will give its follower a displacement according to uniform motion. The worm of a worm-and-wheel pair, and the well known screwdriver of the "Yankee" type, are examples of cylinder cams.

A cylinder cam with a horizontal axis may drive its follower to the right or to the left, and the follower may have straight-line displacement as shown by Fig. 2(*b*), or angular displacement as shown by Fig. 2(*c*). The follower roll should be conical in form, and roll in a groove which has inclined sides, in order to avoid slippage between the roll and the driving edge. The inclination of the sides of the grooves is not constant, varying somewhat according to the slope of the cam curve, and it is usually made to fit the groove at the place where the pressure is the greatest. Due to its cone shape the roll will have a thrust away from the cylinder, and this thrust should be provided for by a washer between the roll and the follower.

92. Cylinder Cam with Straight-line Follower.—Figure 20 shows the development of a cylinder cam which turns counterclockwise, and drives its roll follower in a straight line from *a* to *g* with uniform acceleration for 180 deg. of turn, rests for 60 deg., and

finally drives the follower from *h* to *l* with uniform motion while the cam completes its revolution. The cam chart for a cylinder cam is identical with the developed surface if the cylinder dimensions are plotted to scale.

In Fig. 20 the line *ag*, the displacement of the follower, is laid off in 6 divisions of uniform acceleration and retardation, while the cam displacement is 6 unit angles of 30 deg. each, and the theoretical curve is determined by the intersection of the lines. The chart indicates that when the follower rests for a turn of 60 deg. of the cam, the theoretical curve will be on a circle of the cylinder, as shown by the curve at the points where the line *gh* cuts the lines 6, 7, and 8. The follower moves form *h* to *l* with uniform

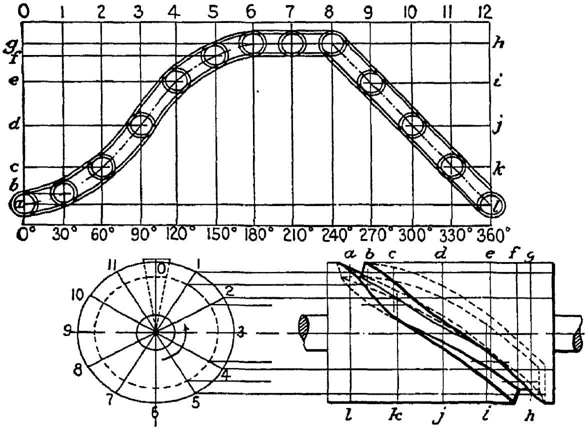


FIG. 20.

motion, and the theoretical curve is found at the intersection of the lines *h*, *i*, *j*, *k*, and *l* with the lines 8, 9, 10, 11, and 0. The developed surface is then wrapped around the cylinder and the curves are transferred by projection and measurement.

93. Cylinder Cam with Swinging Follower.—The design of the cylinder-cam curve for a swinging follower is similar to that described for the straight-line follower in the preceding section. Figure 21 (*b*) shows a cam with the roll at the end of the arm *ob*, swinging about the center *o*. The roll follows the arc *ba* as the roll moves from *b* to *a*.

In laying out the developed surface diagram as shown in Fig. 21(*a*), the center lines of the follower displacements are arcs drawn with *oa* as a radius. It will be noted also that the

center line of the roll coincides with the axis of the cam only when the arm *ob* is in its central position at *c*. When the arm is in its extreme position *ob* or *oa*, the center line of the roll is at a distance *x* away from the axis of the cam, and a correction must be made on the drawing to allow for this displacement.

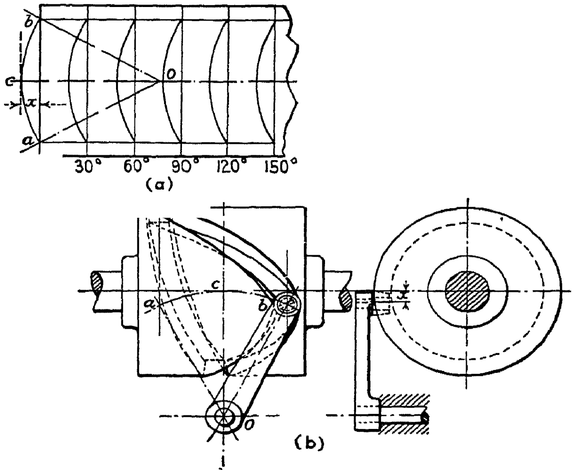


Fig. 21.

94. Making of Cams.—Cams are in common use on automatic machines for timing and controlling various operations. Several cams often function as a unit, and their design is facilitated by the use of a cam-chart diagram. In this way the timing of each movement and the relation of one movement to another is readily depicted.

Cam drawings are made full size and the actual cam curves are sometimes modified by using arcs of circles whenever practicable, in order to machine the cams with ordinary shop tools.

Cams, follower rolls and studs, and surface followers are usually made of a good grade of steel, heat treated and ground. The making of cams requires the skill of an expert toolmaker because the finishing of a cam necessitates handwork.

Problems

1. Show by sketches: (*a*) a single-edge plate cam; (*b*) a double-edge plate cam.
2. Show by sketches three types of cylinder cams.
3. Define and show graphically how the following motions are applied to cams: (*a*) uniform motion; (*b*) harmonic motion; (*c*) uniformly accelerated motion.

4. For uniformly accelerated motion show: (*a*) how the series k, $4k$, $9k$, etc., is developed; (*b*) how the series 0, 3, 5, 7, etc., is found.
5. Assume all circles and curves and show by a sketch a cam and follower, giving the names of all curves and parts.
6. Show by sketches: (*a*) a cam with a reciprocating roll follower; (*b*) a cam with a reciprocating flat-face follower; (*c*) a cam with a swinging-roll follower; (*d*) a cam with a swinging-flat follower.
7. Lay out a cam chart for a cam which is rotating clockwise to give its follower motion as follows:
(*a*) A 3-in. rise in 120 deg. of turn with uniformly accelerated and retarded motion.
(*b*) A rest for 80 deg. of turn.
(*c*) A 3-in. drop to its original position with uniform motion during the remaining angle.
8. Lay out a plate cam with a reciprocating-roll follower which will give the follower motion as follows:
(*a*) A rise of 4 in. with harmonic motion during 120 deg. of clockwise turn.
(*b*) A rest for 90 deg.
(*c*) A drop of 4 in. to the starting position, with uniform motion, during the remaining angle.

Data.—Diameter of base circle is 4½ in.
Diameter of roll is 1¼ in.
Diameter of roll pin is 5⁄16 in.
Width of follower is ⅞ in.
Diameter of camshaft is 1⅜ in.

NOTE: Make a full-size $\left\{\begin{matrix}\text{pencil}\\ \text{ink}\end{matrix}\right\}$ drawing on a 15- by 22-in. sheet, locating the center of the cam shaft 7½ in. from the top border line and in the center of the sheet from right to left.

9. Lay out a plate cam with a reciprocating flat-face follower, the base of the latter making an angle of 90 deg. with the vertical center line of the cam. The specifications are the same as given for the cam in Problem 8. The follower face is to be ¼ in. longer on each end than the length necessary for contact.
10. Lay out the theoretical and working curves for a plate cam which has a reciprocating-roll follower offset $\left\{\begin{matrix}\text{1 in. to the right}\\ \text{1¼ in. to the left}\end{matrix}\right\}$ of the camshaft center.
The follower is to have the following motion:
(*a*) A rise of 2¾ in. with uniformly accelerated and retarded motion during 150 deg. of counterclockwise turning.
(*b*) A rest for 30 deg.
(*c*) A drop of 2¾ in. with uniform motion during the remaining angle.

NOTE: Make a full-size $\left\{\begin{matrix}\text{pencil}\\ \text{ink}\end{matrix}\right\}$ drawing. Employ the method shown in Fig. 15, dividing the large circle into equal angles starting at the point 1. Locate the point c in the center of the sheet.

11. Lay out the theoretical and working curves for a plate cam which has its reciprocating flat-face follower offset $\frac{3}{4}$ in. to the right of the camshaft center. The specifications and note of Problem 10 also apply to this problem.
12. Lay out a cam chart for:
    (*a*) The cam of Problem 10.
    (*b*) The cam of Problem 10, with harmonic motion for the first portion, instead of uniformly accelerated motion.

Note the difference in the theoretical curves for the uniformly accelerated motion and the harmonic motion.

NOTE: Make a pencil drawing full size, the length of the chart to be equal to the circumference of the base circle.

13. Make a $\left\{\begin{matrix}\text{pencil}\\ \text{ink}\end{matrix}\right\}$ drawing of a cam which has a swinging-roll follower similar to the one shown in Fig. 16(*a*). The center *b* moves through 30 deg. from *b* to *d* with harmonic motion for 180 deg. of clockwise rotation of the cam; the center *b* rests for 45 deg. of turn; and *b* then returns to its starting position with uniform motion during 135 deg. of turn. The arm *a*0 is horizontal in its lowest position.

Data.—Diameter of base circle is $5\frac{1}{2}$ in.
Diameter of roll is $1\frac{5}{16}$ in.
Diameter of roll pin is $\frac{1}{2}$ in.
Diameter of $\left\{\begin{matrix}\text{pin } a \text{ is } \frac{5}{8} \text{ in.}\\ \text{hub } a \text{ is } 1\frac{1}{4} \text{ in.}\end{matrix}\right.$
Diameter of $\left\{\begin{matrix}\text{pin } b \text{ is } \frac{1}{2} \text{ in.}\\ \text{hub } b \text{ is } 1 \text{ in.}\end{matrix}\right.$
Diameter of camshaft is $2\frac{1}{2}$ in.
Diameter of cam hub is 4 in.
The key is $\frac{1}{2}$ by $\frac{1}{2}$ in.
The bell-crank angle 0*ab* is 75 deg.
The length *a*0 is $4\frac{1}{2}$ in.
The length *ab* is $4\frac{1}{4}$ in.

NOTE: The drawing is to be full size. Locate the point *c* $8\frac{1}{2}$ in. down from the top border line, and central on the sheet from right and left. Lay off the required motion along the arc *bd*, making the necessary correction.

14. Design a cam with a swinging-arm and crosshead follower like the one shown in Fig. 16(*a*). The specification for the movement of the follower *e* is:
    (*a*) A drop of $2\frac{1}{2}$ in. with harmonic motion during 180 deg. of counterclockwise rotation of the cam.
    (*b*) A rest for 30 deg.
    (*c*) A rise of $2\frac{1}{2}$ in. with uniform motion for 150 deg.

Data.—Diameter of base circle is $4\frac{7}{8}$ in.
Diameter of camshaft is 2 in.
Diameter of camshaft hub is $3\frac{1}{2}$ in.
The key is $\frac{1}{2}$ by $\frac{1}{2}$ in.
The length of link arm *a*0 is $3\frac{3}{8}$ in.

The length of *ag* is 4½ in.
The link 0*g* is horizontal in its original position.
The length of *ge* is 5 in.
The horizontal distance from the center line of cam to the center line of guides is 8¼ in.
Diameter of roll is 1¼ in.
Diameter of roll pin is ½ in.
Diameter of pin at *a* is ⅝ in., at *g* is 7⁄16 in., and at *e* is 7⁄16 in. The hubs are twice the diameter of the pins.
Distance between guides is 1½ in.

NOTE: The drawing is to be full size and in $\left\{\begin{matrix}\text{pencil}\\ \text{ink}\end{matrix}\right\}$. Locate the point *c* 6½ in. from the top border line and 6 in. from the left border line.

15. Design a cylinder cam similar to the one shown in Fig. 21. The swinging-follower arm is 8 in. long and sweeps from *a* to *b* with harmonic motion during 150 deg. of clockwise rotation of the cam, rests for 60 deg., and sweeps from *b* to *a* with uniform motion during the remaining angle.

Data.—Diameter of base cylinder is 5¾ in.
Length of cylinder is 5¼ in.
Chord distance *ab* is 4 in.
Large diameter of roll cone is ¾ in.
Depth of groove is 9⁄16 in.
Shaft diameter is 13⁄16 in.

NOTE: Locate the developed surface in the upper portion of the sheet, and the two other views of the cam, as shown by Fig. 21, are to be drawn as indicated. Make the drawing present a balanced appearance when completed. Show the theoretical and working curves on the developed cam surface, and show the working curves on the other two views. Locate the point *c* off the axis of the cylinder so that the path of the roll will be centrally located with respect to the axis of the cam cylinder.

16. Lay out the theoretical and working curves for a cylinder cam which will move its reciprocating follower 4 in. to the left in two-thirds of a revolution in a clockwise direction, rest for two-thirds of a revolution, and return to its original position in the least number of turns.

Data —Same as for Problem 15.

NOTE: Make a full-sized pencil drawing. Locate the developed surface, and two views so that the drawing will be well balanced on the sheet.

CHAPTER VI

MATERIALS OF CONSTRUCTION

95. The materials of construction used by design engineers are, in the order of their importance: iron in its four forms of cast iron, steel, malleable iron, and wrought iron; copper and the alloys of copper, brass and bronze; light alloys of aluminum; babbitt metals; and miscellaneous materials such as wood, fiber, leather, and mica.

Iron is the most extensively used material for engineering construction. It is one of the most widely distributed metals, but it is not found in its native state except perhaps in meteorites. Iron occurs in the form of various iron minerals, of which the iron oxides (iron ores) are the most important. The iron can be extracted more readily from the iron oxides than from some of the other iron minerals.

It is difficult to realize in an age which regards iron and steel as such important factors in the civilization of the world, that only a few hundred years ago iron was extremely rare, and that as late as the middle of the fourteenth century it was not widely used. Wrought iron, the only iron then used, was probably first made by reducing the ore with charcoal in a furnace similar to the modern puddling furnace, and then hammering it into shape. That wrought iron was known in ancient times is indicated by the fact that a solid column of forged wrought iron at Delhi, India, was probably built in 600 B.C., although the exact date is not known. This column is 16 in. in diameter at the base, 12 in. at the top, and 50 ft. in height. It is in a perfect state of preservation due to its coating of magnetic oxide, and is one of the most interesting relics of the work done by the artisans of antiquity.

96. Classes of Iron.—Pure iron, or ferrite, is an element which is crystalline in structure, and relatively soft. Commercial iron occurs as *pig iron, cast iron, malleable iron, wrought iron,* and *steel.* It is rather difficult to differentiate between these classes, but it may be stated in general that the difference between cast iron and steel is that the latter is malleable when hot. The

difference between steel and wrought iron is that the latter contains some foreign inert material, which is not in combination with the iron, and which in the process of manufacture is drawn into fibers, giving wrought iron a fibrous structure. The foreign materials in steel are in combination with the iron.

97. Pig Iron.—Pig iron is a product obtained from the smelting of iron ores in the blast furnace. Pig iron is a combination, or a

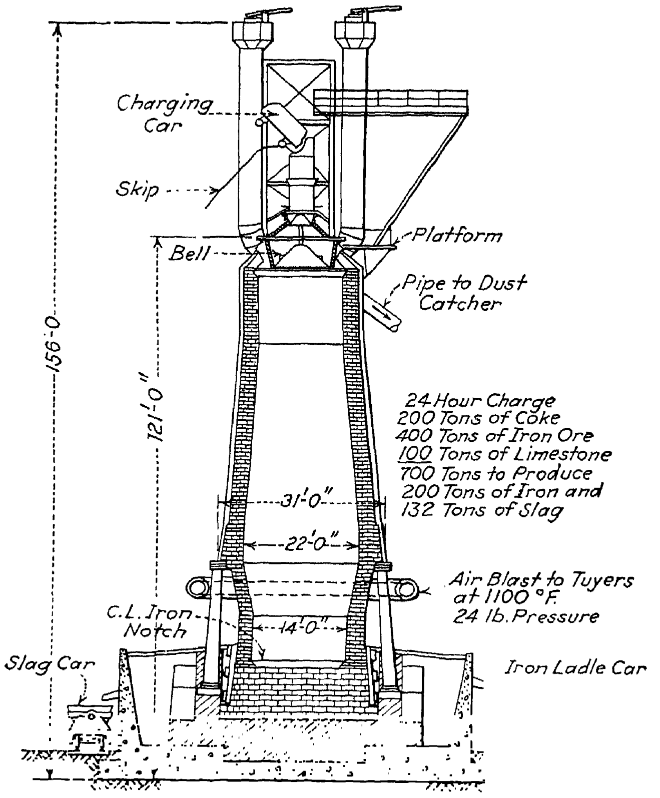


FIG. 1.—Blast furnace.

combination and mixture, of iron, carbon, silicon, manganese, phosphorus, sulphur, and other elements. This product as obtained from the blast furnace is rarely used for any purpose except to be remelted to form cast iron, or to be converted into wrought iron or steel.

98. The Blast Furnace.—The blast furnace is a structure from 60 to 150 ft. high, made of iron and steel, and lined with fire brick,

as shown in Fig. 1. The furnace is charged with alternate layers of coke, lime in the form of limestone, which is used as a flux, and iron ore. The air for the blast, which has been previously compressed and heated, enters near the bottom of the furnace. All of the material which is fed into the furnace leaves at the bottom in fluid form, except the blast-furnace gas which leaves the furnace at the top.

The iron-oxide (ore) is acted upon by the carbon monoxide evolved from the fuel, and is reduced to metallic iron. During the operation more or less carbon and other elements are taken from the fuel, and the iron mass settles to the bottom of the furnace, where it is tapped off at regular intervals into ladle cars, and is converted into steel or is poured into sand molds or into a pig-casting machine. The blast furnace is continuous in operation, and has a capacity of from 100 to 700 tons per day.

The pig-casting machine mentioned above consists of a series of metal molds arranged on a conveyor or on a revolving table. The molds are filled at one station and dumped at a second station. Pig iron made in iron molds or in casting machines is fairly clean, that is, free from sand.

99. Classification of Pig Irons.—In former times iron was graded by the appearance of the fracture of a test bar, and numbered as shown in Table I.

TABLE I.—CLASSIFICATION OF PIG IRON BY NUMBER AND FRACTURE[1]

Number	Name	Appearance of the fracture
	Gray	Dark gray, coarse grain, even and graphitic.
	Gray	Lighter in color than No. 1, with small uneven grain.
	Gray	Lighter in color than No. 2, fine grain.
	Gray	Fine but even grain because of high combined carbon content.
	Mottled	Brilliant white fracture, mottled with gray spots.
	White	Brilliant white, practically free from carbon.

[1] Reprinted by permission from Boylston, H. M. "Iron and Steel," John Wiley & Sons, Inc.

The examination of the fracture gives an indication of the quality of the iron, and is still widely used, but a more rigid classification is by chemical analysis, limiting the percentages of silicon, manganese, sulphur, and phosphorus, according to Table II.

TABLE II.—CLASSIFICATION OF THE CHIEF GRADES OF PIG IRON BY CHEMICAL ANALYSIS (FORSYTHE)[1]

Grade of iron	Silicon, per cent	Sulphur, per cent	Phosphorus, per cent	Manganese, per cent
No. 1 foundry....	2.50 to 3.00	Under 0.035	0.5 to 1.00	Under 1.00
No. 2 foundry....	2.00 to 2.50	Under 0.045	0.5 to 1.00	Under 1.00
No. 3 foundry....	1.50 to 2.00	Under 0.055	0.5 to 1.00	Under 1.00
Malleable........	0.75 to 1.50	Under 0.050	Under 0.20	Under 1.00
Gray forge.......	Under 1.50	Under 1.00	Under 1.00	Under 1.00
Bessemer.........	1.00 to 2.00	Under 0.050	Under 0.10	Under 1.00
Low phosphorus..	Under 2.00	Under 0.030	Under 0.03	Under 1.00
Basic............	Under 1.00	Under 0.050	Under 1.00	Under 1.00
Basic Bessemer...	Under 1.00	Under 0.050	2.00 to 3.00	1.00 to 2.00

[1] FORSYTHE, "The Blast Furnace and the Manufacture of Pig Iron," p. 286, U. P. C. Book Company (*Reprinted by permission*).

100. Chemical Elements in Cast Iron.—After pig iron has been remelted in a cupola or air furnace and cast into any desired form, it is known as cast iron. Cast iron contains from 92 to 96 per cent of iron and from 8 to 4 per cent of other elements. The important elements are those given in Table II, and they occur in combination with the iron, or as a combination and mixture. These elements can hardly be classed as impurities because a number of them give the iron its character and make it of commercial value.

The percentage of *carbon* in cast iron varies from 2½ to 4 per cent. It may be present as combined carbon in the case of white irons, or as combined carbon and free graphite in the case of gray irons. A soft iron may contain as little as 0.20 per cent of combined carbon, but the cast irons suitable for machine purposes contain from 0.40 to 1.00 per cent. Carbon is the most important element present in cast iron, and the hardness of the iron is largely determined by the percentage of combined carbon.

Next to carbon, *silicon* is the most important element in cast iron. In percentages from 0.5 to 3.5 its effect is to cause the carbon to assume the graphitic form, and it therefore acts as a softener. Silicon increases the fluidity of molten iron and the density of castings, and decreases blowholes and shrinkage. When silicon is present up to 5 or 6 per cent it acts as a hardener.

When less than about 0.5 per cent of *phosphorus* is present in cast iron the effect is not marked. If more than 2 per cent of phosphorus is present the cast iron is made weaker and more

brittle. However, high phosphorus makes the iron more fluid and decreases shrinkage, so that castings of intricate shape, such as ornamental stove castings, may be made true to pattern.

Sulphur is considered to be an undesirable element in cast iron, and is usually limited to less than 0.1 per cent. Sulphur tends to make cast iron weak and brittle at high temperatures. Sulphur also promotes the formation of combined carbon, thus causing the iron to become hard and brittle.

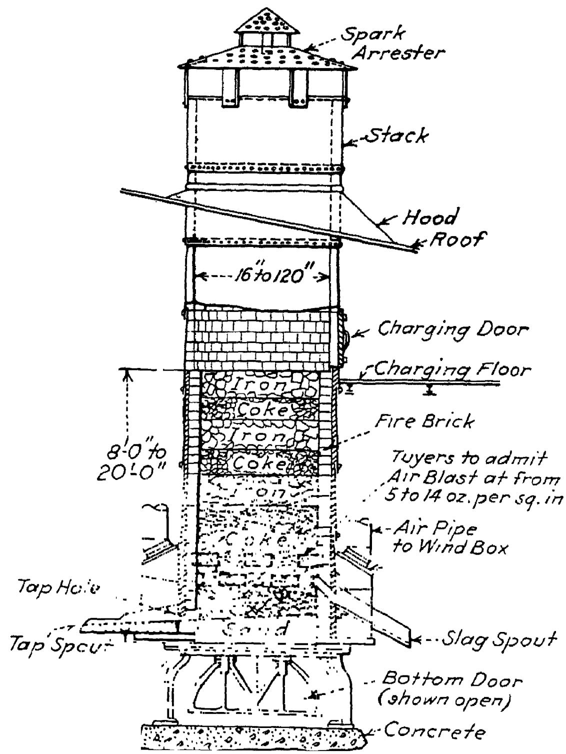


FIG. 2.—Cupola furnace. Manufactured in standard sizes with hourly capacities of ¼ to 30 tons.

Manganese in cast iron, in amounts up to about 2 per cent, combines with sulphur, and thus helps to neutralize the bad effect of sulphur. Manganese promotes the formation of combined carbon, and therefore increases the hardness of cast iron. Since an excess of manganese makes the iron too hard to be machined easily, it is usually limited to 1.00 per cent.

101. Castings.—Iron castings are made by melting pig iron or mixtures of pig iron and scrap cast iron in a cupola or in an air furnace, and then pouring the molten metal into molds. Figure 2 shows a cupola in which alternate layers of fuel and cast iron are charged. Figure 3 shows an air furnace, which, although not as commonly used as the cupola, makes possible a larger quantity of iron for a single tapping. The iron in the air furnace is hotter, under better control, does not absorb sulphur and carbon from the fuel, and is used principally for making castings which are to be malleableized.

102. Advantages and Disadvantages of Cast Iron.—Iron ores are available in such vast quantities, and their reduction to pig iron is done on such a large scale, that the cost of cast iron is relatively low. Castings are easily made and may be given any

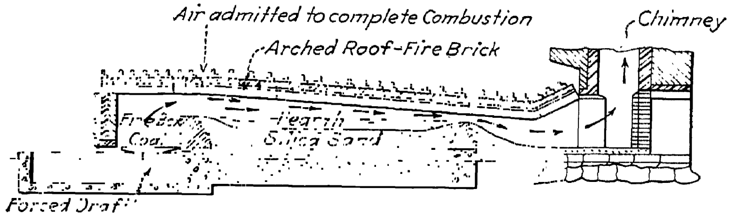


FIG. 3.—Air furnace section 20-ton capacity. Furnace is charged through the removable roof, the hearth is covered and the furnace filled nearly to the roof, leaving a free passage for flame to reach the rear bridge wall.

desired form. Cast iron resists oxidation to some extent, and has a great compressive strength, which allows it to be used to advantage for short compression members.

The disadvantages of cast iron include internal stresses, which are produced when the iron cools and shrinks. Cast iron has a low tensile strength and is brittle, so that it cannot be used under conditions which subject it to much shock. Cast iron cannot be welded or tempered, and in common with all castings, has hidden defects unless the castings are made in a competent manner.

Some of the defects which may be encountered in castings are hard castings, due to improper mixtures or chilling in spots; blowholes and blisters, due to imprisoned gas; cold shuts, due to streams of metal which do not unite properly because of low temperature; checks, due to the prevention of contraction of the casting; internal stresses and warping, due to unequal shrinkage; segregation of sulphur and phosphorus compounds in spots;

coarse grain, due to retarded cooling; and spongy spots, due to too rapid solidification of the iron in the risers.

103. Chilled Castings.—When cast iron is cooled rapidly or *chilled* after pouring, the carbon in the outer surface is retained in the combined form. The outer portions of such castings consist of white iron, which is hard and best suited to resist wear, while the inner portions of the castings are soft and gray, and therefore better able to resist shock.

Chilled castings are made in iron molds, the surfaces being protected by a thin coating of clay wash. The chilling extends to a depth of $\frac{1}{8}$ to $\frac{1}{2}$ in. or more, and chilled castings must be annealed before using to avoid cracking.

104. Annealing of Castings.—*Annealing* consists in bringing a casting to a dull red heat and allowing it to cool slowly. Annealing is practiced for two purposes: (1) to eliminate internal stresses, as in car wheels with chilled rims, and (2) to make gray-iron castings softer so that they may be easily machined.

105. Pickling of Castings.—The scale on the surface of cast iron is hard and destructive to cutting tools, and is frequently removed by *pickling* in a bath of dilute sulphuric acid. A more recent process employs dilute hydrofluoric acid, which removes the silicates which cause the destructive effect on the cutting tools.

106. Sand Blasting.—Pickling is a disagreeable operation, and is sometimes replaced by a *sand-blasting* process. The most recent development for the cleaning of large castings makes use of a hydraulic washer, in which water under high pressure (250 to 400 lb. per square inch) is shot through a nozzle directly against the casting walls, the action of the water and the adhering sand effecting the cleaning. The hydraulic washer has a closed compartment containing the castings, and the operator manipulates the nozzle from the outside while watching the operation through a glass window.

Small castings, when their shape permits and the danger of breakage is not too great, are cleaned in revolving metal drums, in which they are *tumbled* for several hours with hard cast-iron stars, whose abrasive action effects the cleaning.

107. Malleable Cast Iron.—The process of giving white cast iron the desirable property of malleability was sought for by several of the European iron masters. A French physicist, Reaumur, in 1722 described a process which is essentially the

practice followed in Europe by the producers of malleable cast iron. Reaumur's method was to pack castings in pulverized iron oxide (hematite ore), and heat them to a bright-red heat. The heat was maintained for many days and removed all but traces of carbon in the iron. This process could not be applied to castings with thick walls due to the time required for annealing. Castings with thin and thick walls would be completely decarburized in the thin parts but not in the thicker ones. This malleablizing process, called the *white-heart* process, was limited to small castings, and is still used in Germany, France, and England.

In America the malleablizing process does not depend primarily upon the decarburizing process, but upon the fact that annealing changes the combined carbon in white iron to a special form of graphitic carbon called *temper carbon.* The temper carbon is much more finely divided and much more uniformly distributed in the iron than the graphitic carbon, which occurs in flakes in ordinary gray cast iron. The American, or *black-heart* malleable cast iron, is an iron of relatively low carbon and silicon. It has a closely regulated composition, a silvery white fracture, and is glass hard and brittle.

The white-iron castings, after removal of excess material and cleaning by tumbling or otherwise, are packed in annealing pots. The castings may be alone or may be packed with a material like sand, which supports the castings, prevents warping, and to some extent prevents oxidation. The ovens are fired by coal, gas, or oil, and raised to a temperature of from 1550 to 1650° F. The temperature is attained in from 20 to 35 hr., and then maintained for 40 to 75 hr. After slow cooling the castings are removed, the annealing process having made them soft, strong, and ductile.

108. Wrought Iron.—That wrought iron was known in ancient times has already been mentioned, but its production became an industry only during the latter half of the eighteenth century. The industry flourished until the discovery and application of the converter method of making steel caused steel to replace wrought iron for most purposes, because of the relatively low cost of steel.

Wrought iron is produced in the puddling furnace in a pasty condition, and a puddle ball usually weighs about 600 lb. The latest development in the production of wrought iron is the Aston process, which dispenses with the puddle furnace, and produces

the puddle ball by dropping molten steel in a bath of molten slag.

The slag which is not all removed from the puddle ball by squeezing, is well distributed by rolling, and gives the iron its fibrous structure, which distinguishes it from other ferrous products. Due to the presence of sulphur, wrought iron may be brittle or "red short" when hot, and due to the presence of phosphorus it may be "cold short" at ordinary temperatures.

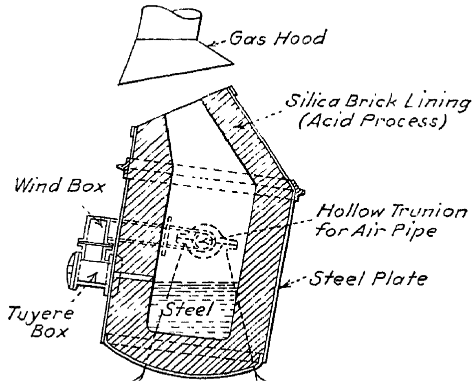
109. Advantages and Disadvantages of Wrought Iron.—Compared with cast iron, wrought iron has a high tensile and transverse strength, and because of its ductility will withstand shock. It can be forged, welded, punched, and riveted. It softens and welds at 1600° F., and can be forged at a still lower temperature. For certain classes of material like pipe and sheets where corrosive resistance is required, wrought iron is sometimes specified in place of steel, although the first cost of steel is lower.

Compared with steel, wrought iron is higher in cost and cannot be hardened like steel by quenching, but it can be case hardened. Wrought iron cannot be cast, because in melting, the slag and iron would not remain intimately mixed, and the product would lose its characteristic properties.

110. The Manufacture of Steel.—The so-called "steel age" began in 1856 with the invention of the converter by the Englishman, Henry Bessemer. An American, William Kelly, had discovered the same process in 1847, but had neglected to apply for a patent. After some litigation Kelly sold his claim to the Bessemer interests. The first Bessemer steel plant was built in 1860 at Sheffield, England, and manufactured high-carbon steel from Swedish pig iron. The Bessemer process employs a converter, shown in Fig. 4, which is a pear-shaped vessel about 20 ft. high and about 10 ft. in diameter at the base. The converter is supported on trunnions, one of which is hollow to permit the injection of an air blast in any position of the converter. The capacity of converters is from 10 to 25 tons.

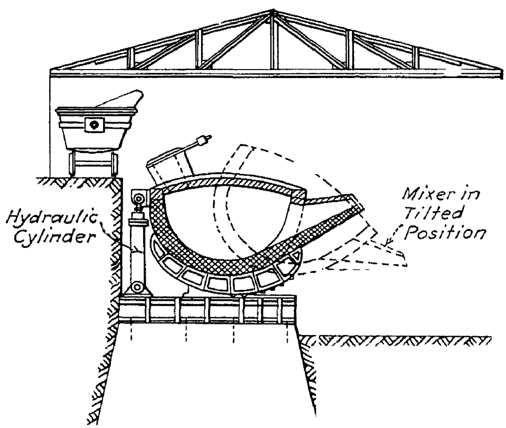
In the manufacture of steel by the Bessemer process the converter is charged, while in a horizontal position, with molten iron taken from a mixer. The mixer, shown in Fig. 5, has a capacity of 200 to 600 tons and is used to equalize the product taken from various casts of the blast furnace. The air blast, at a pressure of 20 to 25 lb. per square inch, is turned on, and the converter is

then turned to an upright position. The "blowing" which then proceeds burns up the carbon, silicon, and manganese.



The converter is turned to the horizontal position to be charged with hot iron from the blast furnace or cupola furnace, it is then turned to the position shown in the figure for blowing

FIG. 4.—Converter.



The mixer is fired by gas to keep the iron hot as it comes from the blast furnaces until the open hearth furnaces or converters are ready to be charged

Some mixers are heated by regenerated heat similar to the open hearth system

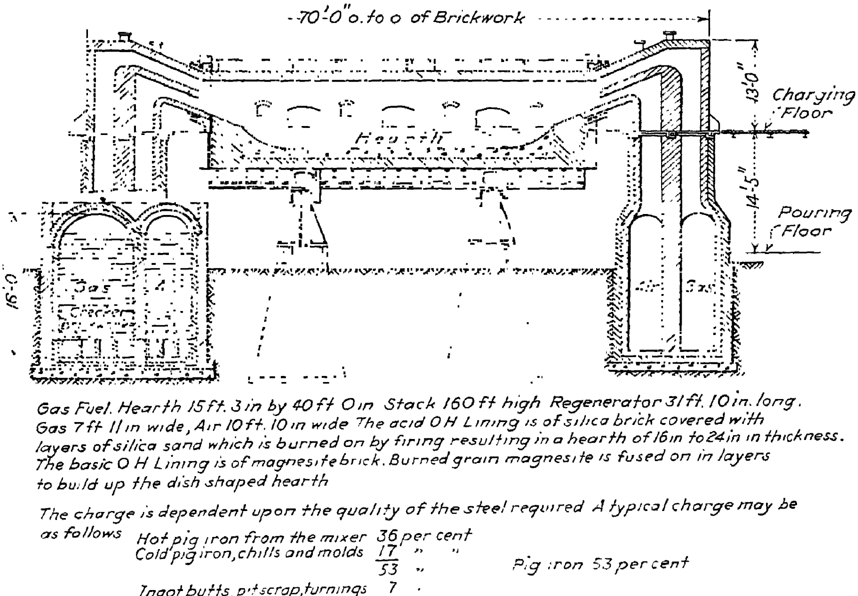
FIG. 5.—200-ton hot metal mixer.

Recarburizers, consisting of a silicon-manganese-carbon alloy, are then added to the melt while the metal is being poured into

ladles, in order that the oxygen may be removed and the metal brought up to the required carbon content.

The original Bessemer process, the *acid Bessemer*, required a pig iron low in phosphorus, because the refractory material used in lining the furnace was acid in character, and the acid slag produced did not eliminate the phosphorus in the iron.

In 1878 Thomas and Gilchrist developed the *basic Bessemer* process, in which the converter is lined with a material basic in character. This process made possible the use of irons high in



Gas Fuel. Hearth 15 ft. 3 in by 40 ft 0 in Stack 160 ft high Regenerator 31 ft. 10 in. long. Gas 7 ft 11 in wide, Air 10 ft. 10 in wide The acid O H Lining is of silica brick covered with layers of silica sand which is burned on by firing resulting in a hearth of 16 in to 24 in in thickness. The basic O H Lining is of magnesite brick. Burned grain magnesite is fused on in layers to build up the dish shaped hearth

The charge is dependent upon the quality of the steel required A typical charge may be as follows

Hot pig iron from the mixer	36 per cent	
Cold pig iron, chills and molds	17 " "	
	53 "	Pig iron 53 per cent
Ingot butts, pit scrap, turnings	7 "	
Sheet Scrap	4 "	
Steel Scrap	35 "	
Ferro-manganese	0.5 "	
Ferro Silicon	0.5 "	Steel scrap 47 " "
	47 0 "	Total metal 100 " "

FIG. 6.—60-ton open-hearth furnace.

phosphorus, because the basic slag unites with the phosphorus and thus effects its partial removal.

111. The Open-hearth Furnace.—Between 1858 and 1868 the English inventor, William Siemens, developed the *open-hearth* process of making steel. The open-hearth furnace, shown in Fig. 6, is charged with pig iron and iron ore, or pig iron, scrap, and iron ore. The metal is melted by the heat obtained from burning gases which have been preheated by passing through checker brick chambers. The regenerative principle is used by

employing a system of valves which permit the waste gases from the furnace to heat these brick chambers, and then allow the gas and air to be preheated by passing through one set of chambers while the waste gases are heating a second set.

The Siemens process employs the principle of oxidation as does the Bessemer process, but in the Siemens process iron ore is used as the oxidizing agent. A greater variety of raw materials may be used in this process, and the elimination of the impurities can be made to take place gradually, thus permitting better control of the product. The capacity of open-hearth furnaces ranges from 40 to 100 tons, and both acid and basic open-hearth furnaces are used.

Bessemer steel finds its principal use in the manufacture of pipe, tubes, plates, and sheets. The open-hearth grade is used for structural steel, and in recent years has supplanted Bessemer steel for nearly all purposes.

112. Crucible Steel.—*Crucible steel* is produced by melting wrought iron and steel scrap in crucibles made of clay and graphite. The metal ingredients are controlled by proper additions, and then cast into ingots or directly made into small castings. Since the crucibles hold only 50 or 100 lb. of metal, this process can be employed economically in the production of only the finest carbon alloy steels, used for tools, cutlery, and springs.

113. Electric-furnace Steel.—The recent application of the electric furnace to the production of steel has resulted in its adoption for making steel where a superior product is required. Any product formerly made by the crucible process can be made by the electric furnace at a lower cost. Complex alloy steels can be made with precision, and also steels free from impurities, which are of great value for electrical apparatus. Large castings can be made from one furnace charge where several crucibles would be necessary if made by the crucible process. Electric-furnace steel is especially adapted for making small castings in which the metal must be free from slags and oxides, and cast at a high temperature. Electric-furnace steel is used extensively by railroads, and in the manufacture of automobiles, aeroplanes, machinery, engines, tools, and guns.

114. Steel.—Steel is iron having various other elements in chemical combination. Some of the differences between cast iron, wrought iron, and steel have been previously mentioned.

Some steels are inherently soft and ductile, and others are hard and brittle. Some steels become hard when cooled rapidly, and others become hard when cooled slowly. In the following sections, steels are classified as carbon and alloy steels.

115. Carbon Steel.—Carbon is the principal hardening element in carbon steel, although silicon and manganese are also present, and may aid in producing desirable qualities. The amount of carbon may vary from a trace to 2.00 per cent, and is usually expressed in "points," a 0.20 per cent carbon steel being called a 20-point carbon steel. Steel increases in tensile strength, hardness, and fusibility with increase of carbon. Ductility, on the other hand, decreases with increase of carbon.

The *silicon* content of carbon steels ordinarily does not exceed 0.50 per cent, and is usually below 0.25 per cent. Silicon is often added to the molten steel to remove oxygen and prevent blowholes. In the manufacture of steel castings the deoxidizing effect of silicon is important because it quiets the melt and insures soundness. Silicon is about one third as effective as carbon in increasing hardness.

The *manganese* content of carbon steel ordinarily does not exceed 1.00 per cent, and usually ranges from 0.30 to 0.60 per cent. Manganese has a strong affinity for oxygen and sulphur, and aids in withdrawing these elements into the slag. Manganese increases hardness and brittleness, and in steel castings promotes soundness. In the higher carbon steels manganese increases the solubility of carbon in iron, and has a tendency to cause the metal to crack when suddenly quenched.

Phosphorus is considered an injurious element in steel, and is usually specified to be below 0.05 per cent. Phosphorus makes steel brittle at ordinary temperatures, and decreases the resistance to shock. The compounds of phosphorus have a tendency to segregate in spots.

Sulphur is also considered to be an undesirable element in steel, and is usually specified to be under 0.05 per cent. Sulphur makes steel brittle when hot, and the presence of sulphur as high as 0.07 or 0.08 per cent makes welding difficult.

116. Alloy Steels.—Alloy steels are very largely used and owe their properties to the combination, with iron and carbon, of elements such as *nickel, chromium, vanadium, manganese,* and *tungsten*. *Nickel* in quantities from 3.5 to 4 per cent increases the ductility and toughness as compared with a plain carbon

steel of equal strength. Nickel steels are very commonly used for machine parts after having received a heat treatment of quenching and tempering. Nickel steels containing from 30 to 35 per cent nickel have high strength and ductility and good resistance to corrosion.

Manganese steel containing from 10 to 14 per cent of manganese can be given some very remarkable properties with proper heat treatment. It is very tough and has great resistance to abrasion. It is used for rails on curves, for frogs and switches, for vaults and safes, and for crusher jaws and rolls.

Chrome steels contain from 0.5 to 2 per cent of chromium. The steel is hard, has a high strength, and good toughness. It is used for chisels, drills, and saw blades. *Chrome-nickel* and *chrome-vanadium* steels are largely used for automobile springs and shafts.

Vanadium steels contain from 0.10 to 0.20 per cent of vanadium, and have great strength and hardness. They are used for forgings, automobile axles, and springs.

The *high-speed steels* are iron alloys of chromium, vanadium, and carbon, containing from 15 to 20 per cent of tungsten. By a proper heat treatment these steels become very hard, and retain their hardness even up to a red heat. High-speed steels are used for lathe and planer tools, milling cutters, and where heavy cuts develop high temperatures in the tools.

117. Copper.—Copper in its native state is widely distributed, and has for this reason been used alone or alloyed with tin or zinc from the earliest times. Commercial copper is never pure copper, but is chemically combined with impurities like iron, red oxide of copper, oxide of antimony, tin, and lead.

Copper is soft, very ductile, very malleable, and can be hammered or rolled into sheets or drawn into wire. It has great conductivity for heat and electricity, it can be forged hot or cold, but it cannot be easily welded except electrically. Joints in this material may be "brazed," and are only slightly weaker than the solid sheet. Cold working increases the strength of copper, but also makes it harder and more brittle. The effects of cold work may be removed by annealing, which consists in heating the copper to a temperature of 500 to 800° F., and quenching in water. Copper does not expand when changing from the molten to the solid state, so that it is difficult to get sharp and sound

castings. A little phosphorus added to the metal before pouring improves the castings and also increases its strength.

118. Brass.—An alloy of copper and zinc is known as brass, and one of copper and tin is known as bronze. Both of these alloys may have other constituents besides the ones mentioned, so that the terms brass and bronze are used somewhat loosely. The percentage of zinc in the brasses varies between 10 and 40 per cent. Brass is harder than copper, very ductile and malleable, and may be rolled into sheets or drawn into wire. It may be "struck up" in dies, or "spun" into various shapes when cold. It is used in the construction of scientific apparatus, cartridge cases, bearings, parts of machines, and in places where resistance to corrosion is a factor.

Delta metal is a brass containing a certain amount of iron. It has great tenacity, may be worked hot or cold, and may be brazed. It is used for making special shapes by hot stamping, and in places where strength and resistance to corrosion are desired.

119. Bronzes.—*Phosphor bronze* is an alloy of copper, tin, and zinc, which has been fluxed with 0.1 to 4 per cent of phosphorus to clear the alloy of oxides. Phosphor bronze has great strength and soundness, and has replaced iron and steel for propellers, pump rods, and valves, where resistance to corrosion is important.

Manganese bronze is an alloy of bronze and ferro-manganese, having qualities similar to those of phosphor bronze. It is about as strong and tough as mild steel, and may be forged at a cherry-red heat. Manganese bronze containing a large proportion of zinc may be forged and rolled hot or cold, and is used for nuts, bolts, rods, plates, and sheets.

Aluminum bronze contains from 7 to 10 per cent of aluminum, and gives the alloy great strength and toughness. The alloy casts readily, can be worked easily, and can be forged at a red heat. It is used for valves, pump rods, propellers, engine gears, springs, and all places where strength and resistance to corrosion are important.

120. Babbitt Metal.—*Babbitt metal* may mean any one of a number of alloys of tin, antimony, lead, copper, iron, and arsenic. The principal use of this alloy is for the lining of bearings.

121. Wood.—The principal woods used in machine construction are *white ash*, *beech*, *white oak*, and *maple*. White ash is tough, elastic, and durable when protected from the weather,

and is used for tool handles and wagon tongues. Beech is smooth and of compact grain, and is used for the teeth of mortise gears. White oak is hard, tough, and strong, and is used for agricultural implements and wagon stock. Hard maple is tough and strong, and is used for ship and car construction.

NOTE: For a specification of any material that is used in design engineering, the student is referred to the specifications of the American Society for Testing Materials, known as the A.S.T.M. Standards.

CHAPTER VII

FUNDAMENTAL MECHANICS

122. Design consists in the determination of the form and size of the various parts of a structure or machine, preceding its fabrication. A *structure* is a combination of resistant bodies so connected that the application of force to one member produces definite forces in the other members. One characteristic of a structure is the absence of appreciable relative motion. A bridge is an example of a structure. A *machine* is a combination of fixed and movable parts, interposed between the source of power and the work to be done, for the purpose of adapting the one to the other. Examples are the steam engine, which converts energy into useful work; and the dynamo, which converts mechanical work into electrical energy.

Kinematics is the name given to the study of motion without regard to the forces which cause the motion. *Motion* means a change of position. The design engineer is interested in the application of the several kinds of motion to machine parts, such as *plane*, *helical*, and *spherical motion*.

The motion of a point or body may be classified as either *rectilinear* or *curvilinear* motion. Rectilinear motion is motion in a straight line, and curvilinear motion is motion in a curved line.

The *position* of a point may be specified by its distance from some fixed origin in its path.

The *displacement* of a point for any interval of time is the distance from its position at the beginning of the interval to its position at the end of the interval. The displacement is a straight line in either rectilinear or curvilinear motion.

The *velocity* of a point is the rate of change of its displacement with respect to time. In a rectilinear motion, velocity is *uniform* when equal displacements take place in equal intervals of time. All other motions have *non-uniform* velocity. A velocity in curvilinear motion cannot be uniform, because the displacements cannot be the same in direction even though they are otherwise equal.

The *acceleration* of a point is the rate of change of velocity with respect to time.

In rectilinear motion displacements, velocities, and accelerations have magnitude, and either a plus or minus direction; while in curvilinear motion displacements, velocities, and accelerations have magnitude, and the direction may be plus or minus at any angle whatsoever.

Translation is a motion of a body such that all lines in the body remain fixed in direction. The side rod of a locomotive moves with a motion of translation. A body sliding in a straight line on a table also moves with a motion of translation. Obviously rectilinear motion is a type of translation.

Rotation is a motion of a body such that one line in the body or its extension remains fixed. This fixed line is the axis of rotation, and all points in the body describe circles about the axis of rotation.

Plane motion of a body is one in which every point in the body remains at a constant distance from a fixed plane. A cylinder rolling down a plane in a straight line is an illustration of plane motion.

Helical motion and *spherical* motion are two special cases of motion in three dimensions. Helical motion is traced by a point which moves uniformerly around an axis and at the same time moves uniformly parallel to the axis. Spherical motion is such that only one point in the body is fixed; each point, excepting the fixed one, moving on the surface of a sphere.

Work is said to be done by a force when it has a component parallel to the displacement which occurs, and the amount of work is the product of the displacement times the working component of the force. The time element does not enter in work.

Power is the rate of doing work with respect to time. The average power would be the work done divided by the corresponding interval of time. The engineering unit of power is called *horsepower*, and is the work done in 1 min. in raising a 1-lb. weight through a distance of 33,000 ft. Horsepower is, therefore, the work done in foot-pounds divided by 33,000. The electrical unit is the *kilowatt*, or 1,000 watts, and there are 746 watts in 1 horsepower.

123. Stresses and Strains.—*Load* is defined as any external force acting upon a machine or structural member. When a

load is steady it is called a *dead* load, while if it is variable it is called a *live* load. A live load may be *suddenly applied* without *impact*, or may be applied with impact by falling through a distance.

When external forces are applied to a body there are set up at various sections of the body internal forces which resist the external forces. The internal force acting on any cross-section is called the *stress* at that section. Figure 1 shows a simple clamp, and when this device is in use there are forces *PP*, acting as shown. At a section such as *AA* there is a force exerted by one portion of the clamp on the adjacent portion, and either of these forces is called the stress at the section.

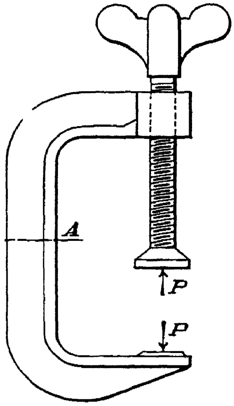


FIG. 1.

Stresses may be simple or they may be quite complex, and they may be caused by a variety of actions. A body may be stressed by a load which it supports, by a load which falls on it, by magnetic forces, and by changes in temperature.

By *intensity of stress* is meant the stress per unit of cross-sectional area. Intensity of stress is usually expressed in pounds per square inch in the United States, and is often called unit stress, although the appropriateness of this term is questionable. In England intensity of stress is expressed in long tons per square inch, and on the continent in kilograms per square

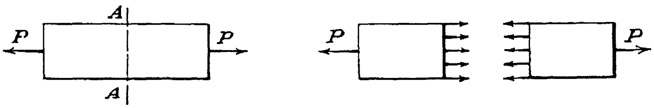


FIG. 2.

centimeter. Intensity of stress may be uniform over a surface, or it may vary from point to point.

When a body is subjected to external forces there is a change in its shape or size, or both, and this change is called *strain*. The general term for the change of form is called *deformation*. By *unit strain* is meant the charge in length or volume divided by the original length or volume in which the strain accurred.

124. Tension and Compression.—In Fig. 2 is shown a case of axial *tension*, in which the body is subjected to forces which tend

to stretch it. For such a case the stress is called tension and the strain is called *elongation* or *tensile deformation*.

In Fig. 3 is shown a case of axial *compression*, in which the body is subjected to forces which tend to squeeze it. For such a case the stress is called compression and the strain is called *shortening* or *compressive deformation*.

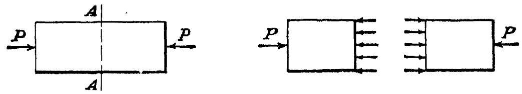


Fig. 3.

In both Figs. 2 and 3, if the load P is applied along the axis of the bar, the stress at a section such as AA will be uniformly distributed, and for equilibrium:

$$P = SA, \tag{1}$$

in which P denotes total load or external force, in pounds.
A denotes area of cross-section at right angles to the external force, in in.^{2}
S denotes unit stress, in pounds per square inch.

125. Shear.—In Fig. 4 the body consists of two plates connected by a rivet. In this case the forces PP tend to cut the

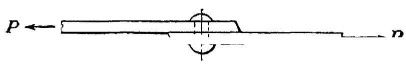


Fig. 4.

rivet in two or shear it, and the stress on the rivet cross-section is called *shear* stress. Here also, for equilibrium:

$$P = S_sA, \tag{2}$$

in which P denotes total load or external force, in pounds.
A denotes area of cross-section, in in.^{2}
S_s denotes average unit shear stress, in pounds per square inch.

It should be noted that for tension and compression the area involved in formula (1) is at right angles to the external force, while for shear the area involved in formula (2) is parallel to the external force.

126. Working Stress and Factor of Safety.—The ultimate strength of the materials used in engineering construction has

TABLE I.—ULTIMATE STRENGTH OF MATERIALS
(Average values)

Material	Tension, pounds per square inch	Compression, pounds per square inch	Shear, pounds per square inch	Weight, pounds per cubic inch
Cast iron, gray, ordinary	18,000	90,000	18,000	0.26
Best grade	25,000	100,000	25,000	
Malleable	45,000	100,000	45,000	0.28
Wrought iron, parallel to fibers	48,000	35,000[a]	36,000	0.28
Perpendicular to fibers	35,000	......	28,000	
Steel, forged or rolled	......	......	......	0.28
0.08 to 0.15 per cent C, structural	55,000 to 65,000	35,000 to 40,000[a]	45,000 to 48,000	
0.15 to 0.40 per cent C, medium	65,000 to 80,000	40,000 to 50,000[a]	60,000	
0.40 to 0.70 per cent C, hard	80,000 to 110,000	50,000 to 55,000[a]	65,000	
0.70 to 1.15 per cent C, spring	110,000 to 150,000	55,000 to 80,000[a]	85,000	
Nickel, heat treated	110,000	90,000[a]	80,000	
Castings	60,000 to 80,000	25,000 to 35,000[a]	50,000 to 60,000	
Brass, soft, rolled	50,000	15,000[b]	......	0.30
Castings	30,000	5,000[b]		
Bronze, cast, ordinary	30,000	10,000[b]	......	0.31
Cast, aluminum	60,000	25,000[b]		
Cast, manganese	70,000	30,000[b]		
Cast, phosphor	35,000	12,000[b]		
Rolled, phosphor	65,000	63,000[b]		
Rolled, Tobin	70,000	42,000[b]		
Copper, castings	22,000	5,000[b]	......	0.32
Rolled, forged	30,000	20,000[b]		
Hard-drawn wire	50,000	30,000[b]		
Aluminum, castings	13,000	5,000[b]	......	0.108
Rolled	20,000	15,000[b]		
Stone	......	6,000	......	0.093
Brick	......	3,000	......	0.069
Concrete	300	3,000	......	0.087
Leather	4,000	......	......	0.035
Wood				
White oak, parallel to grain	14,000	3,500	1,000	0.028
Across grain	500	900[b]		
White pine, parallel to grain	10,000	3,000	600	0.017
Across grain	250	300[b]		
Southern long leaf, parallel to grain	......	4,000	1,100	0.023
Across grain	275	500[b]		

[a] The ultimate strength in compression for ductile materials is usually taken as the yield point of the material. The values marked are yield points. The bearing values for pins and rivets may be much higher, and for structural steel is taken as 90,000 lb. per square inch.

[b] The values thus marked are elastic limits.

been obtained experimentally, and from these values the breaking strength of a machine or structure may be computed approximately. It is evident, however, that the stress induced in any part of a machine or structure should be considerably less than the breaking load. The unit stress which may be safely used under working or operating conditions is called *safe working stress.* Engineering experience has determined what unit stresses may be used as safe working stresses. The ultimate strength of a material divided by the safe working stress is called *factor of safety.* The proper factor of safety to be used in a given case depends upon the reliability and uniformity of the material, and also upon whether the load which is applied is a dead load, repeated load, or an impact load.

Table I gives the ultimate strengths of a variety of materials, and Table II indicates some of the factors of safety used under different conditions.

TABLE II.—FACTORS OF SAFETY
(Average values)

Material	Kind of load		
	Steady or dead, producing stress of one kind only	Varying or live, producing equal alternate stresses of different kind	Shock
Wrought iron, mild steel, structural steel and machinery steel	4	6	10
Hard steel	5	6	15
Cast iron and brittle metals	6	10	15
Timber	6	10	20
Masonry and brickwork	10	15	

127. Shear Stress Produced by Axial Tension or Compression.—It can be shown[1] that when a body is subjected to axial tension or compression there is produced on every oblique plane (see Fig. 5) a shearing unit stress whose value is:

$$S_s = \frac{P}{2A} \sin 2\theta, \qquad (3)$$

[1] MAURER and WITHEY, "Strength of Materials," p. 8.
BOYD, "Strength of Materials," p. 51.

in which P denotes axial load in tension or compression, in pounds.

A denotes cross-sectional area at right angles to P, in in.^{2}

θ denotes the angle which the oblique plane makes with the axis of the body.

S_s denotes unit shear stress parallel to the oblique plane, in pounds per square inch.

The intensity of this unit shear stress reaches a maximum value when θ is 45 deg., and S_s is then equal to $\frac{P}{2A}$.

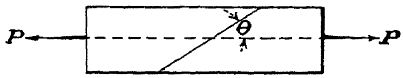


Fig. 5.

In Fig. 5 the unit tensile or compressive stress normal to the oblique plane is:

$$S_t \text{ or } S_c = \frac{P}{A}\sin^2 \qquad (4)$$

This unit stress is always less than that on the plane at right angles to the load P.

128. Tension and Compression Produced by Shear Stress.—When a body such as that shown in Fig. 6 is subjected to shearing forces QQ on the two end faces, it is evident that for equilibrium there must be forces PP on the top and bottom faces. Furthermore, it can be shown[1] that if the unit shear stress due to Q is S_s then the unit shear stress due to P is also S_s. The unit stresses produced on any oblique plane at an angle θ by the tangential force R_s and the normal force R_n are:

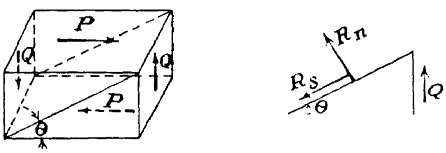


Fig. 6.

$$S_s' = S_s \cos 2\theta, \qquad (5)$$
$$S_n = S_s \sin 2\theta, \qquad (6)$$

in which S_s denotes unit shear stress applied to the body, in pounds per square inch.

θ denotes the angle which the oblique plane makes with the horizontal.

[1] Maurer and Withey, "Strength of Materials," p. 10.
Boyd, "Strength of Materials," p. 55.

S'_s denotes unit shear stress parallel to the oblique plane, in pounds per square inch.

S_n denotes unit normal stress perpendicular to the oblique plane, in pounds per square inch.

It can also be shown that the normal unit stress S_n becomes a maximum when θ is 45 deg. and 135 deg., and its value is then equal to the original shearing unit stress S_s applied to the body. In Fig. 6 the normal unit stress on the 45-deg. plane would be compressive, and on the 135-deg. plane it would be tensile.

A body subjected to twist, as will be shown later, is subjected to shearing unit stresses. A striking test of the above theory may be made by twisting a piece of chalk to destruction by applying a twisting couple at each end. For a brittle material like chalk the breaking strength in tension is less than that in either shear or compression. Since the chalk is subjected to equal unit stresses in tension, compression, and shear, it will fail according to the weakest ultimate strength, which is tension. But if it fails in tension, it should fail on a line making 45 deg. with the axis of the chalk. If the experiment is performed, holding an end of the chalk in each hand, failure will occur as suggested.

129. Stress-deformation Diagram.—In Fig. 7 is shown a *stress-deformation* curve of a ductile material like soft steel subjected to tension. The data for such a curve may be obtained by subjecting a specimen to tension in a testing machine, and measuring the loads and corresponding elongations. From the loads and elongations the unit stresses and the unit strains may be computed.

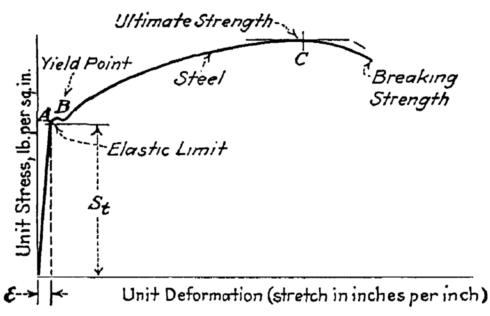


FIG. 7.—Stress-deformation for soft steel.

For a material like steel it is found that the unit stresses are proportional to the unit elongations up to a point A on the diagram. The unit stress corresponding to this point is called the *proportional elastic limit*, and the material is said to be elastic up to this unit stress.

Slightly beyond the elastic limit the material yields when it is ductile, and the deformation increases with no appreciable

increase in unit stress. The unit stress corresponding to this point B on the diagram is called the *yield point.*

Beyond the yield point the stress increases again with increase of deformation until the highest unit stress is reached at C, which is called the *ultimate tensile strength.* For a ductile material the specimen will begin to reduce in cross-section, or *neck down,* at the ultimate, and the final stress at failure, the *breaking unit stress,* may be considerably less than the ultimate strength. All the unit stresses for the stress-deformation curve are calculated on the basis of the original area.

For unit stresses within the proportional elastic limit the ratio of unit stress to unit deformation is constant, and this ratio is called the *modulus of elasticity* or *Young's modulus.*

The fact that for most materials at low stresses the unit stress is proportional to the unit deformation, makes it one of the fundamental assumptions in deriving formulas for calculating the unit stress and the unit deformation. This phenomenon of proportionality is known as *Hooke's Law,* in honor of the man who first observed and reported it.

Within the elastic limit the following relations exist:

$$E = \frac{S}{\epsilon} = \frac{PL}{Ae} = \frac{SL}{e} = \frac{P}{A\epsilon}, \tag{7}$$

in which E denotes modulus of elasticity, in pounds per square inch.
P denotes total load, in pounds.
L denotes length over which deformation occurs, in inches.
e denotes total deformation in the length L, in inches.
ϵ denotes unit deformation, in inches per inch.
S denotes unit stress, in pounds per square inch.

130. Poisson's Ratio.—Within the elastic limit a specimen under tension has its diameter slightly decreased, and under compression slightly increased. The ratio of the unit lateral deformation to the unit longitudinal deformation is called *Poisson's ratio.* This ratio has an average value of about 0.25 for cast iron and steel, and it can be shown[1] that the relation between the

[1] MAURER and WITHEY, "Strength of Materials," p. 22.
BOYD, "Strength of Materials," p. 57.

modulus of elasticity in tension or compression and the modulus of elasticity in shear is:

$$E_s = \frac{E}{2(1 + \lambda)}, \tag{8}$$

in which E_s denotes modulus of elasticity in shear, in pounds per square inch.

E denotes modulus of elasticity in tension or compression, in pounds per square inch.

λ denotes Poisson's ratio.

When Poisson's ratio is 0.25, E_s is two-fifths of E.

Table III shows average values of modulus of elasticity for some of the common metals.

TABLE III.—VALUES OF MODULUS OF ELASTICITY

Material	Modulus of elasticity (pounds per inch²)	
	Tension or compression E	Shear E_s
Aluminum....	9,000,000	3,700,000
Brass.........	13,000,000	5,000,000
Bronze.......	13,000,000	5,000,000
Bronze Mn....	16,000,000	6,000,000
Copper, cast...	15,000,000	
Copper, drawn	17,000,000	6,000,000
Iron, cast.....	15,000,000	6,000,000
Iron, mall.....	22,000,000	8,800,000
Iron, wrought.	27,000,000	11,000,000
Steel.........	30,000,000	12,000,000
Zinc..........	12,000,000	

131. Resilience.—By *elastic resilience* of a material is meant the energy which can be absorbed for stresses below the elastic limit. When the material is stressed to its elastic limit the corresponding resilience per cubic inch is called the *modulus of resilience*.

The elastic resilience in inch pounds is evidently given by the following formula:

$$\text{Resilience} = \tfrac{1}{2}Pe$$

in which P denotes the load applied, in pounds.

e denotes the corresponding deformation, in inches.

Substituting SA for P and ϵL for e it follows:

$$\text{Resilience} = \tfrac{1}{2}S\epsilon \cdot AL.$$

Hence:

$$\text{Resilience per cubic inch} = \tfrac{1}{2}S\epsilon = \frac{S^2}{2E}. \tag{9}$$

132. Flexure or Bending.—When a force acts along the axis of a member it produces *direct stress* in tension or compression. When a force acts with a lever arm, as shown in Fig. 8, it produces a *moment*. The magnitude of the moment with respect to a line is the product of the force and the perpendicular lever arm or moment arm. Since force is usually expressed in pounds, and distance in feet or inches, moment will be expressed in foot-pounds or inch-pounds.

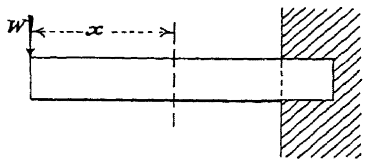


FIG. 8.

In Fig. 8 any section of the bar at a distance of x from W would be subjected to a moment of Wx. It is also evident that in this bar every cross-section is subjected to a shear of W.

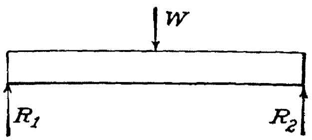


FIG. 9.

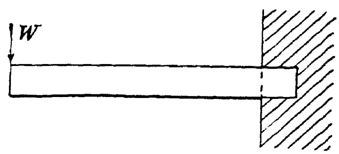


FIG. 10.

A machine or structural element which is subjected to loads and reactions transversely to its long dimension is called a *beam*. Beams may be classified by the manner in which they are

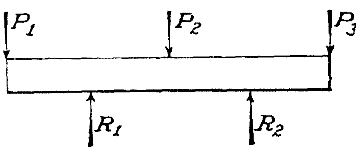


FIG. 11.

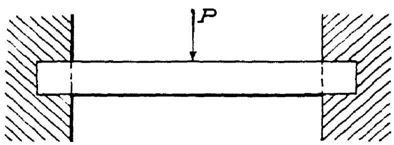


FIG. 12.

supported, thus a beam simply supported at either end, as in Fig. 9, is called a *simple beam*. A beam fixed at one end and free at the other, as in Fig. 10, is called a *cantilever beam*. A beam overhanging its supports, as in Fig. 11, is called an *overhanging*

beam. A beam which is *restrained* or *fixed* at either end, as in Fig. 12, is called a restrained or *fixed-ended beam.* A beam resting on more than two supports, as in Fig. 13, is called a *continuous beam.*

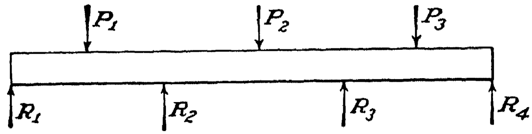


FIG. 13.

Experiment has shown that when a horizontal beam, supported at the ends, is bent under vertical loads and reactions (a common case), the fibers on the upper side of the beam are shortened, and those on the lower side are elongated. Somewhere between the top and bottom fibers is a surface at which the fibers are neither shortened nor lengthened, and this is called the *neutral surface.* The intersection of the neutral surface with any cross-section is called the *neutral axis.* Experiment has shown that the

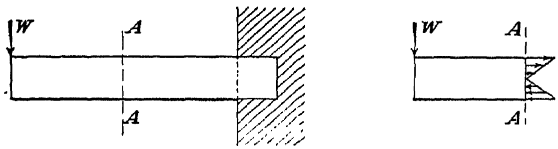


FIG. 14.

deformations in the fibers are directly proportional to their distances from the neutral surface, and therefore, *within the elastic limit,* the unit stresses are also directly proportional to the distances from the neutral surface. Due to bending, therefore, the stress distribution at the cross-section of a beam is as shown in Fig. 14.

It can be shown[1] that the relation between the bending moment in a beam and the unit fiber stress in tension or compression is:

$$M = \frac{SI}{c} \quad \text{or} \quad S = \frac{Mc}{I}, \tag{10}$$

in which M denotes bending moment at the given section where the unit stress is wanted, in inch-pounds.

[1] MAURER and WITHEY, "Strength of Materials," p. 123.
BOYD, "Strength of Materials," p. 120.

c denotes distance from the neutral surface to the extreme fiber, in inches.

I denotes moment of inertia of the cross-section at which the stress is wanted, in in.^{4}. I is taken with respect to the neutral axis.

S denotes unit stress, in pounds per square inch.

It should be remembered that formula (10) may be used for calculating the fiber unit stress at the outer fiber of a beam or at any other fiber. Since the unit stress is proportional to the distance from the neutral surface, if S' and c' are respectively the unit stress at, and the distance to, the fiber in question, then:

$$\frac{S}{S'} = \frac{c}{c'} \quad \text{or} \quad \frac{S}{c} = \frac{S'}{c'}$$

It is evident that formula (10) is of general application to any fiber.

In formula (10) the values of I and of c depend upon the size and shape of the beam cross-section, and the term I/c is called *section factor* or *section modulus*.

133. Shear in Beams.—It is evident in Fig. 14 that a total shear equal to W exists at every cross-section of the beam. It can be shown[1] that the shearing unit stress in a beam is not uniformly distributed over the cross-section, but varies according to the following formula (see Fig. 15).

$$S_s = \frac{V}{Ib}a'y', \tag{11}$$

in which S_s denotes shearing unit stress at the fiber a distance y from the neutral axis, in pounds per square inch.

V denotes total shear at the cross-section where the unit stress is wanted, in pounds.

I denotes moment of inertia of the entire cross-section with respect to the neutral axis, in in.^{4}.

b denotes *net* width of the cross-section at the fiber where the unit stress is wanted, in inches.

a' denotes area above or below the fiber where the unit stress is wanted, in in.^{2}.

[1] Maurer and Withey, "Strength of Materials," p. 131.
Boyd, "Strength of Materials," p. 207.

y' denotes distance from the neutral axis to the centroid of the area a' in inches.

denotes (see Fig. 15) distance from the neutral axis to the place where the unit stress is wanted in inches.

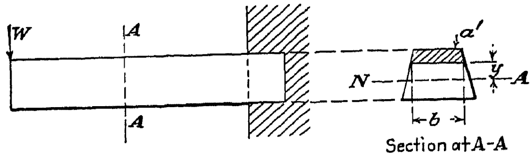
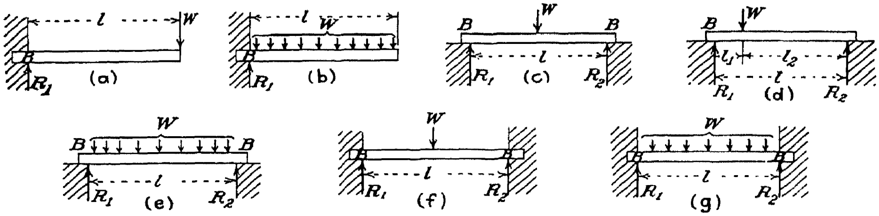


Fig. 15.

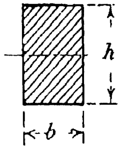
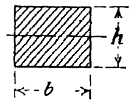
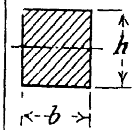
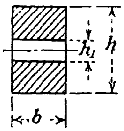
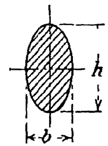
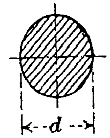
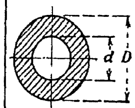
In beams of ordinary length the shearing unit stress is usually not of importance compared with the bending unit stress. It is

Table IV.—Bending Moments and Shearing Forces in Beams



Case	Kind of beam	Maximum bending moment	Maximum shearing force at B	Reaction R_1	Reaction R_2
a	Cantilever, load at the end..................	$M = Wl$	W	W	
b	Cantilever, uniformly loaded.............	$M = \frac{Wl}{2}$	W	W	
c	Simple, load at center	$M = \frac{Wl}{4}$	$\frac{W}{2}$	$\frac{W}{2}$	$\frac{W}{2}$
d	Simple, load off center	$M = W\frac{(l_1 \times l_2)}{l}$	$\frac{Wl_2}{l}$ or $\frac{Wl_1}{l}$	$\frac{Wl_2}{l}$	$\frac{Wl_1}{l}$
e	Simple, uniformly loaded.............	$\frac{Wl}{8}$	$\frac{W}{2}$	$\frac{W}{2}$	$\frac{W}{2}$
f	Restrained, load at center..............	$\frac{Wl}{8}$	$\frac{W}{2}$	$\frac{W}{2}$	$\frac{W}{2}$
g	Restrained, uniformly loaded.............	$\frac{Wl}{12}$	$\frac{W}{2}$	$\frac{W}{2}$	$\frac{W}{2}$

TABLE V.—MOMENT OF INERTIA, SECTION MODULUS, RADIUS OF GYRATION, OF COMMON CROSS-SECTIONAL SHAPES

Shape of section								
Area..............	A	bh	bh	bh	$b(h - h_1)$	$\frac{\pi bh}{4}$	$\frac{\pi d^2}{4}$	$\frac{\pi}{4}(D^2 - d^2)$
Moment of inertia.	I	$\frac{bh^3}{12}$	$\frac{bh^3}{12}$	$\frac{bh^3}{12}$	$\frac{b}{12}(h^3 - h_1^3)$	$\frac{\pi bh^3}{64}$	$\frac{\pi d^4}{64}$	$\frac{\pi(D^4 - d^4)}{64}$
Section modulus...	$\frac{I}{c}$	$\frac{bh^2}{6}$	$\frac{bh^2}{6}$	$\frac{bh^2}{6}$	$\frac{b}{6}\left(\frac{h^3 - h_1^3}{h}\right)$	$\frac{bh^2}{10.2}$	$\frac{\pi d^3}{32}$	$\frac{\pi(D^4 - d^4)}{32D}$
Radius of gyration.	$\sqrt{\frac{I}{A}}$	$0.289h$	$0.289h$	$0.289h$	$0.289 \times \sqrt{\frac{(h^3 - h_1^3)}{h - h_1}}$	$\frac{h}{4}$	$\frac{d}{4}$	$\frac{\sqrt{D^2 + d^2}}{4}$

only when beams are very short that the shear may become the determining factor for strength.

Table IV shows the bending moments, shearing forces, and reactions for some of the more common loadings on beams.

Table V shows the properties of some of the more common beam cross-sections.

134. Beams of Uniform Strength.—A very common design for machine parts which are acting as beams is to make them approximately of uniform strength. In the beams discussed thus far the cross-section has been constant throughout the length of the beam; and the maximum unit fiber stress in a cantilever, for instance, would occur only at the fixed end. Evidently the cross-sections at other places along the length of the beam are larger than necessary. In a beam of uniform strength the maximum unit fiber stress at any section is kept constant, and the cross-section is varied in size according to the requirements of formula (10) for beams.

It will be sufficient to illustrate the principle by discussing cantilever beams of rectangular cross-section carrying the more

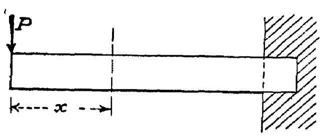


FIG. 16.

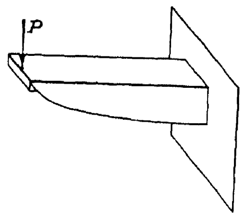


FIG. 17.

common loadings. Figure 16 represents a cantilever beam with a load P at the free end, and the design will be based on a cross-section of constant width and varying depth.

The bending moment at any cross-section distant x from P is Px, and according to formula (10):

$$Px = \frac{SI}{c} = \frac{SBd^2}{6}, \quad d = \sqrt{\frac{6Px}{Sb}}. \tag{12}$$

By assigning various values to x and computing for the depth d the profile of the beam may be determined.

For the beam just discussed the depth at the free end might be zero so far as bending moment is concerned, but this is not the case if shear is to be provided for. For a rectangular section the

maximum shearing unit stress occurs at the neutral axis. and is equal to three-halves of the average, or:

$$S_s = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{P}{Bd}, \qquad d = \frac{3}{2}\frac{P}{BS_s}. \tag{13}$$

The profile of the beam would therefore look like that shown in Fig. 17. The notch at the free end could be left, or the profile could be made a smooth curve.

Figures 18, 19, and 20 show other profiles of cantilever beams of uniform strength and various loadings.

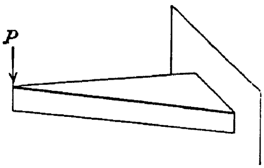


FIG. 18.—Concentrated load.

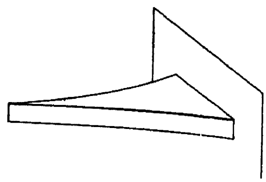


FIG. 19.—Uniform load.

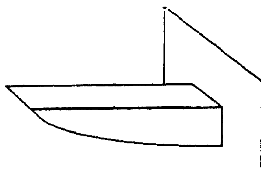


FIG. 20.—Uniform load

Even for cross-sections much more complicated than the rectangular it is possible to determine a beam of approximately uniform strength. Formula (10) may be applied at every half foot or every foot along the length of the beam, thus locating enough points to determine the profile.

135. Eccentrically Loaded Prisms.—A short prism, such as shown in Fig. 21, may be subjected to a tensile or compressive load P, acting at a distance e from the axis of the prism. The action of this load is equivalent to a direct axial load P, and a bending moment Pe.[1]

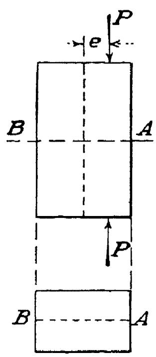


FIG. 21.

The direct axial load produces on the cross-section of the prism a uniformly distributed stress, which, according to Formula (1), equals:

$$S = \frac{P}{A}.$$

The bending moment, Pe, produces a flexural stress, which, according to formula (10) equals:

$$S = \frac{Pec}{I}.$$

[1] MAURER and WITHEY, "Strength of Materials," p. 307. BOYD, "Strength of Materials," p. 240.

This flexural unit stress for the case shown in Fig. 21 would be compressive at A and tensile at B. The combined stress due to both effects would therefore be added at A because both stresses are compressive. At B they would be subtracted from each other, and whether the resultant stress at B was tensile or compressive, would depend on whether the axial stress was smaller or greater than the flexural stress.

Cases of combined flexure and direct stress in which the eccentricity of the load is relatively large may occur as shown in Fig. 22.

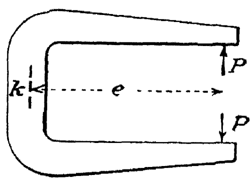


FIG. 22.

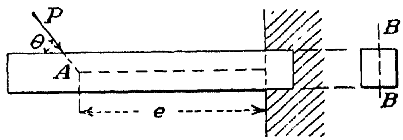


FIG. 23.

Other cases of combined flexure and direct stress, not involving eccentric loads, may occur due to an oblique load acting on a beam as shown in Fig. 23. In this case the load P lies in the plane B B, the component producing axial stress is $P \cos \theta$, while the component producing flexural stress is $P \sin \theta$. The maximum moment would occur at the fixed end of the beam and would be equal to $Pe \sin \theta$.

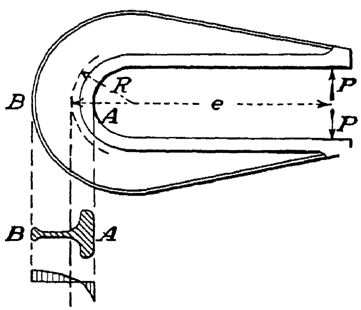


FIG. 24.

136. Curved Beams.—The discussion in the previous section applies correctly to such cases as shown in Figs. 21 and 22 in which the sides of the beam are straight. It is not correct to compute the flexural stress by this method for cases like C frames or hooks in which the sides of the beams are curved, as shown in Fig. 24. The unit flexural stress at a point such as A depends on the radius of curvature R in such manner that the smaller the radius R, the larger is the unit flexural stress.

The reason for this may be illustrated by Figs. 25 and 26. In Fig. 25 two parallel planes such as AB before loading might take the position CD after loading, the deformations being proportional to the distance from the centroidal axis. The deforma-

tion BD and the deformation CA would both occur in an original length which is the same in each case, being the distance between the original planes AB. The unit deformations would therefore be equal for two fibers equally distant from the centroidal axis.

In Fig. 26 the planes AB before loading might take the position CD after loading. The deformation DB, however, would have occurred in an original length BB, while the deformation EF would have occurred in an original length EE, which is smaller. Therefore, even for fibers equally distant from the centroidal axis, the unit deformation would be greater for fibers to the right of the centroidal axis than for fibers to the left. It is evident, therefore, that for curved beams the unit deformations and also the unit stresses will not vary linearly, but will vary somewhat as shown in the lower part of Fig. 24.

For curved beams, therefore, the formula $Mc/I = S$, based upon beams

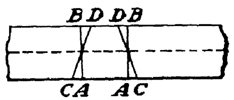


Fig. 25.

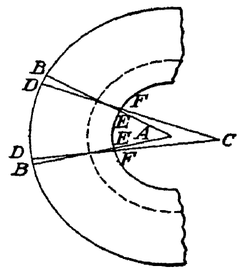


Fig. 26.

with straight sides, gives maximum unit stresses which are too small. It can be shown[1] that the formula for the maximum unit stress in curved beams is as follows:

$$S = \frac{Pe}{R(A' - A)}\left(\frac{R}{R - c} - \frac{A'}{A}\right) + \frac{P}{A}, \tag{14}$$

$$S = \frac{Pe}{R(A' - A)}\left(\frac{A'}{A} - \frac{R}{R + c}\right) + \frac{P}{A}, \tag{15}$$

in which Pe denotes bending moment, in inch-pounds.

A denotes area of cross-section, in in.^{2}.

R denotes radius of curvature for the centroidal axis, in inches.

c denotes distance from centroidal axis to place where unit stress is wanted, in inches.

S denotes the maximum unit stress, in pounds per square inch.

[1] Maurer and Withey, "Strength of Materials," p. 212.
Boyd, "Strength of Materials," p. 350.

A' denotes a factor depending upon the shape and size of the cross-section.

For Fig. 24, formula (14) would be used for computing the tensile stress to the right of the centroidal axis, and formula (15) for computing the compressive stress to the left of the centroidal axis.

It can be shown that the factor A' has the following value:

$$A' = R\int \frac{dA}{R - c'}$$

the notation being the same as above, except that c' is variable.

The value of A' for regular geometrical areas like the rectangle, triangle, or circle, may be computed by the calculus. For irregular areas resort is had to graphical and semi-graphical methods.[1]

137. Deflections of Beams.—For some cases of loaded beams the maximum deflection of the beam rather than the maximum unit stress governs the design, and in many problems it is desirable to be able to calculate the deflection of a beam. Methods have therefore been developed for determining the equation of the elastic curve, which is the curve which the neutral surface assumes in the deflected position of the beam. In developing the relations between the bending moment and the elastic curve it can be shown[2] that the following formula holds:

$$R = \frac{EI}{M}, \tag{16}$$

in which R denotes the radius of curvature of the elastic curve at any point, in inches.

M denotes bending moment at the same point, in inch-pounds.

E denotes modulus of elasticity of the material, in pounds per square inch.

I denotes moment of inertia of the cross-section of the beam with respect to the neutral axis, in inches.[4]

Formula (16) is useful in such problems as the bending of a band saw over a wheel. For this case, since $M = SI/c$, $R = Ec/S$,

[1] Maurer and Withey, "Strength of Materials," p. 218.
Morley, "Strength of Materials," p. 339.

[2] Maurer and Withey, "Strength of Materials," p. 170.
Boyd, "Strength of Materials," p. 143.

and the unit stress existing in the saw for a given wheel radius and thickness of saw may be easily computed.

The equation of the elastic curve in terms of rectangular coordinates may be obtained from the following formula:[1]

$$EI\frac{d^2y}{dx^2} = M. \tag{17}$$

For a specific case of loading on a beam the equation of M in terms of x must be inserted, and the equation must then be integrated twice to obtain the relation between x and y of the elastic curve. Expressions for the maximum deflections of beams under the more common loadings may be found in the various engineers' handbooks.

138. Torsion.—When a machine or structural element is subjected to a twisting moment as shown in Fig. 27, the member

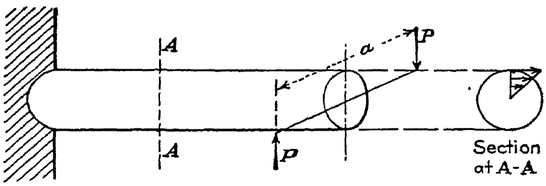


FIG. 27.

is said to be subjected to *torsion.* The stresses set up by torsion are shearing stresses.

Experiment has shown that a longitudinal element on the surface of a shaft will be twisted into a helix, and at any cross-section the unit shear stresses, which are produced by the external *twisting moment* or *torque,* vary as the distance from the axis of the shaft.

It can be shown[2] that the relation existing between torque and unit stress is as follows:

$$T_m = \frac{SJ}{r}, \tag{18}$$

in which T_m denotes torque or twisting moment, in inch-pounds.

J denotes polar moment of inertia of the cross-section, in in.^{4}.

r denotes distance to the outer fiber, in inches.

[1] MAURER and WITHEY, "Strength of Materials," p. 171.
BOYD, "Strength of Materials," p. 144.

[2] MAURER and WITHEY, "Strength of Materials," p. 250.
BOYD, "Strength of Materials," p. 82.

It should be clearly noted that this formula is correct only for solid or hollow circular shafts, which are the most commonly used sections to transmit torque in machines. Textbooks on strength of materials give formulas for the ellipse and rectangle, but for complicated sections there are no theoretical formulas, and resort must be had to experiment or to empirical formulas based on experiment.

Just as was the case with formula (10) for beams, formula (18) may be used to calculate the unit shear stress at any point of the cross-section of a shaft. If r' is the distance from the axis of the shaft to the fiber where the unit stress is S', then $T_m = S'J/r'$.

Another important formula for torsion problems is that which gives the angle of twist of the shaft between two cross-sections a distance L away from each other. The formula[1] is:

$$\theta = \frac{57.3 T_m L}{E_s J}, \tag{19}$$

in which θ denotes the angle of twist, in degrees.

T_m denotes torque acting on the shaft, in inch-pounds.

L denotes length over which twist occurs, in inches.

E_s denotes modulus of elasticity in shear, in pounds per square inch.

J denotes polar moment of inertia of the shaft cross-section, in in.^{4}.

139. Stresses in Columns.—Compression members may be divided into three classes: (1) those which are so short that practically no bending takes place and the stress is uniformly distributed over the cross-section; (2) those which are so long that the bending effect is the most significant action; and (3) those which are intermediate in length, so that the resultant maximum stress is partly due to bending and partly due to direct stress. The word "column" is usually restricted to long compression members which would fall into the classes 2 and 3, above.

The end conditions of a column affect the load which can be carried, by the restraint which the ends offer in preventing the bending of the column. Figure 28 shows various end conditions of columns. Round ends, as shown in Fig. 28(a), do not restrain the column at the ends. Figure 28(b) shows bending in a plane

[1] MAURER and WITHEY, "Strength of Materials," p. 252.
BOYD, "Strength of Materials," p. 80.

perpendicular to the axis of the pins. The restraint would be due to the friction at the pins. For bending in a plane parallel to the axis of the pins (see Fig. 28 (*c*)) the column would be very considerably restrained, and would approach the square-ended or fixed-ended condition. Flat ends (Fig. 28(*d*)) offer considerable restraint at the ends, and fixed ends (Fig. 28(*e*)) offer even greater restraint than flat ends.

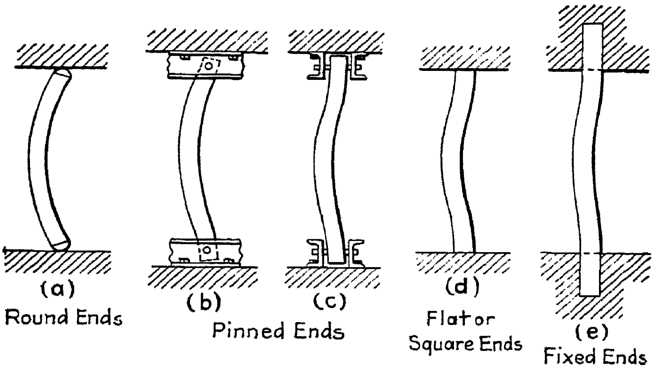


FIG. 28.—End conditions for columns.

It can be shown[1] that Euler's formula for long columns is the appropriate one to apply to class 2, as follows:

$$P = \frac{mEI}{L^2} \text{ or } \frac{P}{A} = \frac{mE}{(L/r)^2}, \tag{20}$$

in which P denotes maximum load which the column will carry, in pounds.

E denotes modulus of elasticity of the material, in pounds per square inch.

I denotes least moment of inertia of the cross-section, in in.^{4}.

L denotes length of the column, in inches.

r denotes least radius of gyration of the column cross-section, in inches.

m denotes a constant depending upon end conditions.

Experiments have shown that the constant m has an average value of about 10 for round-ended columns, 16 for pin or hinged ends, and 25 for square or flat ends.

[1] MAURER and WITHEY, "Strength of Materials," p. 285.
BOYD, "Strength of Materials," p. 258.

Most of the columns used in bridges and buildings fall into class 3, and the formulas which are used in discussing such columns have in them experimental constants determined from tests on columns. The most common formula used in the United States today, and one of the simplest, is the so-called "straight-line formula," first suggested in 1886 by T. H. Johnson. Johnson found that a straight line made tangent to Euler's curve for long columns fitted experimental points quite well.

To determine whether a given column falls into class 2 or class 3, the criterion of slenderness ratio, L/r, is applied as follows:

$$L/r = \sqrt{\frac{3mE}{S}}, \tag{21}$$

in which the notation is the same as before, and S is an experimental constant related to the yield point or ultimate compressive strength of the material. If the slenderness ratio of a given column is greater than that given by formula (21) it falls into class 2 and should be handled by means of the Euler formula. If the slenderness ratio is less than that given by formula (21) it falls into class 3 and should be handled by means of one of the straight-line formulas.

Two of the commonly used straight-line formulas are the American Railway Engineering Association formula and the American Bridge Company formula. Table VI gives these formulas, and shows also some of the limiting conditions of their use.

TABLE VI

Name of formula	Formula	P/A must not exceed	Slenderness ratio limitations
American Railway Engineering Association	$P/A = 15{,}000 - 50\frac{L}{r}$	12,500	$L/r < 200$
American Bridge Company	$P/A = 19{,}000 - 100\frac{L}{r}$	13,000	$L/r > 60$, $L/r < 120$
	$P/A = 13{,}000 - 50\frac{L}{r}$	......	$L/r > 120$, $L/r < 200$

140. Combined Stresses.—There are many cases in practice in which structural or machine members are subjected to combined stresses, due to the simultaneous action of either tensile or compressive stresses combined with shear stresses. In a reinforced

concrete beam there is tension due to flexure, and shear due to beam shear. In many shafts there is tension and compression due to flexure or direct stress, and shear due to torsion.

It can be shown[1] that the maximum unit stresses due to combinations of tensile or compressive stresses with shear stresses are as follows:

$$S_t' \text{ or } S_c' = \frac{S}{2} + \sqrt{\left(\frac{S}{2}\right)^2 + S_s^2}, \tag{22}$$

$$S_s' = \sqrt{\left(\frac{S}{2}\right)^2 + S_s^2}, \tag{23}$$

in which S_t' denotes the maximum unit tensile stress, in pounds per square inch.

S_c' denotes the maximum unit compressive stress, in pounds per square inch.

S_s' denotes the maximum unit shear stress, in pounds per square inch.

S denotes the unit stress in tension or compression due to flexure or direct stress, in pounds per square inch.

S_s denotes the unit stress in shear due to beam shear or torsion, in pounds per square inch.

One of the common cases of combined stresses occurs in shafts subjected simultaneously to a torque and a bending moment. In such a case S may be computed by means of formula (10) for beams, and S_s may be computed by means of formula (18) for torsion. Having these unit stresses, formulas (22) and (23) may be applied to determine the maximum unit stresses due to the combination.

In design problems, S_t', S_c', and S_s' would be fixed by engineering practice, and the design dimensions would be required. For such cases the formulas commonly give third- and fourth-power equations, which are often most quickly solved by trial and error methods, that is, by assuming a dimension and finding out whether the two sides of the equation balance.

141. Other Considerations.—Problems in the design of structures and machines involve stresses which may be conveniently grouped into three classes: (1) stresses due to steady or quiescent

[1] Maurer and Withey, "Strength of Materials," p. 30.
Boyd, "Strength of Materials," p. 294.

loads; (2) stresses due to repeated loads; and (3) stresses due to impact loads.

When members are subjected to steady loads, it is usually sufficient if the design is based upon working stresses which are less than the elastic limit and ultimate strength of the material. Under repeated loads the phenomenon of "fatigue" must be taken into account, and under impact loads the "elastic resilience" is of prime importance. Repeated impact will give rise to conditions which are very closely related to fatigue.

142. Fatigue Phenomena.—The case of repeated loads may be illustrated by a pair of wheels and axle under an ordinary railroad freight car. Consider the portion of the axle projecting beyond the wheels. The upper fibers of the axle are under tensile stress and the lower fibers under the same amount of compressive stress. As the wheels roll, the fiber which was at the top of the axle is at the bottom at the next instant. In such a case the stresses are reversed from tension to compression for each revolution of the wheels, and this is called a *cycle* of stress. In other cases of repeated loads the unit stresses may be only partly reversed, and in still other cases the unit stresses may be of the same kind but changing from a minimum to a maximum value.

Numerous experiments have shown that when a material is subjected to repeated stresses of sufficient magnitude, the action of the stresses is such as to start a microscopic crack, which, under continued application of stress, spreads until failure occurs.

Figure 29 shows typical endurance curves for several steels, in which the unit stress to which the specimen was subjected is plotted as ordinate, and the number of cycles of stress for rupture is plotted as abscissa. The unit stress which the material can withstand apparently indefinitely, which is shown on the diagram by the horizontal part of the curve, is called the *endurance limit.* For materials subjected to repeated stresses it is evident that the endurance limit is a significant property, and machines and structures must be so designed that the working stress is less than this limit.

Figure 30 shows a diagram for wrought ferrous metals which is known as the Goodman diagram, and which was developed independently by both Goodman and J. B. Johnson. The ordinate to the line *EB* represents the static ultimate tensile strength of the metal. The minimum stresses are plotted along

a 45-degree line DOB. The horizontal line through O is the line of zero stress; tensile stress being plotted above the line and compressive stress below it. According to the "dynamic theory" of suddenly applied loads, the minimum or dead-load stress plus

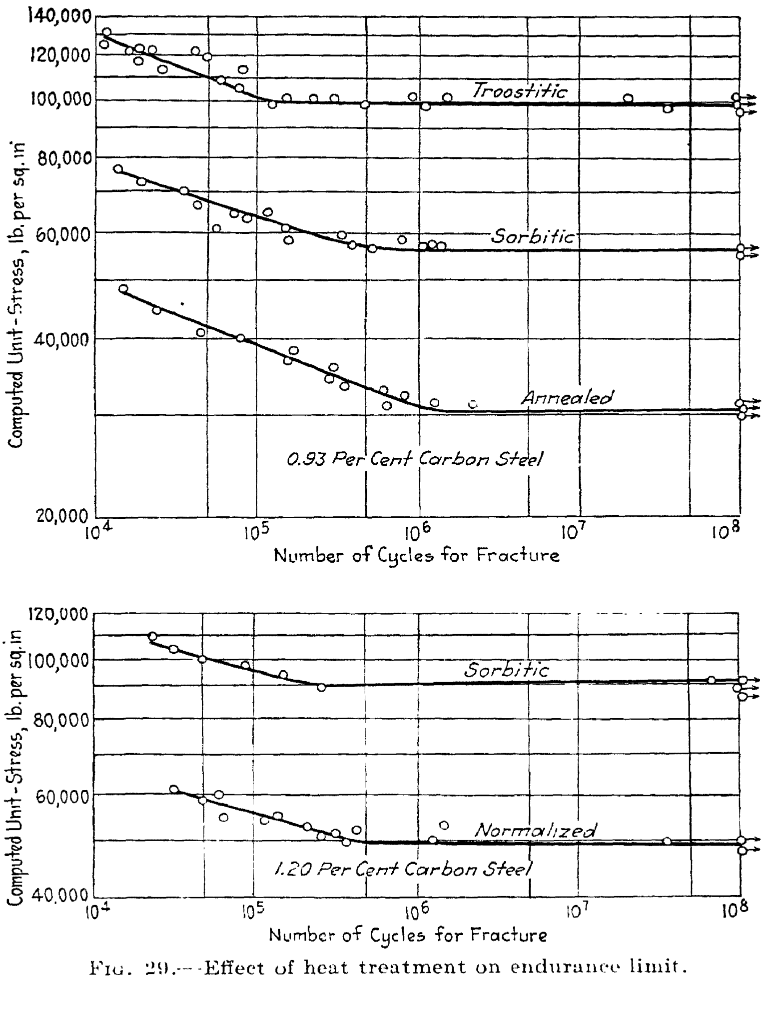


FIG. 29.—Effect of heat treatment on endurance limit.

twice the live-load stress equals the static ultimate strength; and the maximum applied stress should fall on a line CAB, such that the point A is five-tenths of the ultimate static strength. Goodman plotted endurance limits obtained by various investigators after the material has been subjected to over 4,000,000

cycles of stress, and he found that these experimental points fell fairly well on the straight line CAB.

As the diagram shows, when the minimum stress is zero the maximum stress for indefinite endurance should be five-tenths of the ultimate static strength. When the stress is completely reversed at CD, the plus and minus stresses should be one-third of the ultimate strength. Presumably a diagram similar to the one shown in Fig. 30 would hold when the stress above the zero line is compression and that below it is tension. Experimental data, however, are very meager for these combinations of compressive stress.

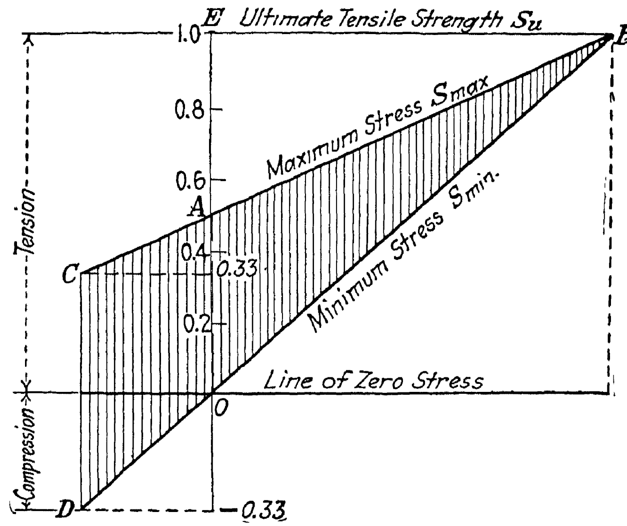


FIG. 30.—Goodman diagram for range of stress.

According to the Goodman diagram the *range* of stress (algebraic difference between maximum and minimum) is greatest for reversed stresses, and decreases as the maximum stress is increased above one-third of the ultimate strength, being actually zero when the maximum stress coincides with the ultimate. In other words, as the maximum stress is increased the minimum stress must be decreased algebraically, in order that the material may be stressed indefinitely without failure.

J. B. Johnson developed a formula, which was later simplified by Barr, based on a diagram similar to Goodman's. The formula is:

$$S_{max} = \frac{0.5S_u}{1 - 0.5r}, \tag{24}$$

in which S_u denotes static ultimate tensile strength, in pounds per square inch.
r denotes range ratio for the cycle of stress, that is, the ratio of the minimum stress to the maximum stress.
$S_{\max}$ denotes the maximum unit stress during the cycle.

It should be remembered that r, the range ratio, is positive if the stress limits of a cycle are both tensile or both compressive, but is negative if one limit is tensile and the other compressive.

Experiments have shown that S_{-1}, the endurance limit for completely reversed stresses, is about five-tenths of the ultimate tensile strength, instead of one-third as indicated by the Goodman diagram.

The Goodman diagram shows that the ratio $S_o : S_{-1}$ has a value of 1.5, S_o being the endurance limit when the minimum stress is zero, and S_{-1} having the same meaning as above. Experiments show that it is reasonably safe to assume that such a ratio exists.

Moore and Kommers[1] have suggested a modification of the J. B. Johnson formula which is not based on any assumed ratio of $S_{-1} : S_u$, but rather on an experimentally determined value of S_{-1} for each metal. The value of 1.5 for the ratio $S_o : S_{-1}$ is retained, and the formula is:

$$S_{\max} = \frac{1.5S_{-1}}{1 - 0.5r} \text{ or } \frac{S_{\max}}{S_{-1}} = \qquad - r \tag{25}$$

in which the notation is the same as before. If $r = 0$, $\frac{S_{\max}}{S_{-1}} =$ 1.5, the Goodman ratio.

It should be remembered that elastic failure is as important as fatigue failure, and therefore $S_{\max}$ as determined by Goodman's diagram or any formula for endurance stresses should never be permitted to exceed the elastic limit of the material.

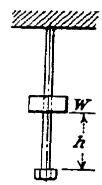


FIG. 31.

"The Fatigue of Metals" by Moore and Kommers gives convenient tables showing what may be expected for the endurance limit of numerous steels and also non-ferrous metals.

143. Impact Stresses.—Figure 31 shows an ordinary tension bar, supported at the upper end, and subjected to the stresses produced when a weight W is allowed to drop from a height

[1] MOORE and KOMMERS, "Fatigue of Metals," p. 185.

h. It can be shown[1] that under these circumstances the stress produced in the member due to the application of the load is as follows:

$$S_1 = S + S \tag{26}$$

in which S denotes unit stress produced in the member if W were a quiescent load, in pounds.
e denotes total deformation in the member due to W acting as a quiescent load, in inches.
h denotes distance through which the load W falls, in inches.
S_1 denotes the unit stress actually produced by the impact, in pounds per square inch.

Formula (26) shows that when h is zero, that is, when the load W is "suddenly applied" without falling through any distance, the stress is twice as great as that due to W applied as a quiescent load. It is also obvious that with increase in h the unit stress S_1 increases very rapidly, and it follows that disastrously high stresses may be produced by impact loads.

If the bar in Fig. 31 is held in a horizontal position and the weight W is projected against the end with a velocity V, it can be shown that the stress produced in the member is as follows:

$$S_1 = S\sqrt{\frac{2h}{e}}, \tag{27}$$

in which S denotes unit stress produced in the member if W were a quiescent load, in pounds per square inch.
h denotes $\frac{V^2}{2g}$, V being the velocity of the weight W in inches per second, and g being the acceleration of gravity in inches per second per second.
e denotes total deformation in the member due to W acting as a quiescent load, in inches.

144. Effect of Shape on the Resilience of a Member.—The shape of a member may affect the amount of energy which can be absorbed safely, for the reason that resilience is the product of stress and deformation. Suppose that in Fig. 32 the bolt must withstand tension, and let the maximum unit stress permitted at the root area of the threads be 16,000 lb. per square

[1] MAURER and WITHEY, "Strength of Materials," p. 32.

inch. A 1-in. bolt has a diameter at the root of the thread of 0.837 in., therefore the unit stress in the shank of the bolt would be 11,200 lb. per square inch. The shank of the bolt could therefore absorb an amount of energy equal to:

$$\text{Energy} = \frac{11{,}200^2}{2 \times 30{,}000{,}000} \times \frac{\pi \times 1^2}{4} \times 6 \quad 9.84 \text{ in.-lb.}$$

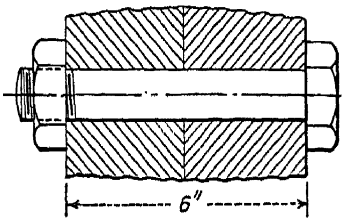


Fig. 32.

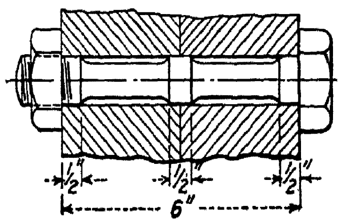


Fig. 33.

If the design of the shank of the bolt were changed as shown in Fig. 33, so that the smaller diameter of the shank were made equal to that at the root of the threads, then the energy that could be absorbed by the bolt would be:

$$\text{Energy} = \frac{16{,}000^2}{2 \times 30{,}000{,}000} \times \pi 0.837^2 \times 4.5 + \frac{11{,}200^2}{2 \times 30{,}000{,}000} \frac{\pi \times 1^2}{} \times 1.5$$

$$= 10.55 + 2.46 = 13.01 \text{ in.-lb.}$$

This shows that the bolt in Fig. 33 can absorb 32 per cent more energy than the bolt in Fig. 32. This is due to the fact that with the higher unit stress in the smaller diameter of the shank goes a greater elongation, which together produce a greater capacity for absorbing energy.

CHAPTER VIII

RIVETED JOINTS AND FASTENINGS

145. *Rivets* are the simplest form of fastener, and are used as connectors in structural and machine work where a permanent fastener is desirable. There are two general classes of riveted connections, one in which strength alone is important, and the other in which a fluid-tight joint is an additional requirement besides strength. Rivets are used generally for fasteners for ductile materials, as it is difficult to form the head in the process of riveting against a brittle material like cast iron.

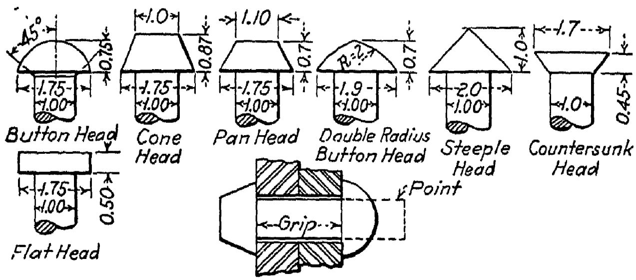


FIG. 1.—Forms of rivet heads.

A rivet is a short cylindrical bar with upset heads. The first head is formed on the rivet in an upsetting machine called a header. The rivet is applied by passing the body of the rivet through the members to be fastened, and then upsetting the protruding point, forming a second head on the other end. The application of a rivet and the forming of the second head is called "riveting." The heads of rivets have various forms, depending upon the use of the rivet. Figure 1 shows some of the types used in practice.

146. Driving of Rivets.—A rivet may be driven hot or cold. When the rivet is driven cold it should accurately fit the hole, as it is used primarily as a fastener to resist shearing action. When the head is being formed in cold riveting, the impact of the hammer or machine may damage the metal in the head being formed. Hot riveting is done while the metal in the rivet is

plastic, allowing the head to be formed with less danger of starting cracks in the metal of the head. In cooling, the rivet will shrink, and this contraction between the heads draws the members tightly together, and in so doing sets up an initial stress in the body of the rivet. This stress is indeterminate, but in some rivets it is of considerable amount. Any additional load on a rivet so stressed might cause the rivet to become overloaded, resulting in the rupture of the rivet. For this reason great caution should be exercised in the use of rivets in locations where the rivet is likely to be subjected to an external tensile load. Figure 2 shows a rivet before and after driving.

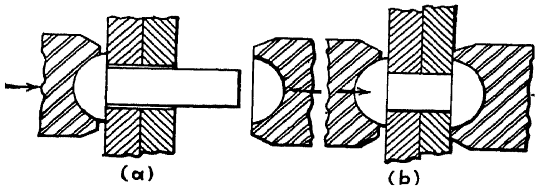


FIG. 2.—Rivet before and after driving.

The diameter of a rivet is one-sixteenth inch less than that of the hole into which it is to fit, to insure easy entrance of a hot rivet. In driving, the rivet metal in the hole spreads out to fill the hole, and after cooling the rivet is slightly smaller, but for all practical purposes the rivet and hole are assumed to be of the same diameter. In calculations for boilers, the bearing, shearing, and net tensile areas are all based on the diameter of the hole. The head formed on a rivet by driving should be concentric with the hole, if the rivet is to carry its full share of the load. In design it is assumed that each rivet is equally loaded, and that the bearing pressure is equally distributed over the plate areas.

Riveting done by hydraulic or pneumatic machines is more uniform and dependable than hand riveting. The pressure that is used for machine riveting varies with the grip of the rivet. If the grip does not exceed the diameter of the rivet, pressures according to Table I are recommended. If the grip is greater than the rivet diameter, the pressure should be increased approximately in proportion to the square root of the grip of the rivet. The power required for cold riveting is about double that which is required for hot riveting.

TABLE I.—PRESSURES FOR HOT RIVETING
(R. D. Wood & Co.)

Diameter of rivet, inches	Pressure in pounds per square inch of rivet area		
	Boiler work	Tank work	Structural work
½	40,000	30,000	18,000
⅝	50,000	36,000	24,000
¾	66,000	44,000	30,000
⅞	90,000	60,000	44,000
1	120,000	90,000	60,000
1⅛	150,000	120,000	76,000
1¼	200,000	140,000	90,000
1½	250,000	170,000	120,000

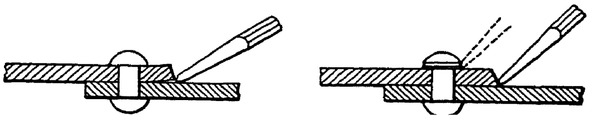
For boiler work the rivet holes should be in perfect alignment and the hole walls true and clean. The plates should be held together by bolts which fill the holes, until they are replaced by the rivets. If the holes are punched, the holes should be cleaned out by reaming, using a reamer that is 1/16 in. larger in diameter than the punched hole. Punched holes are not true cylinders, because the slug volume is less than that of the hole. The explanation is, that, in piercing the sheet, the punch pushes some metal against the hole walls in shearing out the slug, thereby doing some damage to the metal in the hole walls. Reaming the holes removes this damaged metal and at the same time trues up the hole. Drifting, that is forcing holes into alignment by the use of a long taper pin, should be discouraged because such practice crushes the metal by the wedging action of the pin.

When holes are punched there is a loss of strength in the plate unless the holes are reamed, although annealing the plate will restore its strength to some degree. This loss due to punching is estimated at 10 per cent for ¼-in plates, 20 per cent for ½-in. plates, and 30 per cent for ¾-in. plates.

In structural work, other than pressure vessels, punched holes are permitted, punching being cheaper, and the insertion of a few extra rivets in structural joints perhaps more than offsets any weakness due to the use of punched holes. For work in which the plates are punched but not reamed, the practice is to punch the hole 1/16 in. larger than the diameter of the rivert. For purposes of calculation, the bearing and shearing areas are based upon the

diameter of the rivet, while the net tensile or compressive area is based upon the diameter of the rivet plus $\frac{1}{8}$ in.

Before riveting, all fins due to drilling or punching should be removed, to insure good surfaces between the joint. In boiler and tank work the outside edge of the plates is usually beveled. If the rivets do not pull the plates of the joint together tight enough to prevent leakage, the outside edge or edges of the plates are calked, but care should be taken not to cut into the surface of the plate along the seam. For this reason the nose of the calking tool should be rounded and in the hands of a careful workman. Calking as shown in Fig. 3(*b*) is to be recommended,



(a) (b)

FIG. 3.—Caulking seams of riveted joints.

so that the plates along the seam will not spring apart as shown by Fig. 3(*a*).

147. Friction in Riveted Joints.—It is evident that there is considerable frictional resistance between the surfaces of a joint, which resists the slipping of one surface over the other; but as the tension in the rivets is indeterminate, the resultant friction is not taken into account in the calculations for the joint. Therefore, being on the side of safety, friction adds to the strength of the joint. Experiments have shown that the frictional resistance developed in riveted joints is often equivalent to a shearing unit stress on the rivets of 7,000 to 10,000 lb. per square inch.[1]

148. Types of Riveted Joints.—Boiler and tank joints are classified as lap or butt joints. In lap joints the shell plates are overlapped so that one, two, or three rows of rivets can be driven through the double thickness of plate, and are specified as:

(*a*) Single-riveted (Fig. 4).

(*b*) Double-riveted (Fig. 5).

(*c*) Triple-riveted (Fig. 6).

Butt joints are formed by bringing the main plates together in line and using strap plates to overlap for riveting. Butt joints are specified as:

[1] JOHNSON, BRYAN, and TURNEAURE, "Modern Framed Structures," Part III, p. 117.

(*d*) Single-riveted (Fig. 7).
(*e*) Double-riveted (Fig. 8).
(*f*) Triple-riveted (Fig. 9).

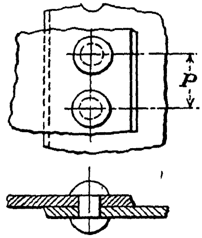


FIG. 4.—Single riveted lap joint.

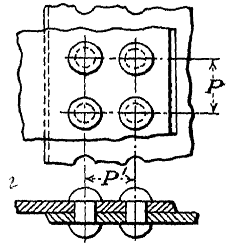


FIG. 5.—Double riveted lap joint.

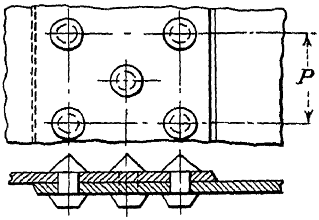


FIG. 6.—Triple riveted lap joint, staggered pattern.

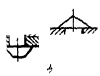


FIG. 7.—Single riveted one strap butt joint.

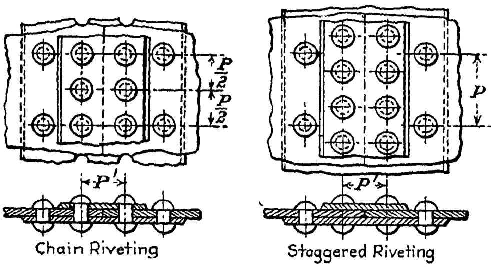
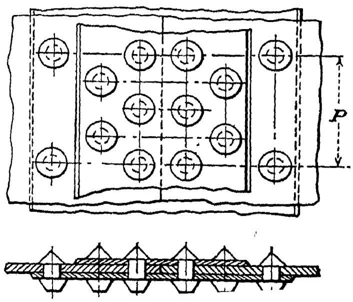


FIG. 8.—Double riveted two strap butt joint.



-Triple riveted two strap butt joint.

(*g*) Quadruple-riveted (Fig. 10).
(*h*) Quintuple-riveted.

Practice recommends that butt joints be used for the longitudinal seams for all boilers with shell diameters over 36 in., and that lap joints be used on boilers of less than 36-in. diameters

and pressures 100 lb. per square inch or less. The girth seams are lap joints because the stress is only one-half in this direction as compared with the stress in the circumferential direction.

A boiler shell is a cylinder with thin walls, and it may be assumed that the intensity of stress is distributed uniformly over the full thickness of the plate. This assumption is justified so long as the thickness of the plate is small compared with the diameter of the shell.

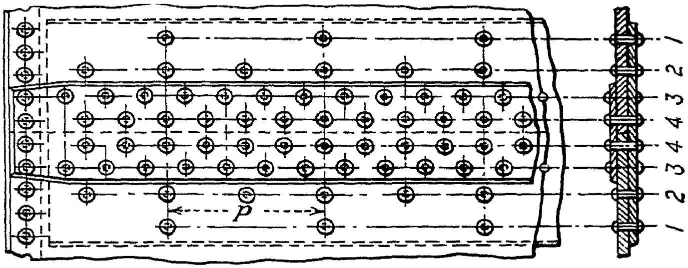


FIG. 10.—Quadruple riveted two strap joint.

The thickness of shell that will resist rupture along a longitudinal line is:[1]

$$t = \frac{p \times D}{2S_t}. \qquad (1)$$

in which t denotes thickness of the metal, in inches.

p denotes internal pressure, in pounds per square inch.

D denotes inside diameter of shell, in inches.

S_t denotes unit stress in the metal, in pounds per square inch.

The force tending to rupture the girth seams is only one-half that which tends to burst the shell longitudinally, hence:

$$t = \frac{p \times D}{4S_t}, \qquad (2)$$

in which the notation is the same as before.

149. Relative Strength of Joints.—The *relative strength* of a riveted joint, often called its *efficiency*, is the ratio of the strength of the joint to the strength of the unperforated solid plate. Table II shows the approximate relative strengths that may be expected from various types of riveted joints.

[1] MAURER and WITHEY, "Strength of Materials," p. 76.
BOYD, "Strength of Materials," p. 75.

TABLE II.—RELATIVE STRENGTH OF RIVETED JOINTS
(In percentage)

	Minimum	Maximum	Average
Single-riveted lap joint...........	52	64	55
Double-riveted lap joint..........	64	78	70
Triple-riveted lap joint...........	64	84	78
Single-riveted butt joint..........	53	64	60
Double-riveted butt joint.........	64	78	75
Triple-riveted butt joint..........	78	88	80
Quadruple-riveted butt joint......	94	96	95

150. Factor of Safety.—The factor of safety that is recommended by the Boiler Code Committee of the American Society of Mechanical Engineers is 5. The allowable stresses for design become:

$$S_t = 55{,}000 \quad 11{,}000 \text{ lb. per square inch.}$$

$$S_c = \frac{95{,}000}{5} = 19{,}000 \text{ lb. per square inch.}$$

$$S_s \quad 44{,}000 = 8{,}800 \text{ lb. per square inch.}$$

151. Types of Failure.—A single-riveted lap boiler joint may fail due to any one of the following causes:

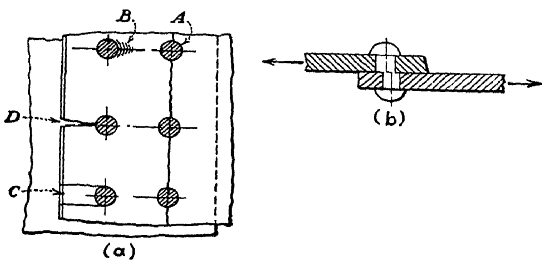


FIG. 11.—Various types of failure of riveted joint.

1. Shearing of the rivet as shown by Fig. 11(*b*).
2. Tearing the plate through the rivet holes as at *A* (Fig. 11(*a*)).
3. Crushing of the plate in front of a rivet as at *B*.
4. Crushing of the rivet.
5. Tearing out the plate as shown at *C*.
6. Splitting the plate as at *D*.

If the joint is a complex one, failure may be due to a combination of the above causes, and for illustration, in Sec. 155 a quadruple-riveted, butt and double strap joint will be checked for strength. This same method may be applied to any joint.

152. Pitch.—The greatest distance between rivets along a seam is called *pitch*. The seams around the boiler are called *girth* seams and the lengthwise seams are called *longitudinal* seams. The distance between adjacent rows of rivets is called *back* pitch. The *margin* is the distance from the edge of the plate to the center line of the nearest row of rivets.

The spacing of the rivets in any seam repeats itself, and the width which will take in all the rivets, making up a pattern, is called a *unit strip*, and this is equal in width to the pitch. The pitch should be chosen so as to result in a good relation between size of rivet and cross-section of perforated plate.

The distance of rivet holes from the edge of the plate is governed by arbitrary rules which experience and experiment have shown to be safe. The A.S.M.E. Boiler Code states that rivet holes in plates for longitudinal joints, except in the ends of butt straps, shall not be less than 1½ nor more than 1¾ times the diameter of the rivet holes from the edge of the plate.

The pitch of the different types of joints is approximately as given in Table III.

TABLE III.—RELATION OF PITCH TO THE SIZE OF RIVET

Type of joint	Pitch in inches		Back pitch, inches
	Longitudinal seams	Girth seams	
Lap, single-riveted	$2d + 0.375$	$1.5d + 0.719$	Not less than $2d$
Double-riveted	$3d + 0.188$	$3d + 0.125$	Same
Triple-riveted	$4.5d - 0.28$	$3d + 0.125$	Same
Butt, single-riveted	$2.76d$	$1.5d + 0.719$	Same
Double-riveted	$4d + 1.25$	$3d + 0.125$	Same
Triple-riveted	$12d - 2.75$ when d = 11/16 to 7/8, d + 7.0625 when d = 15/16 to 1 3/16, $6d$ + 0.875 when d = 1¼ to 1 7/16	. . . . .	$2d + 0.938$ between first and second rows $1.5d + 0.75$ between other rows, if any
Quadruple-riveted	$16d$		

Figures 12 and 13 have been drawn from tables for riveted joints compiled by Haven and Swett.[1] The curve in Fig. 12

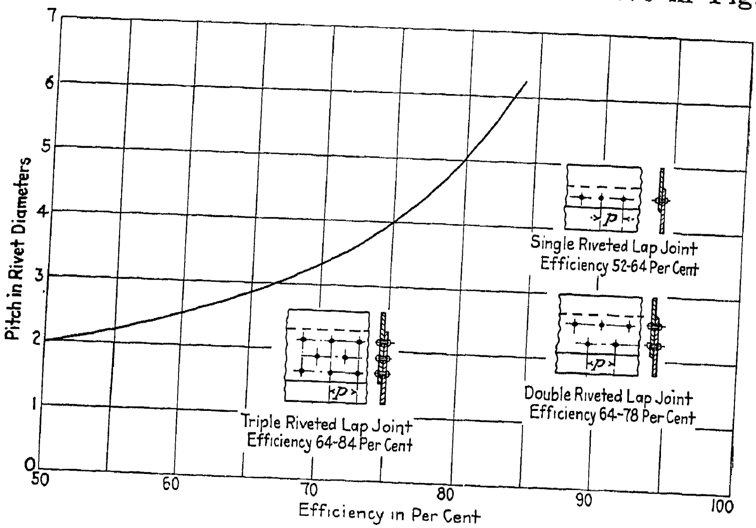


FIG. 12.—Relation of pitch and efficiency for lap joints.

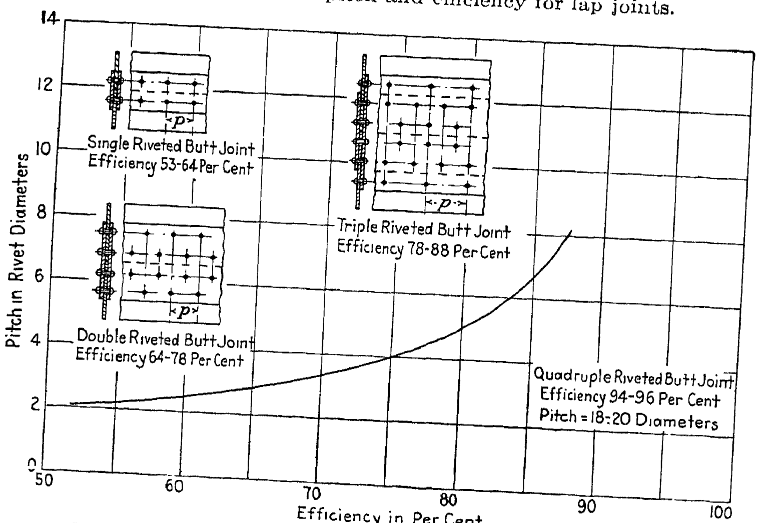


FIG. 13.—Relation of pitch and efficiency of butt joints.

gives the pitch distance in terms of rivet diameter to be used for various types of lap joints to obtain the efficiency desired.

[1] HAVEN and SWETT, "Design of Steam Boilers and Pressure Vessels," p. 157, John Wiley & Sons, Inc., N. Y.

The figure also shows the range of efficiencies which may be expected for various joints. Figure 13 gives the same information for various types of butt joints.

153. Rivet Material, Strength and Sizes.—Rivet steel is open-hearth steel with an ultimate tensile strength of 45,000 to 55,000 lb. per square inch.

The ultimate shear strength is given in Table IV.

TABLE IV.—ULTIMATE SHEAR STRENGTH OF RIVET MATERIAL

Iron rivets......	38,000 lb. per square inch single shear	76,000 lb. per square inch double shear
Steel rivets......	44,000 lb. per square inch single shear	88,000 lb. per square inch double shear

Rivets are made in the following sizes:

From $\frac{3}{8}$ to 1 in., increasing by $\frac{1}{16}$ in.

From 1 to $1\frac{1}{2}$ in., increasing by $\frac{1}{8}$ in.

The usual sizes specified are the ones that are a multiple of $\frac{1}{8}$ in. The handbooks of the steel companies give tables of length of rivets to be used for different grips. The size of rivet that will be required for riveting plates of t thickness is approximately $d = 1.2\sqrt{t}$, but it is recommended that rivets be chosen according to Table V.

TABLE V.—RIVET SIZE BASED ON PLATE THICKNESS

Thickness of plate, inches..........	$\frac{5}{16}$	$\frac{11}{32}$	$\frac{3}{8}$	$\frac{13}{32}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{1}{2}$		
Diameter rivet after driving, inches..........	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{13}{1}$	$\frac{13}{16}$	$\frac{15}{16}$		$\frac{15}{16}$	$1\frac{1}{16}$	$1\frac{1}{16}$

154. Boiler Plates.—The steel plates should be of open-hearth steel, made in two qualities, flange steel and fire-box steel. Steel plates in any part of the boiler, when exposed to the fire, or to products of combustion, and under pressure, should be of fire-box quality steel; for any part under pressure, where fire-box steel is not specified, either quality of steel may be used. The A.S.M.E. specifications call for the following strengths:

Table VI.—Strength of Boiler-plate Steel

	Flange or boiler steel	Fire-box steel
Ultimate tensile strength, pounds per square inch...	55,000 to 65,000	55,000 to 65,000
Yield point, pounds per square inch..............	0.5 × tensile strength	0.5 × tensile strength
Elongation in 8 in. minimum per cent................	1,500,000 ÷ tensile strength	1,500,000 ÷ tensile strength

The compressive strength of mild steel is taken as 95,000 lb. per square inch.

Boiler plate is rolled in the following thicknesses:

$\frac{1}{4}$ to $\frac{9}{16}$ in., increasing in thickness by $\frac{1}{32}$ in.

$\frac{9}{16}$ to $\frac{5}{8}$ in., increasing in thickness by $\frac{1}{16}$ in.

$\frac{5}{8}$ to $1\frac{1}{4}$ in., increasing in thickness by $\frac{1}{8}$ in.

Plates may be bought in sheared sizes in various widths and lengths up to 125 in. wide by 144 in. long.

The thickness of plate used for boilers, based upon the diameter of the shell, is given in Table VII.

Table VII.—Minimum Thickness t of Boiler-shell Plates

Diameter of shell...............	t in inches
36 in. and under...............	$\frac{1}{4}$
37 to 54 in., inclusive...........	$\frac{5}{16}$
55 to 72 in., inclusive...........	$\frac{3}{8}$
Over 72 in.....................	$\frac{1}{2}$

Table VIII.—Minimum Thickness of Butt Strap

Thickness of shell plate	Minimum thickness of butt straps
$\frac{1}{4}$ to $\frac{11}{32}$ in., inclusive.........	
$\frac{3}{8}$ and $\frac{13}{32}$ in.................	
$\frac{7}{16}$ and $\frac{15}{32}$ in................	
$\frac{1}{2}$ to $\frac{9}{16}$ in., inclusive..........	
$\frac{5}{8}$ and $\frac{3}{4}$ in...................	$\frac{1}{2}$
$\frac{7}{8}$ in.........................	
1 and $1\frac{1}{8}$ in..................	$\frac{3}{4}$
$1\frac{1}{4}$ in........................	$\frac{7}{8}$

The thickness of butt straps for different thicknesses of main plates is given in Table VIII.

155. Analysis of a Typical Joint.—Figure 10 shows a double-strap, quadruple riveted joint, which is typical of a joint in a high-pressure boiler. The relative strength of this joint is to be determined. The stresses that are involved in such a joint are tensile, shearing, and bearing stresses, and as has been previously stated, each rivet is assumed to carry its share of the load in shear and in bearing.

S_s = shearing strength of the rivet in single shear, in pounds per square inch.
S_d = shearing strength of the rivet in double shear, in pounds per square inch.
S_c = crushing strength of mild steel, in pounds per square inch.
S_t = tensile strength of the plate, in pounds per square inch.
n = number of rivets in single shear.
N = number of rivets in double shear.
t = thickness of plate, in inches.
b = thickness of butt strap, in inches.
P = pitch of rivets, in inches, on row having the greatest pitch.
d = diameter of rivet after driving, in inches = diameter of rivet hole.

(*A*) F_a = strength of the solid plate $= P \times t \times S_t$. (3)

(*B*) F_b = strength of plate between rivet holes in the outer row.

$$F_b = (P - d) \times t \times S_t. \tag{4}$$

(*C*) F_c shearing strength of rivets in double shear, plus the shearing strength of rivets in single shear.

$$F_c = \frac{\pi d^2}{4} \times N \times S_d + \frac{\pi d^2}{4} \times n \times S_s. \tag{5}$$

(*D*) F_d = strength of plate between rivet holes in second row, plus the shearing strength of rivets in the outer row.

$$F_d = (P - 2d) \times t \times S_t + n \times \frac{\pi d^2}{4} \times S_s. \tag{6}$$

(*E*) F_e = strength of plate between rivet holes in the third row, plus the shearing strength of rivets in the second row

in single shear and the rivets in single shear in the outer row.

$$F_e = (P - 4d) \times t \times S_t + n \times \frac{\pi d^2}{4} \times S_s. \tag{7}$$

(*F*) F_f = strength of plate between rivet holes in the second row, plus the crushing strength of butt strap in front of one rivet in the outer row.

$$F_f = (P - 2d) \times t \times S_t + d \times b \times S_c. \tag{8}$$

(*G*) F_g = strength of plate between rivet holes in third row, plus the crushing strength of butt strap in front of two rivets in the second row and one rivet in the outer row.

$$F_g = (P - 4d) \times t \times S_t + n \times d \times b \times S_c. \tag{9}$$

(*H*) F_h = crushing strength of plate in front of eight rivets, plus the crushing strength of butt strap in front of three rivets.

$$F_h = N \times d \times t \times S_c + n \times d \times b \times S_c. \tag{10}$$

(*I*) F_i = crushing strength of plate in front of eight rivets, plus the shearing strength of two rivets in the second row and one rivet in the outer in single shear.

$$F_i = N \times d \times t \times S_c + n \times \frac{\pi d^2}{4} \times S_s. \tag{11}$$

The efficiency of the joint will be found by dividing (*B*), (*C*), (*D*), (*E*), (*F*), (*G*), (*H*), or (*I*), whichever is the least, by (*A*).

$$e = \frac{\text{minimum strength} \times 100}{\text{strength across solid strip}} = \frac{\min F \times 100}{P \times t \times S_t} \text{ per cent} \tag{12}$$

The various strengths of the riveted joint listed above, except (*A*), are calculations based upon the various ways in which such a joint might fail. The following statements will explain the calculations.

(*A*) The strength *F* will be used in computing the relative strength.

(*B*) The strength F_b is the strength of the joint against failure of the main plate in tension at row 1 in Fig. 10.

(*C*) The strength F_c is the strength of the joint against failure of all the rivets in shear. The rivets in row 1 and 2 would be in single shear, and those in rows 3 and 4 would be in double shear.

(*D*) The strength F_d is the strength of the joint against failure of the main plate in tension at row 2. But if the main plate failed it would have to stretch, and would therefore shear the rivets in row 1. F_d is, therefore, the strength of the main plate in tension at row 2 plus the strength in shear of the rivets in row 1.

(*E*) The strength F_e is the strength of the joint against failure of the main plate in tension at row 3. This would require failure in single shear of the rivets in rows 1 and 2.

(*F*) The strength F_f is the strength of the joint against failure of the main plate in tension at row 2. This failure would require failure either by shear of the rivets in row 1, which is covered by F_d, or failure in bearing of the butt strap at row 1.

(*G*) The strength F_g is the strength of the joint against failure of the main plate in tension at row 3. This failure would require failure either by shear of the rivets in rows 1 and 2, which is covered by F_e, or failure in bearing of the butt strap in rows 1 and 2.

(*H*) The strength F_h is the strength of the joint against failure in bearing at all the rows. The weakest combination is bearing on the main plate at rows 3 and 4, plus bearing on the butt strap at rows 1 and 2.

(*I*) The strength F_i is the strength of the joint against failure in bearing on the main plate at rows 3 and 4, plus shearing of the rivets at rows 1 and 2.

For the above case, there is no need of calculating the possible failure of the main plate in tension at row 4, plus either bearing or shear in the other rows, because row 3 is just like row 4 as far as tension is concerned, and of the two, row 3 is the weaker one.

Example.—Design a longitudinal butt and double-strap joint, quadruple-riveted, for a boiler 60 in. in diameter, to withstand a steam pressure of 165 lb. per square inch.

The efficiency may be assumed as 0.95 from Table II.

From formula (1): $t = \dfrac{165 \times 60}{2 \times \dfrac{55{,}000}{5} \times 0.95} = 0.473$ in. or $\frac{1}{2}$ in.

From Table V: $d = {}^{15}\!/_{16}$ in. for a ½-in. plate $\therefore a = \frac{\pi d^2}{4} = 0.6903$ in.^{2}

From Table III: $P = 16d = 16 \times {}^{15}\!/_{16} = 15$ in.

From Table VIII: b, the thickness of the strap, $= {}^{7}\!/_{16}$ in. for a ½-in. plate.

From Sec. 150: $S_s = 44{,}000$ lb. per square inch
$S_t = 55{,}000$ lb. per square inch
$S_c = 95{,}000$ lb. per square inch

Then from:

(A), $F = 15 \times 0.5 \times 55{,}000 =$ 412,500
(B), $F_b = (15 - 0.9375) \times 0.5 \times 55{,}000 =$ 386,718
(C), $F_c = 8 \times 88{,}000 \times 0.6903 + 3 \times 44{,}000 \times 0.6903 =$ 577,090
(D), $F_d = (15 - 2 \times 0.9375) \times 0.5 \times 55{,}000 + 1 \times 44{,}000 \times 0.6903 =$ 391,310
(E), $F_e = (15 - 4 \times 0.9375) \times 0.5 \times 55{,}000 + 3 \times 44{,}000 \times 0.6903 =$ 400,495
(F), $F_f = (15 - 2 \times 0.9375) \times 0.5 \times 55{,}000 + 0.9375 \times 0.4375 \times 95{,}000 =$ 399,902
(G), $F_g = (15 - 4 \times 0.9375) \times 0.5 \times 55{,}000 + 3 \times 0.9375 \times 0.4375 \times 95{,}000 =$ 426,269
(H), $F_h = 8 \times 0.9375 \times 0.5 \times 95{,}000 + 3 \times 0.9375 \times 0.4375 \times 95{,}000 =$ 473,145
(I), $F_i = 8 \times 0.9375 \times 0.5 \times 95{,}000 + 3 \times 44{,}000 \times 0.6903 =$ 447,369

From formula (12) $e = \frac{386{,}718}{412{,}500} = 0.937$, which is approximately the efficiency of 0.95 which was assumed, so that the strength of the joint is ample.

156. Structural Joints.—Structural joints and connections are made in accordance with standard rules of construction that have grown out of successful practice. For such work the structural handbooks gotten out by the steel companies should be freely consulted. For connecting I-beams and channels, standard angles are recommended, the pitch and size of rivet depending upon the size of the members to be connected. The location of rivet holes in the flanges of all structural shapes should be in accordance with the dimensions given in the handbooks.

The pitch used for structural riveting should not be less than three times nor more than sixteen times the diameter of the rivet. If the pitch is too small, difficulties might arise in forming the heads on adjacent rivets, and if the distance between rivets is great, the plates may be in loose contact, allowing moisture to enter between the surfaces, and causing rust to form, which is detrimental to the joint.

Structural work is prepared in the shop and shipped knocked down in such sized units as can be conveniently handled in the

field. Riveted work that is done in the shop is called *shop* riveting, and the riveting that is done at the place of erection is called *field* riveting. The different elements that are to be connected in the field are given corresponding marks so that the members may be erected, joint by joint, according to a prearranged plan. Field riveting is not considered to be as well done as shop riveting, and for that reason the shear and bearing stresses on field rivets are given a lower value than for shop rivets.

157. Working Rules for Structural Work.—Adjustable members in any part of structures are to be avoided.

Sections should be made symmetrical.

No connection except lattice bars should contain less than two rivets. Where two or more plates are in contact, rivets should be spaced not more than 12 in. apart, to hold the plates together.

Maximum distance from any sheared edge to the nearest rivet should be 1½ in. for ⅞-in. rivets, 1¼ in. for ¾-in. rivets, 1⅛ in. for ⅝-in. rivets, and 1 in. for ½-in. rivets.

It is customary to disregard the friction between the plate surfaces and to proportion the rivets to take the entire stress transmitted. Bearing stresses of rivet and plate must be considered as well as shear stresses. A gusset plate should be the next higher commercial size thicker than the web or leg thickness to which it is connected. Rivets should not be smaller than ½ in. in diameter, and the sizes recommended are ½, ⅝, ¾, and ⅞ in.

158. Stresses Allowed in Structural Work.—For structures carrying cranes, conveyors, or any traveling machinery, the stresses should be considered 25 per cent higher than the stresses resulting for the live load.

	Pounds per square inch
Bearing pressure on shop rivets and pins	24,000
Bearing pressure on field rivets and bolts	20,000
Shear on shop rivets and pins	12,000
Shear on field rivets and bolts	10,000
Shear on webs of girders and rolled sections	10,000

The building codes of cities should be consulted for the allowable stresses which are recommended for different kinds of structural work in a given locality. In general, for dead loads, a factor of safety of 4 is considered ample.

159. Analysis of a Typical Joint.—The structural bracket in Fig. 14 is loaded as shown. The rivets are subjected to a direct shear distributed over the six rivets, and in addition there is a secondary shear over each rivet, due to the tendency of the bracket to rotate about the center of gravity of the rivets at o.

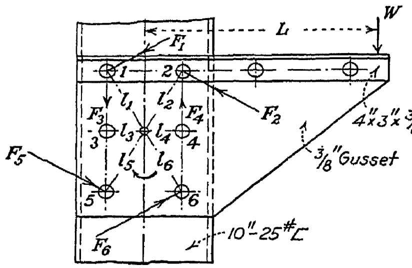


FIG. 14.

(*a*) The direct shear on each rivet $= W/6$.

(*b*) The turning moment is balanced by the sum of the shear moments acting on the six rivets.

$$M = W \times L = F_1l_1 + F_2l_2 + F_3l_3 + F_4l_4 + F_5l_5 + F_6l_6). \quad (13)$$

Since $\frac{F_2}{F_1} = \frac{l_2}{l_1}$ and also $\frac{F_4}{F_1} = \frac{l_4}{l_1}$, then $F_2 = \frac{F_1l_2}{l_1}$ and $F_4 = \frac{F_1l_4}{l_1}$, and substituting the values in formula (13) and simplifying:

$$M = WL = \frac{F_1}{l_1}(l_1{}^2 + l_2{}^2 + l_3{}^2 + l_4{}^2 + l_5{}^2 + l_6{}^2). \quad (14)$$

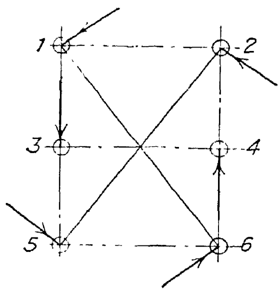


FIG. 15.

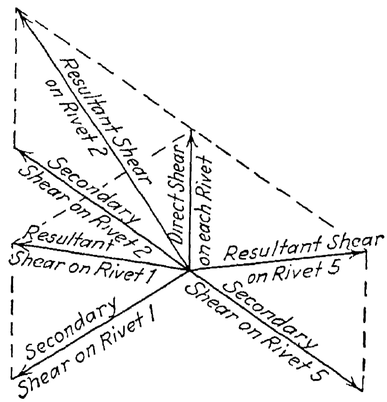


FIG. 16.

(*c*) The force tending to shear off each rivet will be the resultant of the direct and secondary shears, and its magnitude may be found graphically as shown by Figs. 15 and 16.

For this problem $R_1 = R_5$; $R_2 = R_6$; and R_4 and R_3 may be found from the algebraic sum of F_3 and $W/6$, and F_4 and $W/6$, respectively.

Problems

NOTE: For design purposes the ultimate strength of boiler plate steel is taken as $S_s = 44{,}000$, $S_t = 55{,}000$, and $S_c = 95{,}000$ lb. per square inch.

1. Show by sketches the various forms of rivet heads.
2. Determine the pressure required to hot rivet a boiler joint: (*a*) the rivet is $\frac{3}{4}$ in. in diameter and has a $\frac{3}{4}$-in. grip; (*b*) the rivet is $1\frac{1}{8}$ in. in diameter and has a $1\frac{1}{16}$-in. grip.
3. The pressure required for riveting when the grip is greater than the rivet diameter is determined by multiplying the pressure values in Table I by the square root of the grip of the rivet. Using this rule determine the pressure required to form the heads in the following:
   (*a*) A $\frac{3}{4}$-in. rivet with a grip of $1\frac{1}{4}$ in. for a boiler joint.
   (*b*) A 1-in. rivet with a grip of $1\frac{1}{2}$ in. for a structural joint.
   (*c*) A $1\frac{1}{4}$-in. rivet with a grip of 2 in. for tank work.
4. What sized rivets would be used for a double-riveted lap joint for: (*a*) $\frac{5}{16}$-in. boiler plate; (*b*) $\frac{7}{16}$-in. boiler plate?
5. How thick should the strap plates be for a two-strap quadruple-riveted butt joint?
6. Show by a sketch the six ways in which a riveted joint might fail.
7. (*a*) Design the longitudinal and girth lap joints, using boiler plate and steel rivets, for a horizontal return tubular boiler of 48-in. diameter subjected to a maximum working pressure of 125 lb. per square inch. (*b*) What thickness of steel plate would be used for the ends, considering strength only?
8. Design a double-riveted lap joint using steel plate and steel rivets, the plate being $\frac{3}{8}$ in. thick. The joint is to be equally strong in shear, tension, and compression. Find the diameter of the rivets, the pitch, and the relative strength of the joint.
9. A double-riveted butt joint of $\frac{5}{8}$ in. steel plate has two $\frac{1}{2}$-in. butt straps. The pitch of the rivets is 4 in., the diameter of the rivets is $\frac{7}{8}$ in. after being driven, and the relative strength of the joint is 0.77. Determine the resistance which the joint will offer to shear, tension, and bearing.
10. Design a double-riveted lap joint of steel plate and steel rivets, the plate being $\frac{9}{16}$ in. thick, and the relative strength of the joint being 0.75.
11. Make a full-size pencil drawing of a triple-riveted butt joint with two straps, similar to Fig. 9, and show all dimensions. The joint is to be designed according to the following data:
    Thickness of main plate is $\frac{7}{16}$ in.
    Rivet diameter after driving is $\frac{15}{16}$ in.
    Relative strength of joint is 0.83.
12. A 36-in. boiler drum is to carry a steam pressure of 200 lb. per square inch. Design the longitudinal seam and girth seam in accordance with the A.S.M.E. Boiler Code.

13. A water tank has a diameter of 42 in., is 6 ft. long, and is made of plates ¼ in. thick. The longitudinal joints are double-riveted lap joints with staggered riveting of 3-in. pitch, and the girth joints are single-riveted lap joints with a 2-in. pitch. The heads of the tank are flanged and dished to a radius equal to that of the shell. What safe pressure will the tank withstand with a factor of safety of 4? Assume $S_t = 50{,}000$ lb. per square inch.

14. An air tank is made of ⅜-in. steel plate and has a double-riveted lap joint. The pitch of the rivets is 3½ in., the back pitch is 2½ in., and the rivets are ⅞ in. in diameter after driving. Determine the efficiency of the joint and the allowable pressure for a factor of safety of 5, if the tank is constructed according to the A.S.M.E. Boiler Code.

15. The standard connection for a 12-in. I-beam is shown by Fig. 17. The rivets are 13/16 in. in diameter after driving. The beam is 12 ft. long and is loaded at the center with a load of 6,000 lb.

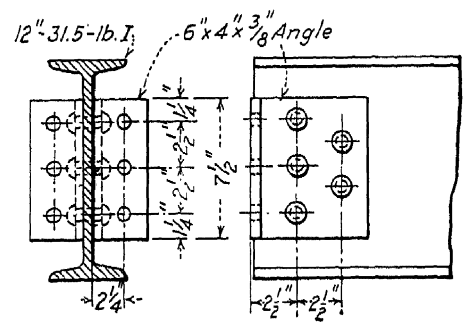


Fig. 17.

(*a*) Determine the maximum deflection of the beam due to the load.

(*b*) Determine the direct shear stress, the secondary shear stress, and the resultant shear stress on each rivet.

(*c*) If the beam tends to rotate about the lower edges of the angles, determine the tensile stress in each rivet and the resultant stress in each rivet.

16. The fabricated structural steel bracket shown in Fig. 14 is fastened to the channel column by six ¾-in. rivets (13/16-in. holes). The vertical spacing is 5 in. and the horizontal spacing is 6 in., center to center of rivets.

Assume $L =$ . . . . . in. and $W =$ . . . . . lb., and determine the following:

(*a*) The values of l_1, l_2, etc.

(*b*) The direct shear on each rivet.

(*c*) The secondary shear on each rivet.

(*d*) The resultant shear on each rivet.

Tabulate the results.

17. If a seventh rivet is placed midway between rivet 1 and rivet 2 in Fig. 14, and all the other data are the same as for Problem 16, make a table and give the results called for under (*a*), (*b*), (*c*), and (*d*) of that problem.

CHAPTER IX

SCREW FASTENINGS AND POWER SCREWS

160.—A *thread* is a helical ridge formed by cutting a groove into the surface of a cylindrical bar or hole, the former being an external thread and the latter an internal thread. The employment of screw threads necessitates the use of a pair of complementary threads, one external and the other internal, engagement being made by turning one threaded part into, or onto, the other.

When the threaded cylinder is turned clockwise to engage the second member, the thread is said to be *right hand*. Engagement of the two threads takes place when either member is turned, convenience in handling determining the choice. Threads may be cut right hand or *left hand*.

When the helical groove is cut so that the advance along the thread, in one turn of either member, is equal to the distance between similar points on adjacent threads, the thread is said to be *single* thread. When a greater advance per turn is required than can be had by a single thread, double, triple, and quadruple threads are used, and such threads are called *multiple* threads.

The *nominal* or *outside diameter* of a screw is the diameter of the crest or outside of the thread, and this is called the *major diameter*. The *core diameter* is the diameter of the bottom of the groove or *root* of the thread and is the measure of the tensile strength of the cylinder. This is called the *minor diameter*. The *depth* of the thread is the radial distance measured from the top to the bottom of the groove.

The *pitch* of the thread is the distance measured parallel to the axis, in inches, between similar points on adjacent threads. For example, a single thread of eight turns per inch has a $\frac{1}{8}$-in. pitch, but is commonly called an 8-pitch thread.

$$\text{Pitch (in inches)} = \frac{1}{\text{number of threads per inch}}.$$

The distance that either member of a threaded couple will travel in one turn along the axis of the thread is called *lead*. The

terms "lead" and "pitch" are often confused, the pitch and the lead of a single-thread couple being equal, while for a double thread the lead is double the pitch, and in a triple thread the lead is triple the pitch.

161. Historical.—In the early stages of the employment of the screw thread as a means of fastening, each shop often had a local adaptation of form and pitch of threads. This necessitated a large number of dies and taps, and caused expense and delay in manufacturing and repair work. In the year 1841, Sir Joseph Whitworth, owner and manager of the Whitworth Works in Manchester, England, proposed a system of screw threads which was adopted in an improved form in 1857, and is called the Whitworth or English Standard thread.

In America, William Sellers, of Philadelphia, made many studies of screw threads, and it is due to his findings and through his efforts that a standard system of screw threads was adopted in this country. The Sellers system was recommended by the Franklin Institute in 1864, and was afterwards adopted with slight modifications as the United States Standard by the Navy and War departments, and is now known under all these names: the Sellers, the Franklin Institute, or the U. S. Standard thread system.

In May, 1924, the American Engineering Standard Committee, sponsored by the Society of Automotive Engineers and the American Society of Mechanical Engineers, approved and adopted the form and profile of the Sellers or United States Standard thread and renamed it the American (National) Form of Thread. The above Committee is working to make the American System and the English Whitworth System of screw threads interchangeable.

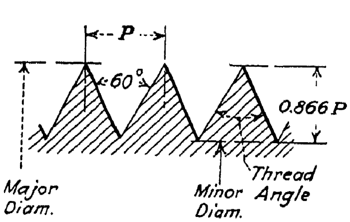


FIG. 1.—Standard full V thread.

162. Forms of Thread.—Figure 1 shows the *V-thread* which is used on bolts and screws of small diameter. This form is mechanically inferior to the American Standard.

Figure 2 shows the *United States Standard*, or *American*, form of thread, which should be used wherever possible for screw-thread work. Table I includes the more important data on the coarse-thread series of the American Standard threads.

Figure 3 shows the *Whitworth* form, which is the English Standard. It is difficult to cut this thread to its true form. It has the crest and groove bottom rounded when cut by a die, but when chased with a round-nose tool the crest is left flat.

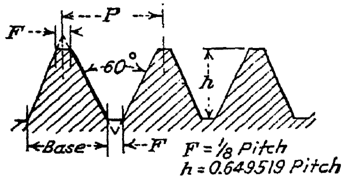


FIG. 2.—U. S. Standard or American thread.

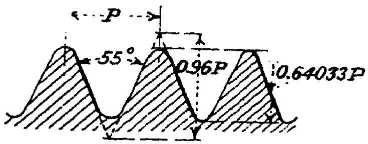


FIG. 3.—Whitworth or English Standard thread.

163. Thread Cutting.—Screw threads used on screws and bolts may be cut with dies, in an engine lathe, or on a milling machine. Threads which are formed on small screws are rolled on as described in Sec. 38. Rolled threads are imperfect because the crest of the thread extends beyond the cylindrical surface of the screw, hence the hole into which the screw is turned must be larger than the screw body. Threaded screws and small bolts are manufactured economically in large quantities by shops specializing in that product; however, if small threaded products are used in quantity in any industry it is more economical for the industry to install automatic screw machines and manufacture its own product.

164. Threaded Fastenings.—The threaded fastenings which are important by common usage are:

Through bolts or bolts.
Stud bolts or studs.
Tap bolts and cap screws.
Machine screws.
Set screws.

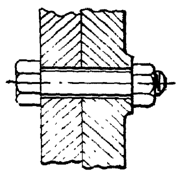
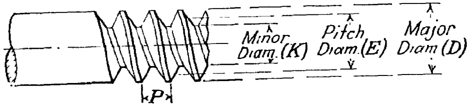


FIG. 4.

165. Through Bolt.—A *through bolt*, shown in Fig. 4, is a round bar, with a head on one end and a thread for a nut on the other. It is the best screwed fastener and passes through the parts which are to be held together. The body or shank of the bolt may or may not be finished, according to the requirements. If the bolt is subjected to any shear action, the body of the bolt should fit snugly into the hole, and when so applied the threaded portion should pass freely through the hole. The use of a through bolt

TABLE I.—COARSE-THREAD SERIES. AMERICAN STANDARD

(For general use in machine construction for bolts and screws)



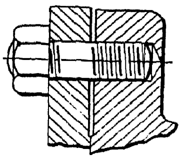
Identification		Basic diameters			Area in square inches		Working stress in pounds per square inch[1]		Bolt heads dimensions						
Number or size	Threads per inch (n)	Major diameter (D), inches	Minor diameter (K), inches	Tap drill sizes	of bolt body	of minor diameter	Ordinary work	Steam tight joints	Rough (Inches)	Fin. (Inches)	Rough (Inches)	Fin. (Inches)	Width across corners (Inches)		Nut thickness, inches
				79 % full thread					Width = 1½D	Width = 1½D except ¼ to 9⁄16, w = 1½D + 1⁄16	Height = ⅔D	Height = ¾D	Width = 1.14 × width across flats (approximate)	Width = 1.37 × width across flats (approximate)	Thickness = ⅞D
1	64	0.0730	0.0527												
2	56	0.0860	0.0628												
3	48	0.0990	0.0719												
4	40	0.1120	0.0795												
[a]5	40	0.1250	0.0925	0.0995											
6	32	0.1380	0.0974	0.1063											
8	32	0.1640	0.1234	0.1324											
[b]10	24	0.1900	0.1359	0.1477											
12	24	0.2160	0.1619	0.1732											
[c]¼	20	0.2500	0.1850	0.1990	0.049	0.0268	1,000	.....	⅜	7⁄16	11⁄64	3⁄16	0.485	0.584	7⁄32
5⁄16	18	0.3120	0.2403	0.2559	0.077	0.0452	1,000	.....	½	9⁄16	13⁄64	15⁄64	0.624	0.751	17⁄64
⅜	16	0.3750	0.2938	0.3110	0.110	0.0679	1,300	400	9⁄16	⅝	¼	9⁄32	0.691	0.832	21⁄64

7/16	14	0.4375	0.3447	0.3642	0.150	0.0935	1,500	500	5/8	3/4	19/64	21/64	0.830	1.000	3/8
1/2	13	0.5000	0.4001	0.4219	0.196	0.126	2,000	750	3/4	13/16	21/64	3/8	0.898	1.082	7/16
9/16	12	0.5625	0.4542	0.4776	0.294	0.162	2,200	1,000	7/8	7/8	3/8	27/64	0.966	1.163	31/64
5/8	11	0 6250	0.5069	0.5315	0.307	0.202	2,500	1,500	15/16	15/16	27/64	15/32	1.033	1.244	35/64
3/4	10	0 7500	0.6201	0.6480	0.442	0.302	3,000	2,000	1 1/8	1 1/8	1/2	9/16	1.240	1.494	21/32
7/8	9	0 8750	0.7307	0.7615	0.601	0.420	3,500	2,500	1 5/16	1 5/16	19/32	21/32	1.447	1.742	49/64
1	8	1.0000	0.8376	0.8723	0.785	0.550	4,000	.....	1 1/2	1 1/2	21/32	3/4	1.653	1.991	7/8
1 1/8	7	1.1250	0.9394	0.9789	0.994	0.693	4,500	3,000	1 11/16	1 11/16	3/4	27/32	1.859	2.239	1
1 1/4	7	1.2500	1.0644	1.1024	1.227	0.889	4,800	.....	1 7/8	1 7/8	27/32	15/16	2.067	2.489	1 3/32
1 1/2	6	1.5000	1.2835	1.3281	1.767	1.294	5,500	.....	2 1/4	2 1/4	1	1 1/8	2.480	2.986	1 5/16
1 3/4	5	1.7500	1.4902	1.5453	2.405	1.744	6,500	4,000	2 5/8	2 5/8	1 5/32	1 5/16	2.893	3.485	1 17/32
2	4 1/2	2.0000	1.7113	1.7717	3.142	2.300	7,000	.....	3	3	1 11/32	1 1/2	3.306	3.982	1 3/4
2 1/4	4 1/2	2.2500	1.9613	....	3.976	3.021	7,500	5,000	3 3/8	3 3/8	1 1/2	1 11/16	3.720	4.480	1 31/32
2 1/2	4	2.5000	2.1752	....	4.909	3.716	8,000	.....	3 3/4	3 3/4	1 21/32	1 7/8	4.133	4.977	2 3/16
2 3/4	4	2.7500	2.4252	......	5.940	4.619	8,500	.....	4 1/8	4 1/8	1 53/64	2 1/16	4.546	5.476	2 13/32
3	4	3.0000	2.6752	......	7.069	5.621	9,000	7,000	4 1/2	4 1/2	2	2 1/4	4.959	5.973	2 5/8

[a] Used in place of 1/8–40.
[b] Used in place of 3/16–24.
[c] Used in place of number 14–20.
[1] For mild steel.

presupposes a clearance for turning on the nut with a wrench. The bolt head and nut may be square or hexagonal, the latter shape allowing more accessibility in drawing up the nut. The wrench used should be the wrench which is designed to be used on the size of nut being applied.

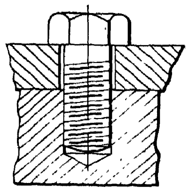
166. Stud Bolt.—A *stud bolt,* shown in Fig. 5, is a round bar with a screw thread formed on each end. It should be used in places where through bolts cannot be applied. The application of a stud bolt requires a tapped hole in one part, a second part through which the body of the stud passes, and a nut screwed down on the outer end, clamping the parts together. Studs are used when the connected parts have to be disconnected frequently; and to provide against the backing out of the stud, the tapped hole and the mating end of the stud are provided with more threads than are on the outside end of the stud, the added threads providing friction which prevents the stud bolt from turning. To accomplish the same purpose the thread end which is screwed into the tapped hole is sometimes made a few thousandths of an inch larger than the thread on the nut end. A special tool should be used for screwing studs into place, avoiding the use of a tool like an alligator-jaw wrench, which will cut into the stud, weaken the body, and disfigure the thread.



Stud Bolt

FIG. 5.

167. Tap Bolts and Cap Screws.—*Tap bolts* and *cap screws,* shown in Figs. 6 and 7, are used to fasten parts together by passing through one part and screwing into the other part, and are used when the parts must be disconnected only infrequently. The head of a tap bolt is usually hexagonal in shape, but may not always be of standard size, because the heads may be formed in bolt-heading machines or by turning the bolt out of hexagonal bar stock.



Tap Bolt

FIG. 6.

Cap screws have heads of various shapes, which are saw slotted so that a screwdriver may be used in applying them. The use of a screwdriver removes the danger of twisting off the heads because the twisting moment which may be applied by a screwdriver is less than that of a wrench. Tap bolts and cap screws are threaded about three-fourths of their length. There is at present no standard shape or size of heads for these bolts

and screws, and this lack of uniformity often causes confusion and inconvenience.

168. Machine Screws.—*Machine screws* are small cap screws and are made in the smaller sizes. The distinguishing feature of machine screws is the designation of size by gage number. The machine screws are measured by the arbitrary machine-screw gage, while cap screws are measured by the inch scale. For example, a 14–20 (No. 14 gage, 20 threads per inch) machine screw is 0.243 in. in diameter, while the same thread is

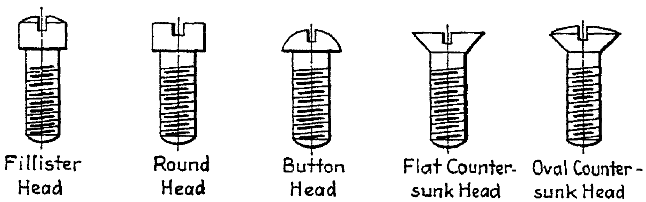
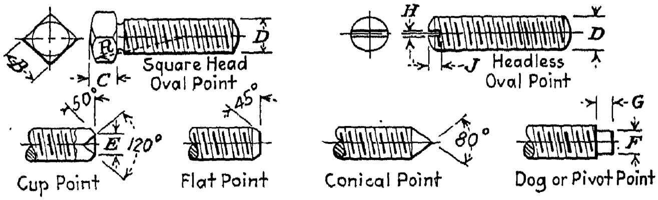


FIG. 7.—Various forms of cap screws.

cut on the 0.250-in. diameter cap screw; the result being that one is frequently sold for the other. The size just mentioned is the usual line of demarcation in the use of these two types of fasteners; above ¼ in. in diameter the cap screw is used and below ¼ in. diameter the machine screw is used.

169. Set Screws.—A *set screw* is a round bar threaded up to the head, and is used to prevent relative movement between two pieces by screwing through one part, the set screw point pressing against the other part. Members so connected can be readily adjusted, but the set screw fastening is not suitable for heavy work because it depends upon friction or a small shearing resistance. Set screws are made with and without heads. The ones with heads are square or hexagonal shaped, the length of the side being equal to the diameter of the threaded part. The headless set screws are slotted for a screwdriver or provided with a socket of various forms for the insertion of plug wrenches. The so-called "safety set screws" are of the latter type. The points of set screws are made in various shapes, and are usually casehardened to prevent upsetting, which might interfere with the removal of the set screw. Set screws are made in stock diameters and lengths, and employ the American screw-thread profile. From a number of forms, those in Table II are selected as being in most common use.

TABLE II.—DIMENSIONS OF SET SCREWS
(in inches)



D	N	B	C	R	E	F	G	H	J
1/4	20	1/4	3/16	1/2	0.166	0.185	1/8	0.049	1/16
5/16	18	5/16	15/64	5/8	0.208	0.240	5/32	0.065	3/32
3/8	16	3/8	9/32	3/4	0.250	0.293	3/16	0.065	3/32
7/16	14	7/16	21/64	7/8	0.291	0.344	7/32	0.083	1/8
1/2	12 or 13	1/2	3/8	1	0.333	0.391	1/4	0.083	1/8
9/16	12	9/16	27/64	1 1/8	0.375	0.454	9/32	0.083	5/32
5/8	11	5/8	15/32	1 1/4	0.416	0.506	5/16	0.109	3/16
3/4	10	3/4	9/16	1 1/2	0.500	0.620	3/8	0.134	7/32
7/8	9	7/8	21/32	1 3/4	0.583	0.730	7/16	0.134	1/4
1	8	1	3/4	2	0.666	0.837	1/2	0.165	9/32
1 1/8	7	1 1/8	27/32	2 1/4	0.750	0.939	9/16	0.165	9/32
1 1/4	7	1 1/4	15/16	2 1/2	0.833	1.064	5/8	0.165	5/16

Std. of the Hartford Machine Screw Co.

170. Holding Power of Set Screws.—The holding power which a set screw must exert at the surface of a shaft to transmit the torque equals:

$$F = \frac{63{,}025 \times \text{hp.}}{N \times r}, \tag{1}$$

in which F denotes the force which the set screw exerts, in pounds.
hp. denotes the horsepower transmitted by the set screw.
N denotes the revolutions per minute of the shaft.
r denotes the radius of the shaft, in inches.

By experiment, B.H.D. Pinkney[1] found the holding power of set screws to be as shown in Table III.

[1] *American Machinist*, p. 687, Oct. 15, 1914.

Table III.—Holding Power of Set Screws

Crest diameter of set screw, inches	Holding power or force, pounds	Crest diameter of set screw, inches	Holding power or force, pounds
1/4	100	5/8	840
5/16	168	3/4	1,280
3/8	256	7/8	1,830
7/16	366	1	2,500
1/2	500	1 1/8	3,288
9/16	658	1 1/4	4,198

Example.—What size set screw is necessary to fasten a pulley to a 2-in. shaft which transmits 7 hp. and rotates at 150 r.p.m.?

$$F = \frac{63,025 \times 7}{150 \times 1} \quad 2,940 \text{ lb.}$$

The table shows that one 1⅛-in. set screw or two ⅞-in. set screws will be ample.

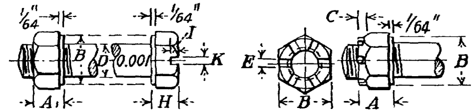
171. S. A. E. Bolts.—With the advent of the automotive industry a new screw-thread system was devised by engineers, which is identical with the American Standard thread profile, but has more threads to the inch. This fine thread is advantageous in that there is less danger of loosening the nut due to vibration (smaller helix angle of the thread), and a smaller displacement of the nut is made in one turn, resulting in finer adjustment. The dimensions of the head and bolt are different also from previous standards, and the nuts used are often castled and usually hardened, a split cotter pin being used for a locking device. The specifications for these bolts call for steel of 100,000 lb. per square inch ultimate tensile strength and 60,000 lb. per square inch elastic limit. These bolts are machine made to a close tolerance, and the dimensions are given in Table IV.

172. Tolerance.—The production in large quantities of an article which is made up of a number of parts, necessitates that the parts from which the article (machine or structure) is assembled, be made interchangeable. Interchangeability demands uniformity of separate parts, so that any part may be replaced by a like part out of stock. It is not possible to manufacture identical duplicate parts economically, hence, by agreement, manufacturers are allowed to deviate from exact dimensions, between limits. These limits, or *tolerances*, are assigned plus

Table IV.—Dimensions of S. A. E. Bolts and Nuts, Fine-thread Series. American Standard

(All dimensions are in inches)

(For automotive, aircraft, and other work requiring strength, reduced weight, and where a fine thread is required.)



Identification					Area in square inches		Dimensions of bolts, castled and plain nuts								
Sizes	Threads per inch (n)	Major diameter (D), inches	Minor diameter (K), inches	Tap drill sizes	of bolt body	of minor diameter	A	A_1	B	C	E	H	Slot I	Slot K	Drill size for pin
0	80	0.0600	0.0438												
1	72	0.0730	0.0550												
2	64	0.0860	0.0657	79 % of full thread											
3	56	0.0990	0.0758												
4	48	0.1120	0.0849												
5	44	0.1250	0.0955												
6	40	0.1380	0.1055												
8	36	0.1640	0.1279	0.1360											
(a) 10	32	0.1900	0.1494	0.1590											
12	28	0.2160	0.1696	0.1800											
(b) 1/4	28	0.2500	0.2036	0.2130	0.049	0.0325	9/32	7/32	7/16	3/32	5/64	3/16	3/32	1/16	1/16
5/16	24	0.3125	0.2584	0.2703	0.077	0.0524	21/64	17/64	1/2	3/32	5/64	15/64	7/64	1/16	1/16
3/8	24	0.3750	0.3209	0.3320	0.110	0.0809	13/32	21/64	9/16	1/8	1/8	9/32	1/8	3/32	3/32
7/16	20	0.4375	0.3725	0.3860	0.150	0.109	29/64	3/8	5/8	1/8	1/8	21/64	1/8	3/32	3/32
1/2	20	0.5000	0.4350	0.4492	0.196	0.149	9/16	7/16	3/4	3/16	1/8	3/8	1/8	3/32	3/32
9/16	18	0.5625	0.4903	0.5062	0.294	0.189	39/64	31/64	7/8	3/16	5/32	27/64	1/8	3/32	1/8
5/8	18	0.6250	0.5228	0.5687	0.307	0.215	23/32	35/64	15/16	1/4	5/32	15/32	1/8	3/32	1/8
3/4	16	0.7500	0.6688	0.6875	0.442	0.352	13/16	21/32	1 1/16	1/4	5/32	9/16	1/8	3/32	1/8
7/8	14	0.8750	0.7822	0.8024	0.601	0.480	29/32	49/64	1 1/4	1/4	5/32	21/32	1/8	3/32	1/8
1	14	1.0000	0.9072	0.9274	0.785	0.646	1	7/8	1 7/16	1/4	5/32	3/4	1/8	3/32	1/8
1 1/8	12	1.1250	1.0167	1.0401	0.994	0.812	1 5/32	63/64	1 5/8	5/16	7/32	27/32	7/32	5/32	1 1/64
1 1/4	12	1.2500	1.1417	1.1651	1.227	1.023	1 1/4	1 3/32	1 13/16	5/16	7/32	15/16	7/32	5/32	1 1/64
1 1/2	12	1.5000	1.3917	1.4153	1.767	1.521	1 25/64	1 5/16	2 3/16	3/8	1/4	1 1/8	1/4	3/16	1 3/64

(a) Used in place of 3/16 − 32. (b) Used in place of 14 − 28.

and minus values specified in 1/1,000 in., and workmen are furnished with limit gages for accurately measuring the dimensions.

173. Nuts.—Nuts are square and hexagonal in shape and in size conform to the bolt head for standard bolts. The thickness of a *full* nut is the same dimension as the diameter of the bolt. For example, a 1-in. nut is 1 in. thick, and the threaded hole is tapped for a 1-in. standard thread. The action of screwing down the nut on a bolt induces a shearing stress in the threads of the nut, but it is seldom that a full nut will fail in this manner; when *stripping* of the threads does occur it is usually due to incomplete or worn threads. Blanks for nuts are usually made by the cold- or hot-press process, in automatic machines. Cold-pressed nuts are of the smaller sizes. Nuts are threaded in a machine by passing a tap through the hole, and are usually faced off on one side to insure a flat-bearing surface, normal to the axis of the hole, or they may be faced and chamfered. A finished nut is finished all over, and is sometimes hardened.

For general design work when special thin adjusting and clamping nuts would be advantageous, a special range of 12-pitch threads are recommended, and are standard in the following diameters (in inches), with the American form of thread: $\frac{1}{2}$, $\frac{9}{16}$, $\frac{5}{8}$, $\frac{11}{16}$, $\frac{3}{4}$, $\frac{13}{16}$, $\frac{7}{8}$, $\frac{15}{16}$, 1, $1\frac{1}{16}$, $1\frac{1}{8}$, $1\frac{3}{16}$, $1\frac{1}{4}$, $\frac{5}{16}$, $1\frac{3}{8}$, $1\frac{1}{2}$, $1\frac{3}{4}$. This pitch and the above range of diameters are sometimes called "railroad sizes," taps being furnished with straight threads or with a taper of $\frac{3}{4}$ in. to the foot.

174. Tapped Holes.—Since commercial tap drills are made to produce approximately 75 per cent of the full depth of cut threads, holes should be tapped deeper than if full threads were cut, to offset the loss in strength. The use of tapered end taps, without using a second bottom tap, would also require the tapped holes to be deeper.

The depth of holes tapped into cast iron should not be less than one and one-half times the diameter of the screw, and if the screw is to be removed often the depth should be two times the screw diameter, because the screw thread in cast iron will wear faster than the steel thread. The depth of tapped holes in steel should not be less than one and one-fourth times the diameter of the screw.

175. Locking Devices for Nuts.—Nuts do not fit accurately because a certain amount of play is always allowed so that the

nut may be easily turned into place, consequently a nut will tend to unscrew due to frequent changes in load or due to vibrations. A *locknut* is used to prevent the gradual unscrewing of nuts. Locknuts should be on the under side, theoretically, as is shown in Fig. 8. However, since the average wrench is thicker than the locknut, this nut is *usually* placed on the outer side. Furthermore it can be shown mathematically that a nut of thickness 0.4 to 0.45 of the bolt diameter is as strong as the bolt thread; which is all that is required. There are many special nut-locking devices on the market, and several are shown in Fig. 9.

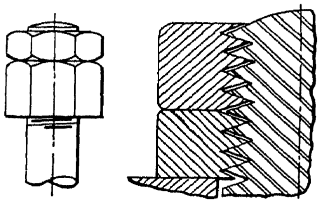


FIG. 8.

176. Stress in Bolts Due to Screwing Up.—It is evident that a stress is set up in the bolt, due to the squeezing action of the nut and bolt head pressing against the parts being held together. When the bolt holds parts together, and no external load tends to separate the parts, the stress in the bolt will be the resultant of the tensile stress due to screwing down the nut, and the torsional

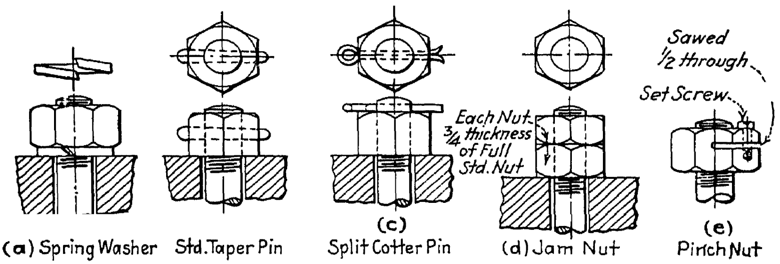


FIG. 9.—Various forms of locking devices.

stress caused by the frictional resistance between the threads of the bolt and nut.

There is no simple practical way to measure the stress set up in a bolt due to tightening the nut, and it is evident that a workman using ordinary wrenches may easily stretch a bolt to the point of rupture. This is especially true with the use of bolts of ½ in. diameter and smaller; hence, great care and good judgment should be exercised in the use of bolts of the smaller sizes.

From experiments at Sibley College, Cornell University,[1] it was found that the load produced by screwing down the nut

[1] BARR, J. H., "The Stress on Bolts in Service," *Sibley Journal of Engineering*, p. 1, October, 1902.

varied about as the diameter of the bolt, and could be estimated at 16,000 lb. per inch of bolt diameter. Expressed as a formula:

$$W = 16{,}000d, \qquad (2)$$

in which W denotes the initial load, in pounds.

d denotes the diameter of the bolt, in inches.

If A is the area at the minor diameter d_1 of the thread then the intensity of the stress is as follows:

$$S_t = \frac{W}{A}.$$

Substituting for the value of W:

$$S_t = \frac{16{,}000d}{A} = \frac{16{,}000d}{\frac{\pi d_1^2}{4}} = \frac{20{,}400d}{d_1^2} \text{ lb. per square inch} \qquad (3)$$

It is evident that this unit stress is excessive for bolts of small diameter, and experience shows that small bolts may actually be broken in the process of screwing up the nut.

177. Tension in Bolts Due to External Load.—When a load is applied which tends to separate the parts which are held together by a bolt, there is a tensile stress induced along the axis of the bolt, which may be in addition to the stress which already exists in the bolt due to screwing up the nut. Design engineers allow for the resultant of the initial and external load in the design of bolts by choosing a sufficiently large factor of safety.

178. Bolts Used for Fluid-tight Joints.—When bolts are used to fasten a joint which is to be tight against fluid pressure, the parts forming the joint must be pulled together more tightly than if the joint were an ordinary connection.

When the surfaces forming the joint are ground together to insure good contact, and the parts are clamped tightly together by screwing down the nuts, the bolts will stretch more than the other members will yield, and under this condition the bolt is designed for the initial load, or the external load, whichever is the greater.

If packing is used between the members joined together, the packing is made of relatively soft material, and consequently the bolts are less yielding than the other members. In this case the resultant load on the bolts is the sum of the initial and external loads.

To prevent leakage, bolts are placed relatively close together, however, there is no rule to follow for spacing, other than that which is practical for such classes of work.

179. Allowable Stresses in Bolt Design.—It is difficult to determine the true resultant stress to which bolts are subjected, but the values as given in Table V have been found satisfactory. These values are for bolts and studs made of mild steel of 60,000 lb. per square inch ultimate tensile strength. Bolts which are made of steel with a higher ultimate tensile strength will have proportionately higher allowable stresses than indicated in the table.

TABLE V.—ALLOWABLE UNIT STRESSES IN BOLT DESIGN
(For steel bolts)

Diameter of bolt, inches	Area at root, square inches	Allowable stress, pounds per square inches	
		Fluid-tight connections	Other than fluid-tight connections
1/2	0.126	700	2,000
5/8	0.202	1,400	2,500
3/4	0.302	2,000	3,000
7/8	0.420	2,500	3,400
1	0.550	2,750	3,900
1 1/8	0.692	3,000	4,300
1 1/4	0.890	3,250	4,700
1 3/8	1.057	3,450	5,100
1 1/2	1.293	3,650	5,500
1 5/8	1.510	3,800	5,800
1 3/4	1.741	4,000	6,300
1 7/8	2.050	4,300	6,600
2	2.300	4,600	7,000
2 1/4	3.030	5,000	7,000

180. Shear Stress in Bolts.—If bolts are to prevent sliding movement between parts, the body of the bolt should completely fill the hole, at least for a short distance on each side of the joint, and should be of sufficient cross-sectional area so that the shear stress is within safe limits. When the stress in bolts is due to the resultant of a shear action and a pull, pins are often used to carry the shear, and the bolts are designed to take the pull only. Such

pins are tapered ¼ in. to the foot, and may be bought from stock in various diameters and lengths.

181. Bolts Designed for Impulsive Load.—The least cross-section of a bolt is subjected to the greatest stress, consequently the threaded part of a bolt is where the greatest stretch occurs, so that when bolts rupture they invariably break in one of the threads unenclosed by the nut. In resisting impact or impulsive load the strength of the bolt depends upon the total extent of distortion due to a given intensity of stress; that is, the stress induced over the cross-section is less for a given energy if the stretch is over a greater length of bolt. Therefore, if the cross-sectional area of the bolt is uniform, the bolt will be of proper design to withstand shock or impulsive loads.

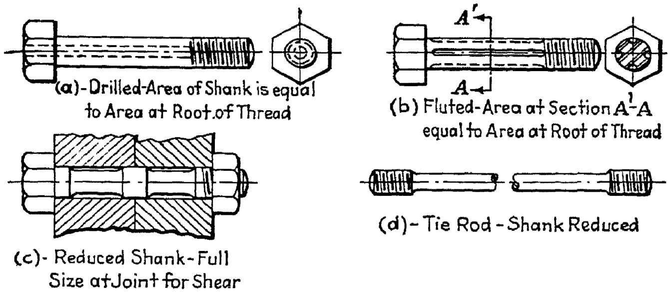


FIG. 10.—Bolts of uniform strength.

Tie rods used on bridges and on passenger and freight cars are made with the threaded ends upset so that the cross-sectional area at the root of the thread at the threaded ends is equal to the cross-sectional area of the rod. Other forms of bolts, designed for uniform strength, are shown in Fig. 10.

Example.—A connecting-rod end is fastened by two 1¼-in. bolts. The pressure in the cylinder causes a load of 5,000 lb. to act through a play of 0.004 in. The bolts are 8 in. long with American thread, the threaded length being 1½ in. For the steel in the bolts $E = 30{,}000{,}000$.

The load on each bolt is $\frac{W}{2} = \frac{5{,}000}{2} = 2{,}500$ lb. It will be assumed that the action of the 2,500-lb. force is equivalent to a weight of 2,500 lb. falling through a distance of 0.004 in. If the portion of the bolt enclosed in the nut does not stretch, then the energy must be absorbed by the stretch of 6½ in. of solid bolt and $1\frac{1}{2} - 1\frac{1}{4} = \frac{1}{4}$ in. of threaded bolt, the nut being assumed to be of full thickness. The unit stress in the threaded portion will be greater than the stress in the solid bolt, and for that reason it is not convenient to apply formula (26) in Chap. VII. The problem will be solved

by assuming that the energy to be absorbed will be divided between the two portions of the bolt in the same manner as under a static load.

The area at the root of the thread is 0.890 in.², and in the body of the bolt it is 1.23 in.². The corresponding unit stresses for a 2,500-lb. load are $S_1 = 2{,}820$ and $S_2 = 2{,}030$ lb. per square inch.

$$\text{Energy of threaded portion} = \frac{2{,}820^2}{2E} \times 0.89 \times 0.25 = 0.0294 \text{ in.-lb.}$$

$$\text{Energy of solid portion} = \frac{2{,}030^2}{2E} \times 1.23 \times 6.5 = 0.55 \text{ in.-lb.}$$

$$\text{Energy to be absorbed} = 2{,}500 \times 0.004 = 10 \text{ in.-lb.}$$

Of this energy the threaded portion will absorb 5.1 per cent, or 0.51 in.-lb.; and the solid portion will absorb 94.9 per cent, or 9.49 in.-lb.

$$\frac{S_1^2}{2E} \times 0.89 \times 0.25 = 0.51. \qquad S_1 = 11{,}700 \text{ lb. per square inch}$$

$$\frac{S_2^2}{2E} \times 1.23 \times 6.5 = 9.49. \qquad S_2 = 8{,}450 \text{ lb. per square inch.}$$

The above calculations are approximate because the total energy to be absorbed is 2,500 times 0.004 plus 2,500 times the stretch of the bolt. The stretch of the bolt on the basis of the above unit stress is:

$$e_1 = \frac{11{,}700}{E} \times 0.25 = 0.0000975.$$

$$e_2 = \frac{8{,}450}{E} \times 6.5 = 0.00183.$$

For a second approximation, therefore, the energy will be taken as $2{,}500 \times 0.0059 = 14.8$ in.-lb. On this basis the unit stresses are found to be $S_1 = 20{,}400$, and $S_2 = 10{,}500$ lb. per square inch. The maximum unit stress of 20,400 lb. per square inch is too large, and it would therefore be desirable to use bolts 1½ in. in diameter. This would reduce the maximum unit stress to 14,300 lb. per square inch.

Second Solution.—Section 144 of Chap. VII pointed out the effect of change of shape on the capacity of a member to absorb energy. This fact will be made use of in solving the above problem by making the area of the shank of the bolt the same as the area at the root of the threads. This can be accomplished by reducing the diameter of the shank.

Since the unit stress may now be taken as uniform throughout the length of the bolt, formula (26) of Chap. VII may be used. Two bolts 1½ in. in diameter will be assumed.

$$S = \frac{2{,}500}{1.29} = 1{,}940 \text{ lb. per square inch.}$$

$$e = \frac{1{,}940}{E} \times 6.75 = 0.000436.$$

$$S_1 = 1{,}940 + 1{,}940\sqrt{1 + 2 \times \frac{0.004}{0.000436}}.$$

$$= 1{,}940 + 8{,}520 = 10{,}460 \text{ lb. per square inch.}$$

It will be noted that the maximum unit stress is considerably lower than that calculated in the previous solution for the same number of bolts. The

lower stress is desirable for several reasons. The bolts would be subjected to a repeated impact and the stresses produced would be repeated stresses. At the root of the thread the unit stress may be three times the nominal calculated value, and such a stress repeated many times would start a crack if the stress were too high. The tests of Moore and Kommers[1] have shown that a specimen with a V-groove may have its endurance limit reduced by 60 per cent.

182. Importance of Location of Bolts.—It is important that a bolt be designed for the conditions of its use. Bolts are used primarily to resist pull, so that by placing them in, or close to, the direct line of action, stresses other than tension are often reduced to negligible values. When a series of bolts is subjected to a variety of loads, the bolt which is resisting the greatest load

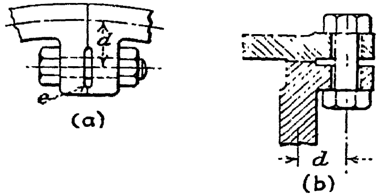


FIG. 11.

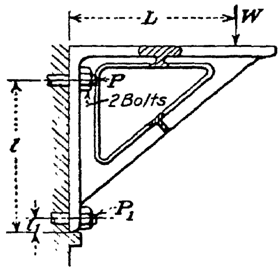


FIG. 12.

is designed, and the others made of like size, since the cost of the excessive material in the bolts is offset by other advantages, such as simplicity, uniformity, and appearance. In Fig. 11(a) the distance d should be as small as is practicable, allowing clearance for turning on the nut. In Figs. 11(a) and (b) the intensity of pressure over the joint is increased by the clearance, e, provided as shown. The surface against which the bolt head and nut bear should be faced for high-class construction. If this is done by a circular bearing concentric with the bolt hole, it is called "spot facing."

Example.—The cast-steel bracket in Fig. 12 is held in place by two bolts at the top and one at the bottom. The pull on the bolts P and P_1 are found as follows:

$$WL = 2Pl + P_1 l_1,$$

also

$$\frac{P}{P_1} = \frac{l}{l_1} \text{ or } P = \frac{P_1 l}{l_1}.$$

[1] MOORE and KOMMERS, "Fatigue of Metals," p. 198.

Substituting for P and simplifying:

$$WL = \frac{2P_1 l^2}{l_1} + P_1 l.$$

Then

$$P_1 = \frac{WLl_1}{2l^2 + l_1^2} \text{ and } P = \frac{WLl}{2l^2 + l_1^2}.$$

If, in the above problem, $W = 2{,}000$ lb., $L = 20$ in., $l = 16$ in., and l_1 3 in., then:

$$P = \frac{2{,}000 \times 20 \times 16}{2 \times 256 + 9} = \frac{640{,}000}{521} = 1{,}230 \text{ lb.}$$

$$P_1 = \frac{2{,}000 \times 20 \times 3}{2 \times 256 + 9} = \frac{120{,}000}{521} = 230 \text{ lb.}$$

It is seen that each upper bolt has about five times the load carried by the lower bolt. If the allowable tensile stress for steel bolts is taken as 2,000 lb. per square inch, each upper bolt will require an area at the root of the thread of $\frac{1{,}230}{2{,}000} = 0.614$ in.^{2}, and the lower bolt $\frac{230}{2{,}000}$ 0.115 in.^{2}. The nearest standard bolts for these values are $1\frac{1}{8}$ in. and $\frac{1}{2}$ in., respectively. However, for simplicity, all three bolts should be made $1\frac{1}{8}$ in.

NOTE: In this problem the shear is carried by the lug at the lower end of the part to which the bracket is bolted. If the bolts carry the shear load as well as the tensile load, then each bolt would have a shear load of $\frac{2{,}000}{3} =$ 667 lb. in addition to the tensile load. The resultant unit stress may be calculated by means of formulas (22) and (23) in Chap. VII.

183. Foundation Bolts.—*Foundation bolts* are used for securely fastening machines and structures to the foundations. Small

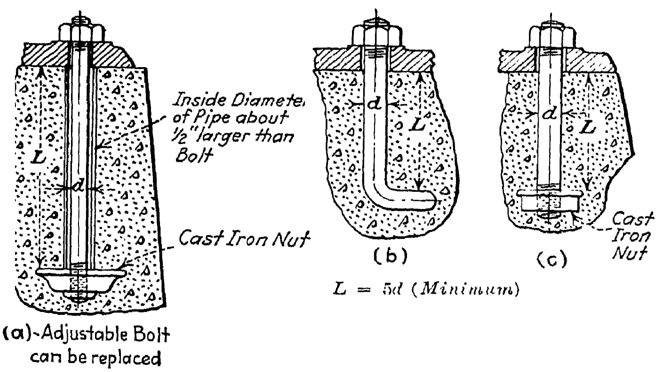


FIG. 13.—Three forms of foundation bolts.

bolts for light work are often anchored by pouring molten lead around the bolt while the bolt is held in the hole which has been drilled to receive it. Bolts used for heavy work are seldom less

than $1\frac{1}{8}$ in. in diameter. Foundation drawings are furnished by the machine builders, locating and specifying the size of bolts. Foundation-bolt holes in bed-plates of machines are cored about $\frac{1}{8}$ in. larger than the bolt size. To allow for core slippage the distances between bolts, as shown on the drawings, call for tolerance, and to insure their correct location, bolts are often set as shown by Fig. 13(a). Other forms of foundation bolts are shown by Figs. 13(b) and (c).

184. Eye Bolts.—*Eye bolts* are used for lifting machines and heavy machine parts. One or two bolts are screwed into tapped holes so located as to balance the load when it is suspended. The bolts are forged of mild ductile steel and the threads are

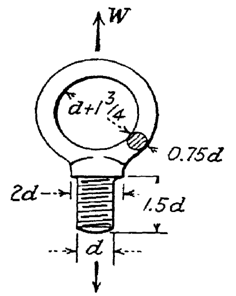


FIG. 14.

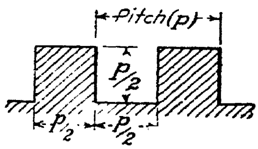


FIG. 15.—Square thread.

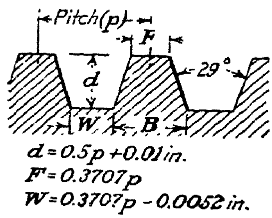


FIG. 16.—Acme thread.

standard. For the proportions of the eye bolt shown in Fig. 14, the safe load which the bolt will lift is:

$$W = 14{,}000A, \tag{4}$$

in which W denotes the load, in pounds.

A denotes the area at the root of the thread, in in.^{2}.

185. Power Screws.—The efficiency of screw threads used on fastenings is relatively low, the profile of the thread being designed for holding rather than for pushing or pulling. For transmitting power, threads with small or no thread angles are used. The ones commonly employed for power screws are the square, Acme, and buttress threads.

The square thread, shown in Fig. 15, is adapted for the transmission of power in either direction. It is a difficult thread to cut with taps or dies, so that it is usually milled or cut in a lathe. It has been replaced to some extent by the Acme thread.

The Acme thread, shown in Fig. 16, is a compromise between the square and the standard American thread, with $14\frac{1}{2}$-deg.

obliquity to the pressure faces, which is the same angle that is generally adopted for cutting worms. The sloping sides allow for the adjustment for wear in the enclosing nuts. This thread is easily cut with taps and dies, and is used for power screws on many machines.

The buttress thread, shown in Fig. 17, is a modification of the square thread, and is used for a push in one direction only. It has the strength of the V-thread and the advantage of the square thread. It is employed as the thread for light jack screws.

186. Torque and Efficiency of Square-threaded Screws.—The torque required to raise or lower a load by means of a screw may be determined by considering the portion of a jack screw shown in Fig.

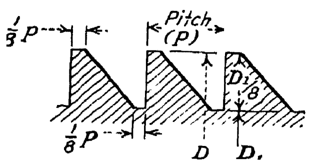


FIG. 17.—Buttress thread.

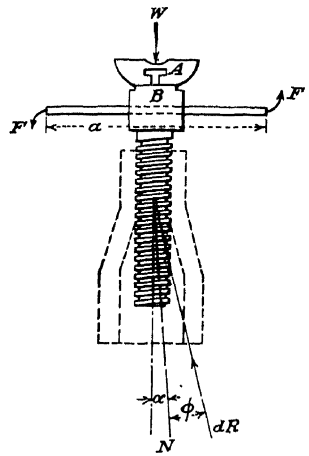


FIG. 18.

18. The couple which moves the screw is applied in a plane perpendicular to the axis of the screw, and has a moment Fa with respect to the axis of the screw. The total reaction of the nut on the screw threads is R and the force acting on an elementary area dA of the screw thread is called dR. As shown in the figure, the normal to the screw thread makes an angle α with the vertical, and dR acts at an angle ϕ with the normal. The angle α is the pitch angle of the screw threads, which has a tangent equal to $p/2\pi r$, in which p is the pitch and r is the mean radius of the threads. The angle ϕ is the angle of friction, whose tangent is equal to μ, the coefficient of friction.

For equilibrium, the sum of the forces parallel to the axis of the screw must equal zero, and the moments of the forces with respect to the axis of the screw must equal zero.

$$\Sigma Fy = -W + \Sigma dR \cos(\phi + \alpha) = 0.$$
$$\Sigma M = Fa - r\Sigma dR \sin(\phi + \alpha) = 0.$$

Hence:

$$Fa = r\Sigma dR \sin(\phi + \alpha).$$
$$W = \Sigma dR \cos(\phi + \alpha).$$

Dividing one equation by the other:

$$\frac{Fa}{W} = r \tan(\phi + \alpha),$$

$$Fa = Wr \tan(\phi + \alpha). \tag{5}$$

In formula (5) Fa is the torque necessary to overcome the frictional resistance at the screw threads. In addition to this torque there would be another torque required to overcome the friction at the collar between A and B in Fig. 18. If the coefficient of friction at the collar is μ_1, and this friction has a moment arm of r_c, then:

$$\text{Total torque} = Wr \tan(\phi + \alpha) + \mu_1 W r_c. \tag{6}$$

When the load is being lowered the force dR, in Fig. 18, falls on the other side of the normal, because friction is reversed, making the angle between dR and the axis of the screw equal to $(\phi - \alpha)$. For lowering the load, therefore:

$$\text{Total torque} = Wr \tan(\phi - \alpha) + \mu_1 W r_c. \tag{7}$$

An examination of this formula shows that when the pitch angle α is equal to the friction angle ϕ, the first term in the formula is equal to zero. If the pitch angle is still further increased the first term becomes negative and can be made to balance the second term. Any increase in the pitch angle over this critical amount would make the screw "overhaul," that is, it would turn under the load.

To determine the critical angle α when the screw will overhaul, the right side of formula (7) is put equal to zero.

$$0 = Wr \tan(\phi - \alpha) + \mu_1 W r_c.$$

Substituting:

$$\frac{\tan\phi - \tan\alpha}{1 + \tan\phi \tan\alpha} = \tan(\phi - \alpha),$$

$$\tan\alpha = \frac{\mu r + \mu_1 r_c}{r - \mu\mu_1 r_c}.$$

For the case when

$$= \mu_1 \text{ and } r \quad r_c:$$

$$\tan\alpha = \frac{2\mu}{1 - \mu^2}. \tag{8}$$

For the case when the coefficient of friction μ is 0.1, $\tan \alpha = 0.2$, and $\alpha = 11° 19'$.

To determine the *efficiency* of the screw, the work done by the load is compared with the energy expended in doing the work. When the load moves through a distance equal to the pitch p, the work done by the load is Wp. The work done by the couple is $2\pi Fa$, or, substituting the value of Fa from formula (6), the work equals $2\pi Wr \tan (\phi + \alpha) + 2\pi\mu_1 Wr_c$.

$$\text{Efficiency } e = \frac{Wp}{2\pi Wr \tan (\phi + \alpha) + 2\pi\mu_1 Wr_c},$$

or, since

$$\frac{p}{2\pi r} = \tan \alpha,$$

$$e = \frac{\tan \alpha}{\tan (\phi + \alpha) + \frac{\mu_1 r_c}{r}}. \tag{9}$$

If the collar friction is negligible or zero:

$$e = \frac{\tan \alpha}{\tan (\phi + \alpha)}. \tag{10}$$

Formula (8) showed that when $\mu = 0.1$ the screw will overhaul when α is greater than $11° 19'$. Using formula (9) and these data, and assuming further that $\mu_1 = \mu$ and that $r = r_c$, the efficiency will be about 49 per cent. It can be shown that any screw which does not overhaul will have an efficiency which does not exceed 50 per cent. Neglecting collar friction and examining formula (10) it is seen that when $\phi = \alpha$ the efficiency will be slightly less than 50 per cent, because the tangent of 2α is slightly greater than twice the tangent of α. Then as α decreases the efficiency will decrease, being always less than 50 per cent.

187. Torque and Efficiency of V-threaded Screws.—The derivation of the torque formula for screws with V-threads is not as simple as that for square threads, because the reaction dR on an elementary area of the thread makes an angle with the axis of the screw which is not very simply related to the angles α, ϕ, and β, the latter being half of the thread angle, or the angle between the adjacent faces of a thread.

Figure 19 is an attempt to show these angles in space. The element of area of the thread on which the force dR acts is at A, and AB is a line parallel to the axis of the screw. The line MAN (Fig. 19(a)) is the development of the screw thread. In

Fig. 19(b) the line AB is parallel to the axis of the screw, and AE is laid off in the plane $AFEB$ at an angle of $(\phi + \alpha)$ with AB, ϕ being the angle of friction, and α the pitch angle. The angle β between AE and AD is one-half of the included angle of a V-thread, and must be laid off in a plane at right angles to $AFEB$, as shown, DE making 90 deg. with the line FE. The final angle between the force dR and the axis of the screw is called γ.

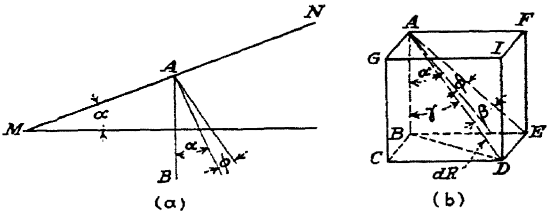


FIG. 19.

Summing up forces along the axis of the screw and taking moments about the axis of the screw:

$$\Sigma Fy = -W + \Sigma dR \cos\gamma = 0.$$
$$\Sigma M = Fa - r\Sigma dR \cos\beta \sin(\phi + \alpha) = 0.$$

Dividing the second formula by the first:

$$\frac{Fa}{W} = \frac{r \cos\beta \sin(\phi + \alpha)}{\cos\gamma}. \tag{11}$$

In Fig. 19(b) if AE is taken as unity, AB is equal to $\cos(\phi + \alpha)$ and AD is equal to $\sec\beta$, hence:

$$\cos\gamma = \frac{AB}{AD} = \cos(\phi + \alpha)\cos\beta.$$

Substituting in formula (11):

$$\text{Torque} = Fa = Wr\tan(\phi + \alpha). \tag{12}$$
$$\text{Total torque} = Wr\tan(\phi + \alpha) + \mu_1 W r_c. \tag{13}$$

$$\text{Efficiency} = e = \frac{\tan\alpha}{\tan(\phi + \alpha) + \frac{\mu_1 r_c}{r}}. \tag{14}$$

The efficiency of a 1-in. standard bolt will be examined. The assumption will again be made that $\mu = \mu_1$, and that $r_c = r$. The coefficient μ will be taken as 0.1, and for this thread, $\tan\alpha = 0.044$. Substituting in formula (14) the efficiency is 18 per cent. It is evident from this calculation that the efficiency

of standard bolts is very low, and this is an advantage, because it helps to prevent them from unscrewing.

For lowering the load on a V-thread screw, formula (13) may be used, except that $(\phi - \alpha)$ is used instead of $(\phi + \alpha)$.

McBride[1] reported some experiments on a lubricated V-thread screw, 2 in. in diameter and having a pitch of 0.22 in. The average efficiency was found to be about 10 per cent.

188. Relation of Efficiency to Pitch Angle.—In Fig. 20 are plotted a number of curves showing the variation of the efficiency of square-thread and V-thread screws with change of the pitch

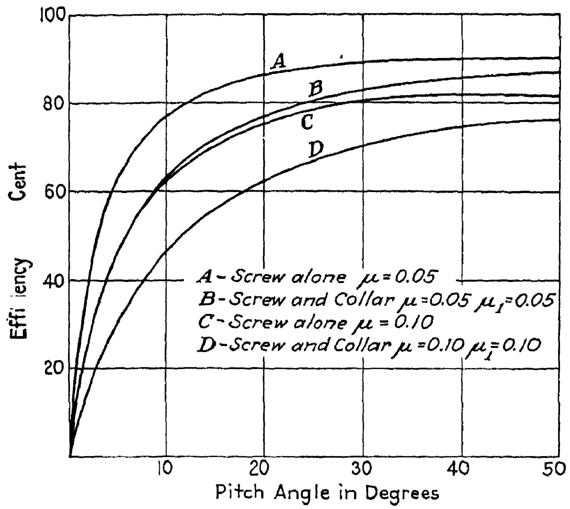


FIG. 20.—Efficiencies of square-thread and V-thread screws.

angle. Two different values of screw and collar coefficients of friction have been used. The curves show that high efficiencies are not obtained except with pitch angles which are so large as to be impracticable. For this reason power screws used in practice will be found to have low efficiencies.

189. Mechanical Advantage.—By *mechanical advantage* is meant the ratio of the resisting force to the driving force when a machine is operating at constant speed. Solving formula (5) for the load F:

$$F = \frac{Wr \tan (\phi + \alpha)}{a}.$$

[1] *Trans. A.S.M.E.*, Vol. XII, p. 781, 1891.

This equation shows that the driving force is decreased if the pitch angle α is decreased, thus increasing the mechanical advantage. On the other hand, Fig. 20 shows that the efficiency is increased by an increase of the pitch angle α. Since the use of large pitch angles to obtain high efficiencies is usually not practicable, the pitch used for a given screw will be a compromise between the two conflicting elements of mechanical advantage and efficiency.

190. Coefficients of Friction and Bearing Pressures.—The experiments of Kingsbury[1] on lubricated square-thread screws, working under pressures ranging from 3,000 to 10,000 lb. per square inch, and having low velocities, showed that the coefficient of friction varied from 0.03 to 0.25. Most of the results were above 0.11, so that an average value of 0.15 may be assumed. It is probable that a value of 0.10 would be a fair value to assume for low pressures and fairly high velocities.

The bearing pressure on the threads of a square-thread screw may be computed as follows:

W denotes the load on the screw, in pounds.
S_b denotes the bearing unit stress, in pounds per square inch.
n denotes the number of threads per inch.
t denotes the length or thickness of the nut, in inches.
d denotes the major diameter of thread, in inches.
d_1 denotes the minor diameter of thread, in inches.

In Fig. 18 the normal pressure N on the screw threads may be found by dividing the total reaction into a normal pressure and a frictional force;

then: $\Sigma Fy = -W - \mu N \sin \alpha + N \cos \alpha = 0.$

$$N = \frac{W}{\cos \alpha - \mu \sin \alpha}$$

$$S_b = \frac{N}{A} = \frac{W \cos \alpha}{(\cos \alpha - \mu \sin \alpha)\frac{\pi}{4}(d^2 - d_1^2)tn}$$

Since the term $\mu \sin \alpha$ will be negligible:

$$S_b = \frac{4W}{nt\pi(d^2 - d_1^2)} \tag{15}$$

[1] *Trans. A.S.M.E.*, Vol. XVII, p. 96, 1896.

For V-threads the component of the friction along the vertical will again be neglected. The normal pressure N makes an angle θ with the axis of the screw (see Fig. 19) such that:

$$\tan \theta \quad \frac{\sqrt{\tan^2 \beta + \sin^2 \alpha}}{\cos \alpha}$$

$$\Sigma Fy = -W + N \cos \theta = 0 \text{ (neglecting friction component).}$$

$$N = \frac{W}{\cos \theta}.$$

$$S_b = \frac{N}{A} = \frac{W \cos \alpha \cos \beta}{nt\frac{\pi}{4}(d^2 - d_1^2) \cos} \qquad \frac{4W \cos \alpha \cos \beta}{nt\pi(d^2 - d_1^2) \cos \theta}.$$

The above formula gives practically the same answer as that obtained by dividing the load W by the projected area of the threads normal to W:

$$S_b = \frac{4W}{nt\pi(d^2 - d_1^2)}. \qquad (16)$$

191. Axial and Torsion Stresses.—From Fig. 18 it is evident that the load W causes an axial unit stress P/A at the root of the thread, which may be either tensile or compressive in a power screw. The screw must also transmit a torque which is Fa minus the friction at the collar. If the collar is at the end opposite to the power end, the torque transmitted by the screw is Fa.

The axial unit stress is calculated by using the area at the root of the thread, and the torsional unit stress may be calculated by means of formula (18) in chap. VII, using the diameter at the root of the thread. The resultant unit stresses due to combined axial and torsion stress may be calculated by using formulas (22) and (23) in Chap. VII.

Example.—What force will be required at a radius of $2\frac{1}{2}$ in. to raise and lower a 2,400-lb. cross-bar of a planer? The bar is raised and lowered by two $1\frac{1}{2}$-in. square-threaded screws, having 3 threads per inch. Use will be made of a steel screw, a bronze nut $1\frac{1}{2}$ in. thick, and a steel collar having an outside diameter of 3 in. and an inside diameter of $1\frac{1}{2}$ in. The coefficient of friction at the threads is assumed as $\mu = 0.11$, and at the collar as $\mu_1 = 0.13$.

Diameter of screw at root of thread = 1.167 in.
Radius of screw at root of thread = 0.584 in.
External radius of screw = 0.75 in.

$$\text{Mean radius of screw} \quad \frac{0.75 + 0.584}{2} = 0.667 \text{ in.}$$

Pitch of thread $= 0.333$ in.

$$\text{Pitch angle:}\quad \tan\alpha = \frac{0.333}{2\pi \times 0.667} = 0.0795 \quad \alpha = 4^\circ\ 33'.$$

$$\text{Friction angle:}\quad \tan\phi = 0.11 \quad \phi = 6^\circ\ 17'.$$

$$\phi + \alpha = 10^\circ\ 50'. \quad \tan(\phi + \alpha) = 0.191.$$

The moment arm of the collar friction may be computed from the following formula:

$$r_c \quad \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) = \frac{2}{3}\left(\frac{1.5^3 - 0.75^3}{1.5^2 - 0.75^2}\right) = 1.17 \text{ in.}$$

Instead of using the above exact formula, the moment arm of the friction is often taken as the mean radius of the collar:

$$r_c = \frac{1.5 + 0.75}{2} = 1.13 \text{ in.}$$

Substituting in formula (6):

$$\begin{aligned}\text{Torque} &= 1{,}200 \times 0.667 \times 0.191 + 0.13 \times 1{,}200 \times 1.17 \\ &= 153 + 183 = 336 \text{ in.-lb.}\end{aligned}$$

The force which produces the torque may be applied with any moment arm which is convenient. If the moment arm is taken as 2.5 in., then:

$$F = \frac{336}{2.5} \quad 134 \text{ lb.}$$

From formula (9) the efficiency of the screw is:

$$\frac{0.0783}{0.191 + \frac{0.13 \times 0.667}{1.17}}$$

$$\frac{0.0783}{0.265} \times 100 \quad 29.6 \text{ per cent.}$$

To determine the torque necessary to lower the arm apply formula (7):

$$\begin{aligned}(\phi - \alpha) &= 1^\circ\ 44', \text{ and } \tan(\phi - \alpha) = 0.0302. \\ \text{Torque} &= 1{,}200 \times 0.667 \times 0.0302 + 183. \\ &= 24 + 183 = 207 \text{ in.-lb.}\end{aligned}$$

$$F = \frac{207}{2.5} = 83 \text{ lb.}$$

The unit stresses which are produced in the screw are as follows:

$$\frac{P}{A} = \frac{1{,}200 \times 4}{\pi \times 1.167^2} = 1{,}120 \text{ lb. per square inch.}$$

$$\text{Torsion stress } S_s = \frac{153 \times 16}{\pi \times 1.167^3} = 493 \text{ lb. per square inch.}$$

The resultant unit stresses from the combination of direct stress and shear stress are:

$$S_s' = \sqrt{\left(\frac{1{,}120}{2}\right)^2 + 493^2} = 745 \text{ lb. per square inch.}$$

$$\begin{aligned}S_c' &= \frac{1{,}120}{2} + \sqrt{\left(\frac{1{,}120}{2}\right)^2 + 493^2} \\ &= 560 + 745 = 1{,}305 \text{ lb. per square inch.}\end{aligned}$$

These stresses are very low and indicate that a smaller screw could have been used, but it must be noted that wear and column action must also be taken into account.

The bearing unit stress is:

$$S_b = \frac{4 \times 1{,}200}{3 \times 1.5 \times \pi(1.5^2 - 1.167^2)} = 382 \text{ lb. per square inch.}$$

Problems

1. Make a sketch showing the V-thread of the American form, and name the parts.
2. A rod 1 in. in diameter is suspended in a vertical position. The upper end of the rod projects through a steel plate and is supported by a 1-in. nut on a standard thread. The rod is of steel having an ultimate tensile strength of 60,000 lb. per square inch. What is the length of this rod if its weight is just enough to cause rupture?
3. The rod of Problem 2 was supported as described, but locked by a second nut screwed up against the under side of the plate. The second nut sets up an initial tension in the threaded end of the rod of 1,600 lb. per square inch. What is the length of this rod if its weight is just enough to cause rupture?
4. Sketch and give the general application and the advantages of:
   (*a*) A through bolt and nut.
   (*b*) A stud bolt and nut.
   (*c*) A tap bolt.
5. Sketch and give the general application and the advantages of:
   (*a*) Tap bolts.
   (*b*) Cap screws.
   (*c*) Machine screws.
6. Show by sketches the various shapes of cap screw heads.
7. Sketch and give applications of the several forms of set screws.
8. An 18-in. pulley on a 2-in. shaft which must transmit 8 hp. at 180 r.p.m., is to be fastened to the shaft by set screws. What size and how many set screws are required? Why would this form of fastening be considered for temporary work only?
9. Sketch and describe four kinds of nut-locking devices.
10. On the basis of the experiments at Cornell University: (*a*) what unit stress might be induced in a bolt ⅞ in. in diameter due to the screwing down of the nut? (*b*) What would be the initial load in pounds?
11. From the Cornell University experiments on bolts, show that by screwing down the nut the ordinary mechanic will cause excessive unit stress to be set up in bolts ½ in. in diameter and smaller.
12. A 16- by 30-in. Corliss engine cylinder is to have its cylinder head fastened by 16 stud bolts. The joint is ground, and the initial steam pressure is 165 lb. per square inch. According to the allowable unit stresses given in Table V, what should be the diameter of the bolts? Make a sketch of the stud bolt, approximately full size, giving all information and dimensions. The cylinder head is 1⅛ in. thick at the bolt circle.

13. A locomotive cylinder head is clamped on with twenty $1\frac{1}{8}$-in. stud bolts. An annealed copper gasket is used to prevent leakage which had developed. The cylinder dimensions are 20 by 24 in., and the boiler pressure is 200 lb. per square inch. What probable unit stress will be developed in the studs: (a) due to screwing down the nut; (b) due to the external load? (c) What is the factor of safety of the studs if the bar steel from which the studs were made has an ultimate tensile strength of 64,000 lb. per square inch?

14 A $1\frac{1}{2}$-in. tie rod with upset ends is to have American standard threads. What will be the pitch of the threads, and what will be the real strength of the rod?

15 How large a hole should be drilled into a bolt, similar to the one shown by Fig. 10(a), if the body of the bolt is $1\frac{1}{2}$ in. in diameter, and the tensile strength of the bolt is uniform for its full length? Does the hole conform to a size for standard drills? What size drill should be used?

16. The solid steel-flanged joint of an 8-in. extra-heavy pipe line is bolted by twelve $\frac{7}{8}$-in. bolts on a bolt circle of 13 in. diameter. The steam pressure is 250 lb. per square inch. The bolt material is high-strength steel having an ultimate tensile strength of 72,000 lb. per square inch.

(a) If no packing is used between the flanges, what is the probable unit tension in each bolt?

(b) If an elastic packing is used between the flanges, what is the probable unit stress in each bolt?

17. Two bolts 1 in. in diameter are subjected to an impulsive load of 2,000 lb. which acts through a distance of 0.006 in. because of lost motion in the parts. The bolts are each $6\frac{1}{2}$ in. long and enclose parts of a total thickness of $4\frac{3}{4}$ in. Two nuts, each $1\frac{3}{16}$ in. thick, secure and lock each bolt in place, the top of the locknut being flush with the end of each bolt. Assuming that the threaded portion of the bolts which is enclosed by the nut does not stretch, what probable unit stress is induced in the bolts?

18. A pillar crane is bolted to its foundation by 8 bolts equally spaced on a bolt circle 36 in. in diameter. The diameter of the flange of the pillar base is 40 in. Determine the diameter of the bolts if a load of 12,000 lb. is lifted at a maximum radius of 14 ft., using an allowable unit tensile stress of 10,000 lb. per square inch. The working radius of the crane is 360 deg.

19 A cast-iron wall bracket similar to the one shown in Fig. 12 is fastened to the wall by two $\frac{3}{4}$-in. through bolts, one bolt at P and one at P_1, respectively. A load W, at a distance of 24 in. from the wall, caused the upper bolt to rupture. If l_1 is 3 in., and L is 24 in., what was the value of the load? Assume 0.20 per cent carbon steel for the bolts, and that the bolts were subjected to tensile loading only.

20. A cast-steel bracket similar to the one shown in Fig. 12 is fastened to a steel column by two bolts at P and one bolt at P_1. A taper pin located midway between the bolts is assumed to carry the shear.

Data.—$W = 6{,}000$ lb.
$L = 36$ in.
$l = 28$ in.
$l_1 = 3$ in.

Allowable unit tensile stress = 8,000 lb. per square inch.

(*a*) Determine the size of the taper pin. (The pin is 3 in. long, and tapers ⅛ in. in 12 in.)

(*b*) Determine the size of the upper bolts.

(*c*) Determine the size of the lower bolt, assuming that it would be made the theoretical size.

21. A motor weighing 3,700 lb. is to be lifted by an eye bolt similar to the one shown in Fig. 14. If soft steel of 58,000 lb. per square inch ultimate tensile strength is used for the forging, design the eye bolt and show by a shop sketch all information and dimensions necessary to forge and finish it. Use a factor of safety of 10.

22. What is the diameter and depth of the tapped hole in the cast-iron motor frame of Problem 21? The motor frame is made of a good grade of cast iron having an ultimate tensile and shearing strength of 20,000 lb. per square inch. Use a factor of safety of 12.

23. An eye bolt has a stem 1½ in. in diameter and 2 in. long. The thread is of American standard form. If the bolt is proportioned correctly, what is the greatest load which could be lifted if the tapped hole in the part to be lifted is:

(*a*) Cast iron, with ultimate tensile and shear strength equal to 22,000 lb. per square inch.

(*b*) Cast steel, with ultimate tensile and shear strength of 64,000 and 48,000 lb. per square inch, respectively.

24. A 2-in. jack screw with 2 square threads per inch is turning with a force of 30 lb. acting on a 24-in. jack lever. The friction collar is 4 in. outside diameter, and 2 in. inside diameter. The coefficient of friction between the screw threads is 0.15, and at the collar it is 0.10.

(*a*) What load is being lifted?

(*b*) What is the efficiency of the screw and collar?

25. A sluice gate weighing 80 tons is raised and lowered by means of two square-thread screws 3 in. major diameter, 1¾ in. minor diameter, and with 1¾ threads per inch. The radius of the friction collar is 1½ in., the coefficient of friction at the threads is 0.13, and at the collar it is 0.11. The two screws are driven by one motor which has a speed of 600 r.p.m. The reduction-gear mechanism has a combined mechanical efficiency of 86 per cent.

(*a*) What is the speed of the power screws if the gate is to lift 5 ft. in 2 min.?

(*b*) What is the motor horsepower required to raise the gate?

(*c*) What is the motor horsepower required to lower the gate?

CHAPTER X

KEYS

192. *Keys* are steel fastenings extending into two machine or structural parts to prevent relative movement between them by means of frictional, shearing, or compressive resistance. Keys are of circular or rectangular cross-section, of various shapes and lengths. Within recent years the American Society of Mechanical Engineers has attempted to eliminate some of the shapes and sizes of keys, and to specify standard cross-sections of keys and standard keyseats. Keys are used to fasten pulleys and toothed gears to shafting, and the kind and size of key is dictated by the load to be transmitted. Keys are easy to replace and cost little, so that in some assemblages of parts which may be subjected to sudden loads or accidental stresses, the key is designed to rupture, thereby functioning as a safety device for the other elements.

193. Common Forms of Keys.—When the load is light, a *saddle key* may be used, as shown in Fig. 1(*a*). The key fits into the

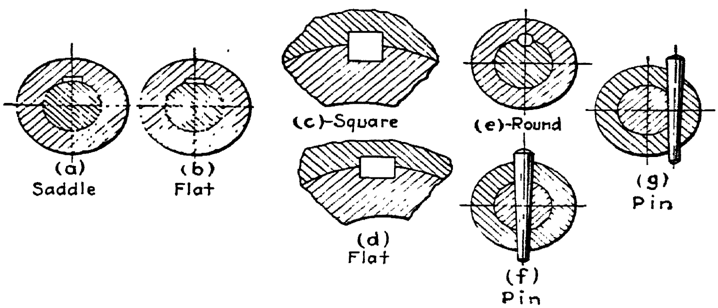


FIG. 1.—Forms of keys.

keyway of the hub and presses against the shaft, forcing the two surfaces apart. This key holds by frictional resistance only. The shaft may be flattened slightly, as shown in Fig. 1(*b*), and a *flat key* used. This key will transmit more power than the saddle key, but here also the key depends upon frictional resistance for its holding power.

The *square key*, Fig. 1(*c*), and the *flat key*, Fig. 1(*d*), are the more common forms of key fasteners. The *round key*, shown in Fig. 1(*e*), is used with excellent results by some manufacturers, the key being tapered and fitting a reamed hole. Keys as shown in Figs. 1(*f*) and 1(*g*) are often used for fastening the hubs of gears and wheels to a shaft. The hole is tapered and the key is a standard tapered pin, which will usually shear if the load becomes excessive, with no damage to the hub or shaft.

A *straight key* with parallel sides is the most desirable one, but since it cannot take up any play between the shaft and hub, it requires accurate fitting of the hub and shaft.

The advantage of the *taper key* is that its wedging action will take up play between the parts, but it may throw them out of alignment unless skill is used in assembling. The taper is usually 1/8 in. per foot, and should be on the top or hub side. It is more difficult to cut a taper keyseat in the shaft, and the cross-section of the shaft at the deep end of the keyway is weakened by cutting a taper keyway. Taper keys should fit accurately on the sides as well as on the top and bottom.

The *square key* has parallel sides and parallel top and bottom. It transmits force by its resistance to shear because it usually has a slight clearance at the top and bottom. The hub is closely fitted to the shaft which insures accurate concentricity of parts. Square keys are often used for parts which may have to be frequently disconnected.

Most keys are modified square keys with the width greater than the height, and are called *flat keys*. They are often tapered, and fit on all sides, but if driven in too tightly may cause the connected parts to spring.

A *draw key* has a lug formed on one end of the key for easy withdrawal by inserting a wedge between the lug and the hub. These keys should not be used in places where the protruding head, while rotating with the shaft, might become a hazard.

A *feather* or *spline key* prevents relative rotation of connected parts, but allows free axial sliding of one part over the other. It is a key of rectangular section, usually slightly higher than wide. It fits only on the sides, allowing one member to slide, and is fastened either to the shaft or to the hub. It may be secured to the shaft by countersunk machine screws as in Fig. 2(*a*), or it may move with the hub, secured as shown in Fig. 2(*b*). The length of feather keys is usually made greater than that of

sunk keys, for the same size of shaft, in order to reduce bearing pressure and ensuing wear on the sliding surfaces.

The *Woodruff key,* shown in Fig. 3(*b*), is a product of the Pratt and Whitney Manufacturing Company, of Hartford, Connecticut. The keyseat in the hub is of the usual form, while that in the shaft is of circular outline, made in a single cut by sinking the cutter into the shaft, as shown in Fig. 3(*a*). The

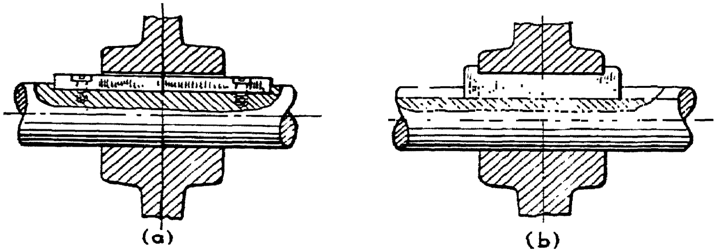


FIG. 2.—Feather keys.

maximum depth of the keyway is much greater than that of ordinary keys, consequently the shaft is weakened. However, the key is set so firmly that it cannot tip. On account of its circular bottom it will accommodate itself to an inclined keyway in the hub, while an ordinary key, in such a case, would perhaps bear on one point only. Several such keyways may be inserted in line, as shown in Fig. 3(*b*), matching the same keyseat in the

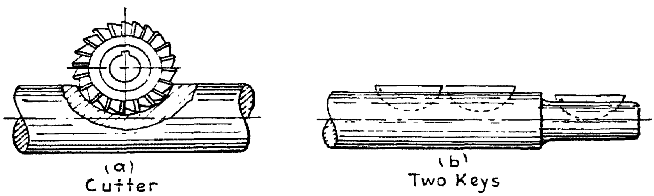


FIG. 3.—Woodruff key.

hub, to avoid cutting too deeply into the shaft, as would be the case if but one key were used.

A *round key* is a pin, either cylindrical or tapering. When using taper pins it is best to use standard taper pins, because these may be purchased more cheaply than they can be made in small lots. Furthermore, reamers may be readily obtained to correspond to the various sizes. Such keys are used only for light and small work, as a rule, and provide a cheap and accurate fastener for hubs to shafts. When taper pins are located on the

parting line between parts as shown in Fig. 1(*e*), the key offers more resistance to shear than if the pin penetrates the hub and shaft, as in Fig. 1(*f*), and when placed on the parting line may be used for relatively heavy loads.

194. Material of Keys.—Small keys are made of mild or machinery steel, procured from market stock in square and rectangular shapes and in various sizes. Large keys are usually forged from what is known as "cheap toolstock," machined to size, and fitted accurately by hand.

195. Stresses in Keys.—In Fig. 4(*a*) the key has a length L, a thickness t, and a width b. On the basis of shear, if S_s is the allowable unit shearing stress for the key material, the force which the key can transmit is:

$$F_s = b \times L \times S_s. \tag{1}$$

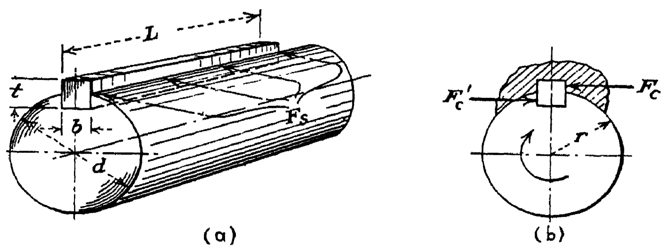


Fig. 4.

On the basis of compression or bearing, if S_c is the allowable unit bearing stress, the force which the key can transmit is:

$$F_c = \frac{t}{2} \times L \times S_c. \tag{2}$$

Since the material in the shaft or the hub may offer less resistance to crushing than the key, S_c should always be the least safe allowable unit stress for the combination.

The torque or twisting moment M_s which the key can transmit on the basis of shear is:

$$M_s = S_s \times b \times L \times r, \tag{3}$$

in which r denotes the radius of the shaft.

The torque which the key can transmit on the basis of bearing is approximately:

$$M_c = S_c \times \frac{t}{2} \times L \times r. \tag{4}$$

The resisting moment of a solid circular shaft is:

$$M_t = \frac{\pi d^3}{16} S_s',$$

in which d denotes the diameter of the shaft, in inches.

S_s' denotes the allowable shearing unit stress for the shaft, in pounds per square inch.

196. Dimensions of Keys.—There is no fixed standard for the dimensions of keys, the usual practice being to make the width of the key about one-quarter of the shaft diameter.

If the twisting moment of the key is based upon shear and is to be equivalent to the resisting moment of the solid shaft, then:

$$S_s \times b \times L \times \frac{d}{2} = \frac{\pi d^3}{16} S_s'. \tag{5}$$

If the quality of the steel is the same in the shaft and in the key, so that $S_s = S_s'$, then:

$$b \times L \times \frac{d}{2} = \frac{\pi d^3}{16}. \tag{6}$$

If, furthermore

$$\frac{d}{4},$$

then:

$$L \times \frac{d^2}{8} \quad \frac{\pi d^3}{16},$$

$$L = \frac{\pi d}{2} \quad 1.57d. \tag{7}$$

This value conforms closely to the minimum length for hubs. Experience shows that hubs less than $1.5d$ in length are likely to rock on the shaft. The above calculation shows that keys which are $0.25d$ in width and $1.5d$ in length are satisfactory to resist shearing of the key.

The key must also be designed to resist the bearing stress safely. Assuming that the width of the key is $0.25d$ and the length $1.5d$, then, on the basis of bearing:

$$S_c \times \frac{t}{2} \times 1.5d \times \frac{d}{2} = \frac{\pi d^3}{16} S_s'. \tag{8}$$

For mild steel, $S_s' = 0.8S_c$, and substituting this for S_s':

$$t = 0.4d, \text{ approximately}. \tag{9}$$

If

$$b = \frac{d}{4},$$

then:

$$t = 1.6b. \tag{10}$$

These calculations show, therefore, that a mild steel key which will transmit the full torque in the shaft, should have the following proportions:

$$b = \frac{d}{4},\ t = 1.6b, \text{ and } L = 1.5d.$$

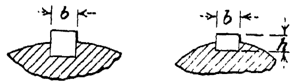
The height or thickness of keys is often less than the width, with the result that such keys are liable to bearing failure. This may be overcome by making the key longer if the key is to transmit the full strength of the shaft.

Table I shows the dimensions for square and flat stock keys.

TABLE I.—SQUARE AND FLAT STOCK KEYS

(American Standard)

(To be cut from cold-finished stock and to be used without machining)

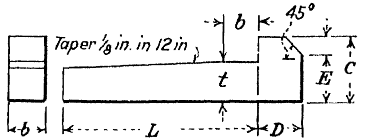


Shaft diameter (d), inches, inclusive	Square stock key b inches	Flat stock key $b \times h$ inches	Shaft diameter (d), inches, inclusive	Square stock key b inches	Flat stock key $b \times h$, inches	A negative (−) tolerance is allowed as follows: Key stock	Tolerance, inches
$\frac{1}{2}$ to $\frac{9}{16}$	$\frac{1}{8}$	$\frac{1}{8} \times \frac{3}{32}$	$2\frac{5}{16}$ to $2\frac{3}{4}$	$\frac{5}{8}$		$\frac{1}{8}$ to $\frac{3}{8}$ incl.	0.0020
$\frac{5}{8}$ to $\frac{7}{8}$	$\frac{3}{16}$	$\frac{3}{16} \times \frac{1}{8}$	$2\frac{7}{8}$ to $3\frac{1}{4}$	$\frac{3}{4}$		$\frac{1}{2}$ to $\frac{3}{4}$ incl.	0.0025
$\frac{15}{16}$ to $1\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4} \times \frac{3}{16}$	$3\frac{3}{8}$ to $3\frac{3}{4}$	$\frac{7}{8}$		$\frac{7}{8}$ to $1\frac{1}{2}$ incl.	0.0030
$1\frac{5}{16}$ to $1\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	$3\frac{7}{8}$ to $4\frac{1}{2}$	1			
$1\frac{13}{16}$ to $2\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	$4\frac{3}{4}$ to $5\frac{1}{2}$	$1\frac{1}{4}$			
			$5\frac{3}{4}$ to 6	$1\frac{1}{2}$	$1\frac{1}{2} \times 1$		

Table II shows the dimensions for gib-head taper stock keys, and Table III those for plain taper stock keys.

197. Friction in Sliding Keys.—When a gear or clutch slides along a shaft to which it is keyed, it is good practice to use two keys spaced 180 deg. apart. Experiments have shown that the friction of two keys so located is one-half of that for one key.

TABLE II.—GIB-HEAD TAPER STOCK KEYS
(American Standard)



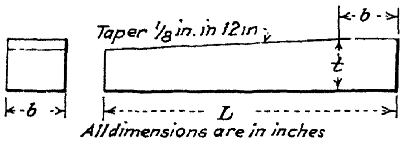
All dimensions are in inches

Diameter of shaft, inches, inclusive	Square type					Flat type					Stock length of key
	Key		Gib head			Key		Gib head			
	b	h	C	D	E	b	h	C	D	E	L
1/2 to 9/16	1/8	1/8	1/4	7/32	5/32	1/8	3/32	3/16	1/8	1/8	1/2 to 2 increment 1/4 in.
5/8 to 7/8	3/16	3/16	5/16	9/32	7/32	3/16	1/8	1/4	3/16	5/32	3/4 to 1 1/8 increment 3/8 in.
15/16 to 1 1/4	1/4	1/4	7/16	11/32	11/32	1/4	3/16	5/16	1/4	3/16	1 to 4 increment 1/2 in.
1 5/16 to 1 3/4	3/8	3/8	11/16	15/16	15/32	3/8	1/4	7/16	3/8	5/16	1 1/2 to 2 1/4 increment 3/4 in.
1 13/16 to 2 1/4	1/2	1/2	7/8	19/32	5/8	1/2	3/8	5/8	1/2	7/16	2 to 8 increment 1 in.
2 5/16 to 2 3/4	5/8	5/8	1 1/16	23/32	3/4	5/8	7/16	3/4	5/8	1/2	2 1/2 to 3 3/4 increment 1 1/4 in.
2 7/8 to 3 1/4	3/4	3/4	1 1/4	7/8	7/8	3/4	1/2	7/8	3/4	5/8	3 to 12 increment 1 1/2 in.
3 3/8 to 3 3/4	7/8	7/8	1 1/2	1	1	7/8	5/8	1 1/16	7/8	3/4	3 1/2 to 14 increment 1 3/4 in.
3 7/8 to 4 1/2	1	1	1 3/4	1 3/16	1 3/16	1	3/4	1 1/4	1	1 3/16	4 to 16 increment 2 in.
4 3/4 to 5 1/2	1 1/4	1 1/4	2	1 7/16	1 7/16	1 1/4	7/8	1 1/2	1 1/4	1	5 to 20 increment 2 1/2 in.
5 3/4 to 6	1 1/2	1 1/2	2 1/2	1 3/4	1 3/4	1 1/2	1	1 3/4	1 1/2	1 1/4	6 to 24 increment 3 in.

TABLE III.—PLAIN TAPER STOCK KEYS SQUARE AND FLAT
(American Standard)

The height of the key is measured at b distance from the large end.

All dimensions for these keys are the same as those shown for gib-head keys in the preceding table.



There are many forms of keys used, especially on large shafting, or where the service is severe.

198. Multiple Splines.—When the force to be transmitted is large in proportion to the size of the shaft, as in automobile transmissions and sliding-gear transmissions, a multiple spline is used, the spline being integral with the shaft. Such shafting is cut with four, six, ten, or sixteen splines.

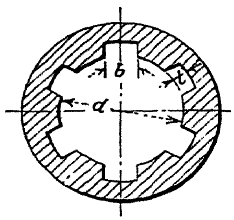


FIG. 5.—Six-spline hub.

The Society for Automotive Engineers has standardized the fittings for this kind of work, and the splines are designed from empirical formulas. Figure 5 shows a six-spline fitting, which, for sliding under no load, has dimensions as follows:

$$d = 0.850D.$$
$$t = 0.075D.$$
$$b = 0.250D.$$

For sliding when under load the dimensions are:

$$d = 0.800D.$$
$$t = 0.100D.$$
$$b = 0.250D.$$

The maximumd iameters $D(D = d + 2t)$ vary from $\frac{3}{4}$ to 3 in., inclusive, advancing by $\frac{1}{8}$-in. increments up to $1\frac{3}{4}$ in., and by $\frac{1}{4}$-in. increments up to $2\frac{1}{2}$ in.

The strength of spline fittings in transmitting torque, per inch of bearing length, and using a bearing unit stress of 1,000 lb. per square inch, may be found by the following formula:

$$T = 1{,}000NRt \tag{11}$$

in which T denotes torque in inch-pounds per inch of length.
N denotes number of splines.
R denotes mean radius, in inches.
t denotes depth of spline, in inches.

199. Cotter Keys.—*Cotter keys*, shown in Fig. 6, are used as a fastener for the joints of crossheads, valve yokes, valve rods, or for any similar application. Cotter keys are not standardized, but the taper is usually small so that the key will not tend to back out When used in the crankpin end of connecting rods, the cotter keys should be locked in place to prevent the centrifugal force from throwing the key out of position.

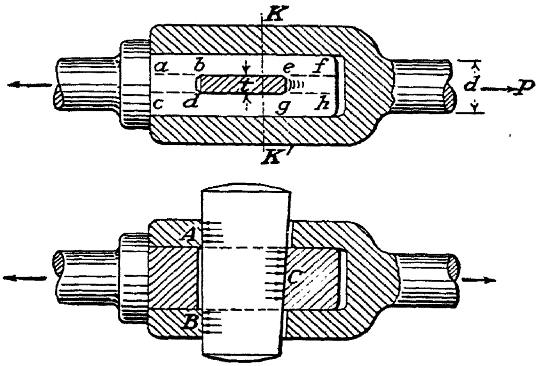


FIG. 6.—Cotter joint with key.

To design a joint of the cotter-key type, shown in Fig. 6, the following possibilities of failure should be investigated:

(*a*) Failure of the rod in tension.
(*b*) Tensile failure of the rod at KK', or of the socket at KK'.
(*c*) Shear failure of the rod at *efgh*, or of the socket at *abcd*.
(*d*) Compressive failure of the rod at *eg*, or of the socket at *bd*.
(*e*) Shear failure of the key at the joint (double shear).
(*f*) Compressive failure of the key where it bears against the socket at A and B and against the rod at C.

The load which may be safely carried for the above cases, or the unit stress produced by a given load, may be calculated as follows:

(*a*) F = tensile area of rod $\times S_t = \frac{\pi d^2}{4} \times S_t$.

(*b*) F_1 = tensile area of rod at slotted hole $\times S_t$.
F_2 = tensile area of socket at slotted hole $\times S_t$.

(c) F_3 = 2 (area in shear of the rod, back of the slot) $\times S_s$.
F_4 = 2 (area in shear of the socket back of the slot) $\times S_s$.
(d) F_5 = compressive area in socket back of the key $\times S_c$.
F_6 = compressive area in rod back of the key $\times S_c$.
(e) F_7 = 2 (shearing area of key) $\times S_s$.
(f) F_8 = compressive area of the part of the key that bears against either the rod or the socket $\times S_c$.

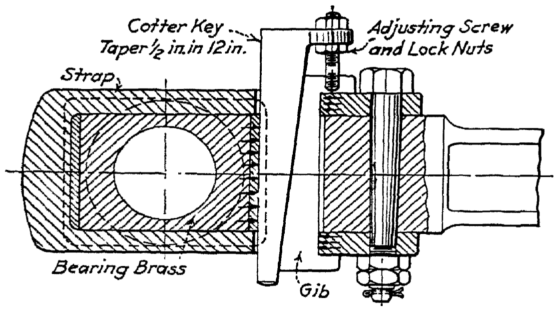


FIG. 7.—Connecting rod end, showing cotter key and adjusting gib.

Cotter keys, as shown in Fig. 7, are used as adjusting wedges as well as for fastening the straps at the ends of connecting rods. To prevent the cotter key from working loose as a result of the reversing action of the rods, a locking device is sometimes used as shown by the screw and locknuts in Figs. 7 and 8(b), and by the setscrew in Fig. 8(a).

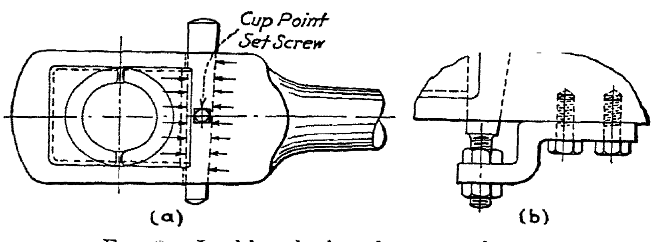


FIG. 8.—Locking devices for cotter key.

Problems

1. Show by sketches the following types of keys and their application: (a) saddle key, (b) flat key (friction), (c) square key, (d) round key (three applications).
2. Show by sketches the following types of keys and their application: (a) draw key, (b) feather key (two forms), (c) Woodruff key (for short and long hubs).

3. A steel key has the following dimensions.

$$b = \tfrac{1}{2} \text{ in.}$$
$$t = \tfrac{7}{8} \text{ in.}$$
$$L = 3\tfrac{1}{2} \text{ in.}$$

What resistance will the key offer to shear if the allowable shear strength of the steel is 12,000 lb. per square inch?

4. A square key is made of steel having an ultimate tensile, compressive, and shear strength of 60,000, 60,000, and 46,000 lb. per square inch, respectively. It has the following dimensions:

$$b = \tfrac{3}{4} \text{ in.}$$
$$t = 1 \text{ in.}$$
$$L = 6 \text{ in.}$$

(a) What resistance will the key offer in shear and in compression?
(b) In what manner will the key probably fail under an excessive load?

5. Show that a key having the following dimensions is approximately as strong as the torsion strength of the shaft, if key and shaft are made of the same quality of steel.

$$b = \tfrac{1}{4}d, \; L = 1.5d.$$

6. A key is $\frac{5}{8}$ in. wide and $2\frac{3}{4}$ in. long, and is made of machinery steel having an ultimate shearing strength of 44,000 lb. per square inch. If this key is replaced with one having the same cross-section but made of steel having an ultimate shearing strength of 58,000 lb. per square inch, determine the length of the second key.

7. A shaft 2 in. in diameter is turning at 200 r.p.m., and delivers equal amounts of power to each of three machines by pulleys keyed to the shaft. What should be the length of the required keys if a factor of safety of 4 is used? The shaft and key are of the same quality of steel.

8. A line shaft rotates at 200 r.p.m. and transmits 50 hp. at a constant rate. If the working shear stress in the shaft is 6,000 lb. per square inch, determine the shearing unit stress and the factor of safety in a key which has the following proportions:

$$b = 0.25d, \; L = 3.5d.$$

9. A cast-iron flanged shaft coupling has proportions as follows:
Bore for shaft = 3 in.
Outside diameter = $11\frac{1}{2}$ in.
Diameter of bolt circle = $8\frac{1}{4}$ in.
Number of bolts = 6.
Diameter of bolts = $\frac{13}{16}$ in.
Length of hub of each half = $4\frac{1}{2}$ in.
Key for each half: $b = \frac{7}{8}$ in., $t = \frac{7}{8}$ in.
Diameter of hub = $2d$.
Determine the maximum horsepower at 150 r.p.m. which can be transmitted by:
(a) The bolts.
(b) The key (shear).
(c) The key (compression).

(*d*) The shaft.
(*e*) The coupling.
Assume that the ratio of the shear strength of the steel to the tensile and compressive strength is 0.8.

10. Make a half-size drawing of a cotter-joint design similar to the one shown in Fig. 6. The design is to be based upon the diameter of the rod, proportioning all parts so that they are equally strong or approximately so. The taper of the fit is to be $\frac{1}{16}$ in. per foot. Detail and fully dimension each part. The diameter of the rod is . . . in

CHAPTER XI

SHAFTING AND SHAFT COUPLINGS

200. A *shaft* is a revolving machine part, supported in bearings, and loaded outside the bearings. An *axle* is a machine part, subjected principally to bending, and carrying a load which is supported at the bearings. Either the axle or the bearing rotates. Shafts are used for driving machines, pulleys, and gears, and by means of shafting power is transmitted from one machine or machine part to another.

Shafts may be classified according to their use as follows:

(*a*) For prime movers.
  1. Engine shafts.
  2. Generator shafts.
  3. Turbine shafts.

(*b*) For transmission of power.
  1. Line shafts.
  2. Jackshafts.
  3. Countershafts.

(*c*) Machine spindles.

A *line shaft* is made up of more than one length of shafting, joined together by couplings. A *jackshaft* is one which obtains its motion directly from the source of power, and in turn imparts the motion to other shafts or machine parts. A *countershaft* is a short shaft, placed between a line shaft and a machine. The term *spindle* is a general term applied to a number of short shafts used on machines. Spindles may be solid or hollow, of uniform cross-section or tapering.

Shafts may be classified according to form as follows:

(*a*) Solid shafts.
(*b*) Hollow shafts.
(*c*) Flexible shafts.

A *solid shaft* is usually of circular cross-section. Square shafts with turned bearings, while at one time common, are now seldom used. *Hollow* circular shafts are made in the larger sizes, where the value of the installation justifies their use. *Flexible* shafts

are used to transmit rotary motion to any desired place, bringing the power to the work instead of the work to the power. Flexible shafting is limited in its application to light work and short lengths.

201. Torsion in Solid Shafting.—Short shafts, and long shafts with well-placed bearings, are subjected principally to torsion. A case of torsional stress alone is rare, for nearly all shafting is subjected to bending stresses due to belt pull, gear-tooth pressure, and weight of pulleys and gears. Since the stresses due to bending are often difficult to determine in advance, and because such calculations are complicated, they are sometimes omitted. The shaft in such cases is designed for torsion and angular deflection only, the bending being taken into account by a large factor of safety.

Formula (18) in Chap. VII, for torsion in shafts is:

$$T_m = \frac{SJ}{r}. \tag{1}$$

For solid circular sections $J = \frac{\pi d^4}{32}$, and substituting in formula (1):

$$T_m = \frac{S\pi d^3}{16}$$

and

$$= \sqrt[3]{\frac{\quad}{\pi S}} \tag{2}$$

When a force F acts with a moment arm r (Fig. 1) the work done in one revolution of the shaft is $2\pi rF$ in.-lb. (F in pounds and r in inches). If the speed of the shaft is N revolutions per minute, the work done per minute is $2\pi rFN$, and this v lue divided by the number of inch-pounds per minute in a horsepower, gives the formula for horsepower.

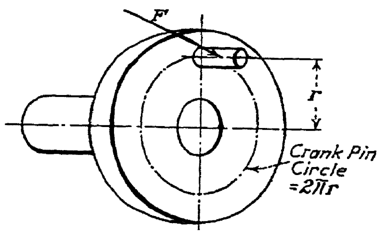


FIG. 1.

$$\text{horsepower} = \frac{2\pi rFN}{33{,}000 \times 12}.$$

Since

$$T_m = Fr,$$

$$\text{horsepower} = \frac{T_m N}{63{,}024},$$

$$T_m = \frac{63{,}024 \text{ hp.}}{N}. \tag{3}$$

Substituting this value in formula (1):

$$d = 68.4\sqrt[3]{\frac{\text{hp.}}{NS}}. \tag{4}$$

202. Twist or Angular Deflection of Shafting.—Formula (19) in Chap. VII, for the angle of twist in a shaft, is:

$$\theta = \frac{57.3T_mL}{E_sJ}. \tag{5}$$

In practice the angle θ is usually limited to 1 deg. in 20 diameters, therefore, substituting in formula (5), letting $\theta = 1$, $L = 20d$, and

$$T_m = \frac{2SJ}{d}:$$

$$1 = \frac{57.3 \times 2S \times 20d}{E_sd}$$

$$S = \frac{E_s}{2{,}292}. \tag{6}$$

For steel $E_s = 12{,}000{,}000$, therefore $S = 5{,}230$ lb. per square inch.
For cast iron $E_s = 6{,}000{,}000$, therefore $S = 2{,}620$ lb. per square inch.

203. Hollow Shafting.—Large shafts, especially those for marine work, are often made hollow. Hollow shafts are stronger

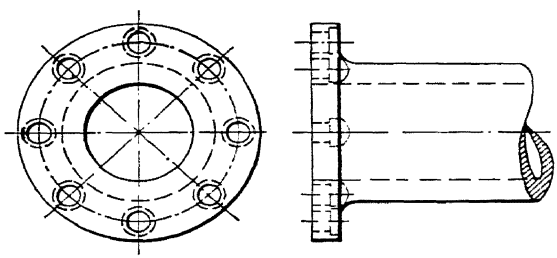


FIG. 2.—Hollow shaft with solid flange.

per pound of material, and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. Hollow shafts are relatively short and have the collar for connections made integral with the shaft as shown in Fig. 2.

If a hollow shaft is to be equal in strength to a solid shaft, the resisting moment of the one must be equal to that of the other. Using the notation shown in Fig. 3:

$$\frac{\pi(d_1^4 - d_2^4)}{16d_1} \times S = \frac{\pi d^3}{16} \times S'.$$

If the two shafts are of like material so that $S = S'$:

$$d_1^3 - \frac{d_2^4}{d_1} = d^3. \tag{7}$$

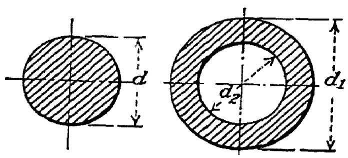


FIG. 3.

204. Transverse Deflection of Shafts.—A shaft subjected to bending will deflect as a beam, and the formulas for the deflections of beams may be applied as follows:
Cantilever with a concentrated load at the free end (Fig. 4(*a*)):

$$y = \frac{PL^3}{3EI}, \tag{8}$$

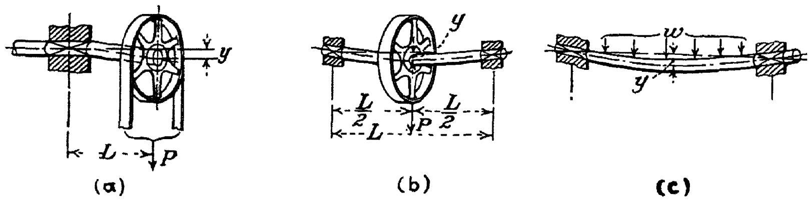


FIG. 4.

Simple beam with a concentrated load at the middle (Fig. 4(*b*)):

$$y \quad \frac{PL^3}{48EI}, \tag{9}$$

Simple beam with a uniformly distributed load (Fig. 4(*c*)):

$$y = \frac{5WL^3}{384EI}, \tag{10}$$

in which y denotes the maximum deflection of the beam, in inches.

P denotes the concentrated load, in pounds.

W denotes the uniform load, in pounds.

L denotes the length of the beam, in inches.

E denotes the modulus of elasticity in tension or compression, in pounds per square inch.

I denotes the moment of inertia of the cross-section, in in.^{4}.

For the simple beam it is usual to consider the length of the beam to be from the middle of one bearing to the middle of the next bearing. For cantilever action the length is taken from the middle of the hub of a pulley or gear to the middle of the bearing.

205. Design of Transmission Shafting.—The following sections on the design of shafting will be based upon the "Code for Design of Transmission Shafting," recommended by the American Society of Mechanical Engineers, and approved by the American Engineering Standards Committee in 1927.

206. Maximum Permissible Working Stresses.—For *simple flexure* the maximum permissible unit stress either in tension or compression is as follows:

16,000 lb. per square inch for "commercial steel" shafting without allowance for keyways.

12,000 lb. per square inch for "commercial steel" shafting with allowance for keyways.

For shafting purchased under definite physical specifications use 60 per cent of the elastic limit in tension, but not more than 36 per cent of the ultimate tensile strength.

For *simple torsion* the maximum permissible unit shearing stress is as follows:

8,000 lb. per square inch for "commercial steel" shafting without allowance for keyways.

6,000 lb. per square inch for "commercial steel" shafting with allowance for keyways.

For shafting purchased under definite physical specifications use 30 per cent of the elastic limit in tension, but not more than 18 per cent of the ultimate tensile strength. It will be noted that the unit stresses permitted in shear are just one-half those permitted in tension or compression.

For any case of *combined stresses* the maximum permissible intensity of shearing unit stress is equal to those given above for simple torsion. It can be shown that when the ratio of the

permissible unit tensile stress to the permissible unit shear stress equals 2, the diameter as determined by the shearing unit stress will govern the design. Since in all the above cases of permissible unit stress this ratio equals 2, it is clear that the design will be based upon shear.

207. Shock and Fatigue Factors.—A machine part subjected to shock or fatigue is much more likely to fail than a part subjected to steady loads only. The following factors, K_t for torque, and K_m for bending moment, are recommended for various types of loading, and they are to be applied in every case to the twisting and bending moments as shown in Table I.

TABLE I

Nature of loading	Values for	
	K_m	K_t
Stationary shafts:		
Gradually applied load	1.0	1.0
Suddenly applied load	1.5 to 2.0	1.5 to 2.0
Rotating shafts:		
Gradually applied or steady loads	1.5	1.0
Suddenly applied loads, minor shocks only	1.5 to 2.0	1.0 to 1.5
Suddenly applied loads, heavy shocks	2.0 to 3.0	1.5 to 3.0

208. Axial Loading Factors.—The factor α applied to cases of axial loading, is the amount by which the unit stress P/A is increased to take care of column action. For axial tensile loads, and for axial compressive loads when the shaft is rigidly supported at frequent intervals so that the shaft acts virtually as a short column, $\alpha = 1$.

For long columns in which the slenderness ratio L/r is less than 115, the factor α is computed from the formula:

$$1 - 0.0044^{L} \tag{11}$$

These values of α are given in Table II.

TABLE II

Slenderness ratio, $\frac{L}{r}$	α
0	1.00
25	1.12
50	1.28
75	1.49
100	1.78
115	2.02

For very long columns whose slenderness ratio is greater than 115, the factor α is computed from Euler's formula for long columns:

$$\alpha = \frac{S_c L^2}{C\pi^2 E r^2} \tag{12}$$

in which α denotes the ratio of the maximum unit stress to the average unit stress.

L denotes the unsupported length of the shaft, in inches.

r denotes the least radius of gyration, in inches.

S_c denotes the compressive yield point of the shaft material, in pounds per square inch.

E denotes the modulus of elasticity of the shaft material, which is about 30,000,000 lb. per square inch for steel.

C denotes the coefficient in Euler's formula depending upon end conditions, being 1 for hinged ends, and 2.25 for fixed ends.

The authors suggest the following values of C:

Both ends round, $C = 1$ (from experiments).

Both ends pinned, ends guided and partly restrained, $C = 1.6$ (from experiments).

Both ends flat, $C = 2.5$ (from experiments).

Both ends fixed, $C = 4$ (Theoretical).

One end free, the other end fixed, $C = 0.25$ (theoretical).

One end round, the other end fixed, $C = 2.25$ (theoretical).

One end round and guided, the other end fixed, $C = 2.05$ (theoretical).

209. Shafts in Pure Torsion.—For a *solid circular shaft:*

$$K_tT = 0.1963Sd^3.$$

$$K_t \times \text{hp.} = \frac{Sd^3N}{321{,}000},$$

$$d = \sqrt[3]{\frac{321{,}000K_t \times \text{hp.}}{SN}}. \tag{13}$$

in which d denotes diameter, in inches.
K_t denotes the constant given in Table I.
hp. denotes horsepower.
S denotes the permissible unit shearing stress, in pounds per square inch.
N denotes revolutions per minute.

Formula (13) is the same as formula (4) when K_t 1.

For a *hollow circular shaft:*

$$K_tT = \frac{0.1963S(d_1^4 - d_2^4)}{d_1}.$$

$$K_t \times \text{hp.} = \frac{SN(d_1^4 - d_2^4)}{321{,}000d_1}, \tag{14}$$

in which d_1 denotes outside diameter, in inches.
d_2 denotes inside diameter, in inches.

The other notation is the same as above.

210. Shafts Subjected to Bending.—For a *solid circular shaft:*

$$K_mM = \frac{\pi}{32}d^3S,$$

$$d = \sqrt[3]{\frac{32K_mM}{\pi S}}, \tag{15}$$

in which M denotes bending moment, in inch-pounds.
K_m denotes the constant given in Table I.
S denotes the permissible unit flexural stress, in pounds per square inches.

The other notation is the same as before.

For a *hollow circular shaft:*

$$K_mM = \frac{\pi S(d_1^4 - d_2^4)}{32d_1}, \tag{16}$$

211. Shafts Subjected to Combined Torsion and Bending.—Formula (23) of Chap. VII gives the maximum shearing unit

stress for a case of tensile or compressive stress combined with shear stress as follows:

$$S_s' = \qquad \frac{S}{2}\Big) + (S_s)^2.$$

For the case of combined torsion and bending in a solid shaft:

$$S = \frac{32M}{\pi d^3} \text{ and } S_s = \frac{16T}{\pi d^3}.$$

Substituting in the formula for S_s':

$$S_s' = \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}.$$

For a *solid circular shaft* the A.S.M.E. code gives the following formula:

$$d = \sqrt[3]{\frac{16}{\pi S}\sqrt{(K_m M)^2 + (K_t T)^2}}. \qquad (17)$$

$$d = \sqrt[3]{\frac{16}{\pi S}\sqrt{(K_m M)^2 + \left(\frac{396{,}000K_t \times \text{hp.}}{2\pi N}\right)^2}} \qquad (18)$$

For a *hollow circular shaft:*

$$\frac{d_1^4 - d_2^4}{d_1} = \frac{16}{\pi S}\sqrt{(K_m M)^2 + (K_t T)^2}. \qquad (19)$$

$$\frac{d_1^4 - d_2^4}{d_1} = \frac{16}{\pi S} \quad (K_m M)^2 + \left(\frac{396{,}000K_t \times \text{hp.}}{2\pi N}\right. \qquad (20)$$

Example.—A machinery shaft is subjected to a maximum torsional moment of 6,900 in.-lb., and a maximum bending moment of 11,500 in.-lb. The loads are steady and the allowable shear stress is fixed at 8,000 lb. per square inch.

From Table I, $K_m = 1.5$ and $K_t = 1$.

$$K_m M = 1.5 \times 11{,}500 = 17{,}250 \text{ in.-lb.}$$
$$K_t T = 6{,}900 \text{ in.-lb.}$$

Using formula (17):

$$d = \sqrt[3]{\frac{16}{\pi \times 8{,}000}\sqrt{17{,}250^2 + 6{,}900^2}} = \sqrt[3]{11.82}.$$

$$d = 2.28 \text{ in.}$$

Therefore, a shaft 2.25 in. in diameter will be used.

Example.—Determine the diameter of a hollow shaft, with the ratio $d_2/d_1 = 0.80$, capable of transmitting 400 hp. at 215 r.p.m. when subjected at the same time to a maximum bending moment of 49,000 in.-lb. Assume

the allowable unit shear stress is 8,000 lb. per square inch, that the shaft is to be subjected to suddenly applied loads with minor shocks only for the torque, and to steady loads for the bending moment.

From Table I, $K_m = 1.5$, and $K_t = 1.5$.

Using formula (20):

$$\frac{16}{\pi 8{,}000}\sqrt{(1.5 \times 49{,}000)^2 + \left(\frac{396{,}000 \times 1.5 \times 400}{2\pi \times 215}\right)^2}.$$

$$\text{Since } \frac{d_2}{d_1} = 0.80,$$

$$\frac{d_1^4 - 0.409 d_1^4}{d_1} = 121.$$

$$0.591\, d_1^3 = 121.$$

$$d_1 = 5.89 \text{ in.}$$

$$d_2 = 4.72 \text{ in.}$$

212. Shafts in Bending over Short Spans with Heavy Transverse Shear.—For a *solid circular shaft:*

$$V = \frac{3\pi S d^2}{16},$$

$$d = \sqrt{\frac{16V}{3\pi S}}, \tag{21}$$

in which V denotes the total shear on the cross-section, in pounds.

S denotes the unit shear stress, in pounds per square inch.

For a *hollow circular shaft:*

$$V = \frac{3\pi S(d_1^4 - d_2^4)(d_1 - d_2)}{16(d_1^3 - d_2^3)}.$$

$$\frac{d_1^3 - d_2^3}{(d_1^4 - d_2^4)(d_1 - d_2)} = \frac{3\pi S}{16V}. \tag{22}$$

213. Shafts Subjected to Combined Torsion and Heavy Transverse Shear.—For a *solid circular shaft:*

$$S = \frac{16T}{\pi d^3} + \frac{16V}{3\pi d^2}. \tag{23}$$

For a *hollow circular shaft:*

$$S = \frac{16Td}{\pi(d_1^4 - d_2^4)} + \frac{16V(d_1^3 - d_2^3)}{3\pi(d_1^4 - d_2^4)(d_1 - d_2)}. \tag{24}$$

214. Materials for Shafts.—Ordinary shafts are made of Bessemer or open-hearth steel of 0.15 to 0.40 per cent carbon, which

is often called *medium steel* or *machinery steel.* *Cold-rolled* steel is of the same quality as machinery steel. It is turned to even sizes, and is then passed through rapidly revolving bevel rolls, which straightens the shaft, renders it uniform in size, and gives it a glazed finish which requires no further finishing for most uses. Cold-rolled shafting is about 20 per cent stronger than hot-rolled shafting of the same material, but it is no stiffer. Cold-rolled shafting is used extensively for line shafting.

The aeroplane and the automobile industries have employed alloy steels to a very considerable extent in the design of shafting.

Large shafts are forged approximately to the required size from the ingot, and finished in a lathe. When great strength as well as lightness is required, shafts are made of nickel steel. Very large nickel-steel shafts are made hollow. Some engineers specify cold-rolled shafting up to a diameter of 4 in. and forged shafting for the larger sizes.

The old standard of undersized diameters has never been changed, and it should be noted that a 2-in shaft is only $1^{15}/_{16}$ in. in diameter. For this reason, when specifying standard shaft hangers, boxes, and couplings, it is *important* to *specify* the *exact diameter* of the shaft.

215. Sizes of Shafting.—The Committee on Shafting Standards of the American Society of Mechanical Engineers recommends the use of the following diameters: From $1^{3}/_{16}$ to $2^{15}/_{16}$ in., varying by increments of $^{1}/_{4}$ in.; and from $3^{7}/_{16}$ to $5^{15}/_{16}$ in., varying by increments of $^{1}/_{2}$ in. For machinery shafts they recommend that the size increase by $^{1}/_{16}$ in. up to $2^{1}/_{2}$ in., by $^{1}/_{8}$ in. from $2^{1}/_{2}$ up to 4 in., and by $^{1}/_{4}$ in. from 4 to 6 in. Table III gives the dimensions and tolerances for cold-finished shafting.

Commercial shafting is obtainable in lengths of 10 to 24 ft. varying by increments of 2 ft. If longer lengths are required they may be obtained, but it should be remembered, that, from the standpoint of transportation, lengths are usually limited to the inside dimension of a box car.

216. Change of Shaft Diameter.—It is well known that abrupt changes of cross-section in a member subjected to stress induce high local stresses, and under the action of repeated or reversed stresses cracks may develop causing ultimate failure. For this reason a shaft which has been turned with various diameters along its length, should have the different diameters joined by generous fillets.

TABLE III.—COLD-FINISHED SHAFTING. DIAMETERS AND LENGTHS
(American Standard)
(The standard stock lengths for cold-finished shafting are 16, 20, and 24 ft.)

Transmission shafting, inches (d)	Machinery shafting, inches (d)	Tolerance on diameters, (−) inches	Transmission shafting, inches (d)	Machinery shafting, inches (d)	Tolerance on diameters, (−) inches
	1/2	0.002	2 3/16	2 3/16	0.004
	9/16	0.002	.....	2 1/4	0.004
	5/8	0.002	.....	2 5/16	0.004
	11/16	0.002	.....	2 3/8	0.004
	3/4	0.002	2 7/16	2 7/16	0.004
	13/16	0.002	.....	2 1/2	0.004
	7/8	0.002	.....	2 5/8	0.004
15/16	15/16	0.002	.....	2 3/4	0.004
	1	0.002	.....	2 7/8	0.004
	1 1/16	0.002	2 15/16	3	0.004
	1 1/8	0.003	.....	3 1/8	0.004
1 3/16	1 3/16	0.003	.....	3 1/4	0.004
	1 1/4	0.003	.....	3 3/8	0.004
	1 5/16	0.003	3 7/16	3 1/2	0.004
	1 3/8	0.003	.....	3 5/8	0.004
	1 7/16	0.003	.....	3 3/4	0.004
	1 1/2	0.003	.....	3 7/8	0.004
	1 9/16	0.003	3 15/16	4	0.004
	1 5/8	0.003	.....	4 1/4	0.005
1 11/16	1 11/16	0.003	4 7/16	4 1/2	0.005
	1 3/4	0.003	.....	4 3/4	0.005
	1 13/16	0.003	4 15/16	5	0.005
	1 7/8	0.003	.....	5 1/4	0.005
1 15/16	1 15/16	0.003	5 7/16	5 1/2	0.005
	2	0.003	.....	5 3/4	0.005
	2 1/16	0.004	5 15/16	6	0.005
	2 1/8	0.004			

217. Keyseats in Shafting.—Keyseats are usually milled in the shaft, and Fig. 5(a) shows the keyseat bottom rounded to the cutter radius, as formed by the cutter. For some classes of work the flat-bottom keyseat cut by an end-mill cutter is specified, as shown by Fig. 5(b). All keyseats should stop short of the bearing.

218. Effect of Keyseats Cut into Shafts.—Cold-rolled shafting will sometimes warp, due to cutting keyseats into the surface, because the cold rolling has set up internal stresses in the shaft.

Since the cutting of a keyseat produces an abrupt change of cross-section, this in turn produces high localized stress, which may become dangerous under repeated or reversed loading. For ordinary shafting the weakening of the shaft due to the cutting of keyseats is taken into account by the factor of safety.

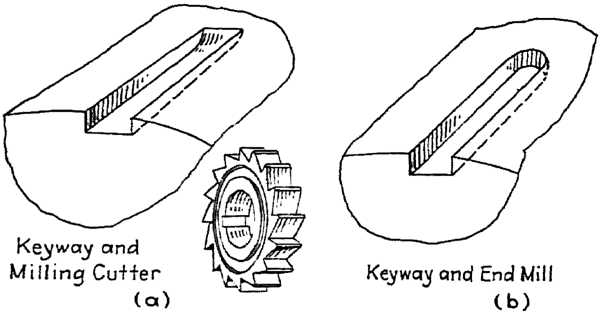


FIG. 5.—Key seats in shaft ends.

The static tests of H. F. Moore[1] on shafting of $1\frac{1}{4}$ to $2\frac{1}{4}$ in. in diameter, showed that the rigidity of the shafts was reduced, and that the angle of twist increased over the lengths in which the keyseats were cut.

Gough[2] carried out static and fatigue tests on shafts of Armco iron and a 0.65 per cent carbon steel. Tests were made using

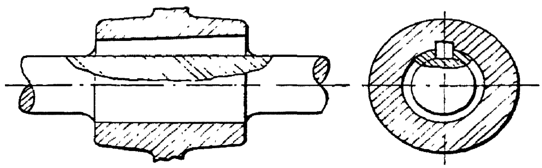


FIG. 6.—Shaft enlargement of hub bearing.

standard keyseats, and also keyseats with the same depth but only half the width of the standard keyseats. The static tests showed that both forms of keyseats had the elastic strength reduced 23 per cent, while the elastic stiffness was reduced 6 and 10 per cent, respectively, for the narrow and wide keyseats. In the fatigue tests under reversed torsional stresses the endurance limit was reduced the same amount for narrow and wide keyseats. This reduction amounted to 21 per cent for the 0.65

[1] *Univ. Ill., Bull.* 42, 1909.

[2] *Brit. Aero. Research Com. Repts.*, Vol. II, p. 488, 1924–1925.

per cent carbon steel, and 12 per cent for the Armco iron.. Keyed shafts of Armco iron showed a total reduction of 33 per cent.

Large shafts are often enlarged along the keyed portion of the hub so that the bottom of the keyseat is flush with the shaft, as shown in Fig. 6. This method allows the part which is to be keyed on to slip easily over the smaller portion of the shaft.

219. Critical Speed of Shafting.—When a body mounted on a shaft rotates, the center of gravity of the body must be at the center of the shaft to be perfectly balanced. Because of the weight of the body, lack of straightness of the shaft, and vibrations, perfect balance is seldom attained, so that the center of gravity of the body will be at a slight distance from the center of rotation of the shaft. When the shaft is rotating, the centrifugal force generated by this unbalanced condition causes the shaft to bend, and the center of the body will rotate in a small circle, causing a vibration called *whirling*, and the *critical speed* is that speed at which the vibrations are a maximum. If the speed is further increased the vibrations disappear, because the frequency does not coincide with the natural period of vibration of the shaft.

Critical speeds develop at speeds which, with few exceptions, are above the normal running speeds of machines; but with the ever-increasing speeds at which modern machinery is expected to function, critical speed has become an important problem, especially in the design of rotor shafts of turbines. Machines, in starting, pass through the period of whirling so rapidly that the vibrations do not cause failure; nevertheless, provisions must be made at the bearings and clearances must be given, to allow the machine to pass through this condition with the least damage.

The critical speed depends upon the kind and spacing of bearings, and upon the distribution of the shaft loading. There are various formulas for arriving at the critical speeds under different conditions.[1]

220. Shaft Couplings.—*Shaft couplings* are fastenings used to join lengths of shafting together, so that motion from one section may be transmitted to another. A shaft coupling should be capable of being easily attached and detached, should transmit the full power of the shaft, should hold the sections of shafting

[1] "Machinery Handbook," 6th Ed., p. 325.

in alignment, and should have no projecting parts. Some forms of couplings called *muff couplings*, are bored out slightly smaller than the shaft, grip the shaft ends equally on either side of the joint, and transmit power by friction. Other forms, shown in Fig. 7, are keyed to the shaft, and are considered to be superior to muff couplings in transmitting power.

Couplings may be classified according to the function which they are performing, as follows:

(*a*) Permanent couplings, which act positively as power-transmission devices.

(*b*) Releasing couplings, which transmit power at the will of the operator.

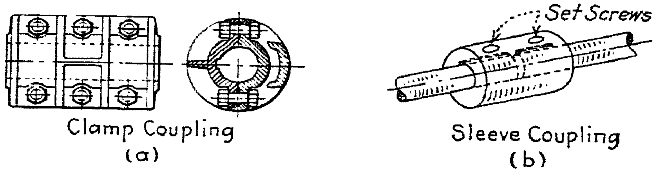


FIG. 7.—Permanent couplings.

Permanent couplings may be used for shafts having the same axis, for parallel shafts, or for shafts with intersecting axes. Releasing couplings may have a positive drive, or the drive may be variable, as in friction and magnetic couplings. Of the large number of commercial couplings which are readily obtainable from the stock of transmission-machinery manufacturers, those described in the following sections have been selected as typical.

Permanent Coupling.—The coupling shown in Fig. 7(*b*) is a simple form of coupling. It is a cast-iron sleeve, bored to fit the shaft ends which are to be connected. The key which holds the coupling in place is more easily applied if it is made in two parts. A sleeve coupling is sometimes shrunk on, and if this is done it is necessary to break it when occasion arises to detach it.

Clamp Coupling.—The coupling shown in Fig. 7(*a*) is made in two parts which are bolted together by through bolts and holds by friction only, or by a combination of a key and friction. It is properly reinforced by ribs, and the bolt heads and nuts are protected to comply with safety laws.

Cone Coupling.—A *cone coupling*, shown in Fig. 8, is made up of a cast-iron sleeve, bored tapering from either end to the middle,

so that conical-shaped split wedges may be inserted and drawn together by bolts, thus squeezing the wedges against the shaft. Keys are used or not, depending upon the amount of power to be transmitted.

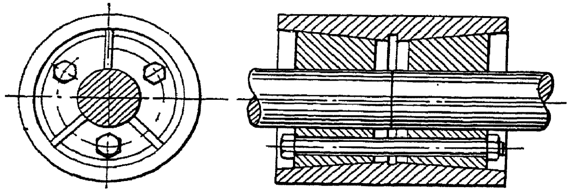


FIG. 8.—Cone wedge coupling.

Flange Coupling.—A *flange coupling* is adapted for severe service and when pressed and keyed onto the shaft it is practically a part of the shaft and its use insures ease of alignment and permanence. This form of coupling is used almost exclusively on shaft sizes 4 in. in diameter and larger. The coupling consists of two cast-iron flanges which are bolted together, each part being keyed to a shaft end. Alignment is secured by extending one shaft end a short distance beyond one part and into the

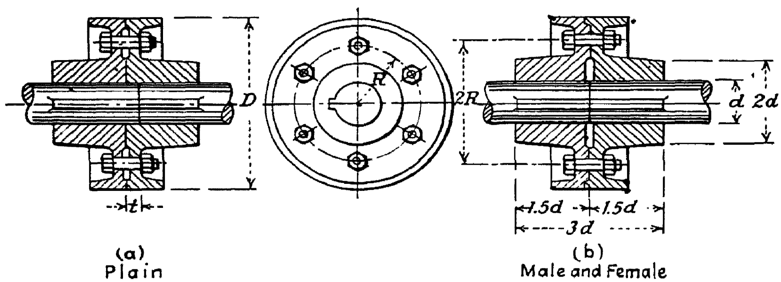


FIG. 9.—Flange coupling.

second part of the coupling, as shown in Fig. 9(*a*); or the faces may be provided with a concentric ring and recess as shown in Fig. 9(*b*). In either case each half of the coupling is keyed to the shaft, the projecting bolt heads and nuts being covered by a safety flange, which also adds to the rigidity of the coupling.

In applying these couplings each half is pushed onto a shaft end, a dummy key being used to maintain the alignment of the keyway in the shaft and hub. After the coupling is in position on the shaft, the dummy key is withdrawn and a well-fitted

key is driven in. For precise work the couplings are faced off after being placed on the shaft, so that the faces will be normal to the axis of rotation.

Example.—Design a flange coupling that will transmit the full power of a 2-in. cold-rolled steel shaft.

In Fig. 7:

d denotes the diameter of the shaft, in inches.
R denotes the radius of the bolt circle, in inches.
n denotes the number of bolts.
d_1 denotes the diameter of the bolts, in inches.
S_s denotes the allowable unit shear stress of the steel in the bolts, in pounds per square inch.
S_s' denotes the allowable unit shear stress of the steel in the shaft, in pounds per square inch.
S_c denotes the allowable unit compressive stress of the steel in the bolts, in pounds per square inch.
S_c' denotes the allowable unit compressive stress of the cast iron in the flange, in pounds per square inch.

(*a*) For equal strength the resisting moment of the bolts must equal that of the shaft:

$$\frac{\pi d^3}{16} S_s' = \frac{\pi d_1^2}{4} nRS_s. \qquad (25)$$

If the same quality of steel is used in the shaft and in the bolts then:

$$S_s' = S_s, \text{ and,}$$

$$d_1 = \frac{1}{2}\sqrt{\frac{d^3}{nR}}. \qquad (26)$$

The usual practice with this class of coupling is to use five bolts for all couplings for from 1 to 8 in. in diameter. The outside diameter of the coupling is approximately equal to $4d$, the diameter of the hub about $2d$, and the radius of the bolt circle is $\frac{D + 2d}{4}$, D being the outside diameter of the coupling. For this problem, therefore, $n = 5$, $R = \frac{8 + 4}{4} = 3$ in., and formula (26) becomes:

$$d_1 = \frac{1}{2}\sqrt{\frac{8}{5 \times 3}} = 0.365 \text{ in., say a } \tfrac{3}{8}\text{-in. bolt.}$$

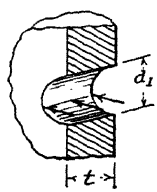


FIG. 10.

(*b*) The cast-iron flanges should be of sufficient thickness to be safe against failure by compression. In Fig. 10:

t denotes the thickness of the web, in inches.
S_s denotes the allowable shearing unit stress of the bolt material, in pounds per square inch.
S_c' denotes the allowable compressive unit stress of the flange material, in pounds per square inch.

The shearing force which each bolt transmits produces bearing stress between bolt and flange, therefore:

$$\frac{\pi d_1^2}{4} \times S_s = d_1 t S_c'. \qquad (27)$$

For this problem S_s may be taken as 6,000, and S_c' as 10,000 lb. per square inch.

$$t = \frac{\pi \times 0.375 \times 6,000}{4 \times 10,000} = 0.177 \text{ in.}$$

It would probably not be desirable to make the thickness less than ½ in., to avoid abrupt changes of the wall thickness of the casting.

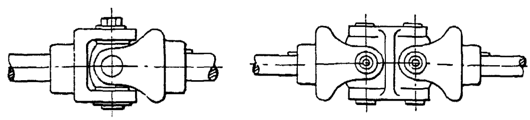
Since the strength of the steel bolts in bearing is at least equal to the strength of cast iron in bearing, it will be unnecessary to investigate the bolts for bearing.

The chapter on screw fastenings has shown that the use of bolts ⅜ in. in diameter would not be considered good practice, because of the initial stress induced in the bolts by screwing down the nuts. It is therefore desirable to use bolts at least ½ in. in diameter, and ⅝-in. bolts would be better.

The above design, has indicated that the flanged coupling should include the following:

Five bolts ⅝ in. in diameter.
Web of flange at least ½ in. thick.
Outside diameter of coupling, 8 in.
Diameter of the bolt circle, 6 in.
Diameter of the hub, 4 in.
The length of each hub should be at least 3 in.
The safety flanges should extend over the bolt heads and nuts.
Standard keys should be used.

221. Universal Coupling.—*Universal couplings* are used to transmit rotary motion between shafts, when the axes of the



(a) (b)

FIG. 11.—Universal couplings.

rotating members are not colinear. Hooke's Joint, shown in Fig. 11(a), is a type of universal coupling that is used extensively. The velocity ratio between the driving and the driven shaft is not constant for all parts of a revolution. It can be shown[1] that the minimum speed of the driven shaft is equal to the speed of the driver multiplied by the cos α, and the maximum speed is equal to that of the driver divided by the cos α, the angle α being the angle between the axes of the two shafts. This variation in angular velocity may be prevented by placing an inter-

[1] BARR and WOOD, "Kinematics of Machinery," p. 214.

mediate shaft, such as shown in Fig. 11(*b*), between the two main shafts, making the same angle α with each.

The universal coupling has been used on the transmission shaft of automobiles to some extent, but its many parts, its position under the car in its enclosure to protect it from dirt, and its inaccessibility for lubrication and repairs, are the principal reasons why it has been replaced by a flexible coupling. Figure 12 shows a semi-universal coupling which affords an easy and

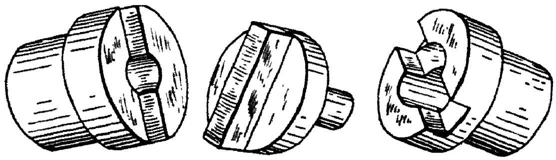


Fig. 12.—Semi-universal coupling.

effective means of connecting shafts that are subjected to misalignment and shocks. The center piece has two tongues at right angles to each other, which engage loosely with slots in the end pieces. The tongues have a continuous bearing across their length, giving ample wearing surfaces. This coupling is applicable for use with any speed up to 1,500 r.p.m., and is made in sizes to fit shafting from ½ to 4½ inches in diameter.

222. Flexible Couplings.—There are a number of flexible couplings of the type shown in Fig. 13, the main features being the number and composition of the disks. The flexibility of

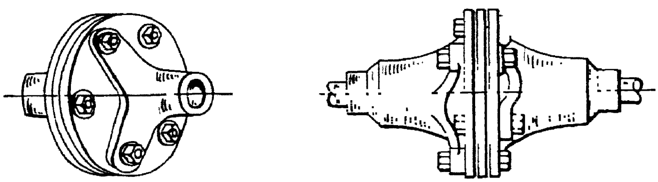


Fig. 13.—Flexible coupling.

the rubber or fabric disks allows for the non-alignment of the sections of the driving shaft, and the variation in distance from the front to the rear axles in an automobile is allowed for by a slip joint at the coupling.

More rigid couplings are also made which allow for a slight deviation in the alignment of connected shafts, and are used for the connection between the shafts of a direct-connected engine and generator, and between the motor and generator of a set.

Such a coupling consists of two solid disks which carry a series of projecting pins on their inner faces. Each pin of one flange is connected to a pin in the mating flange by some flexible material, such as rubber or leather, which allows the flanges to conform to slight inaccuracies in the alignment of the shafts.

Figure 14(a) shows the Francke coupling, which uses thin strips of steel; and Fig. 14(b) shows a flexible coupling which uses links of leather.

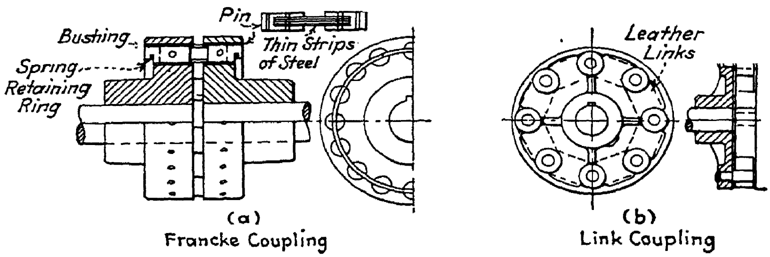


FIG. 14.—Flexible couplings

The flexible coupling shown in Fig. 15 consists of two outer cast-iron flanges which are keyed to the shaft ends. Between these flanges is a leather disk with lugs cemented and riveted on each side. The driving disk being of non-conducting material, the coupling when used on electric machines forms a good insula-

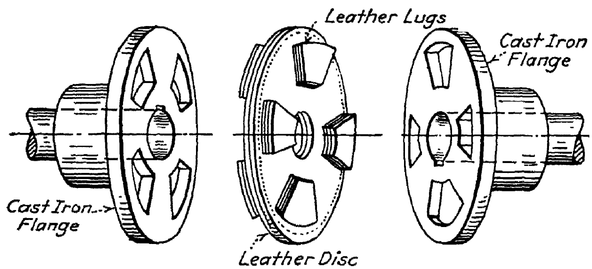


FIG. 15.—Flexible coupling.

tor. This form of coupling is designed for steady loading, and may be used to connect all sizes of shafting.

223. The Falk-Bibby Coupling.—The Falk-Bibby coupling consists of two flanges keyed to the shaft ends. A flat steel spring fits into the space between the teeth, as shown in Fig. 16. The springs are packed in grease and fit easily into the slots, permitting both misalignment and angular movement

of the shafts. The coupling is protected by a cover ring which is easily removed. This coupling is designed to allow a rotating shaft to pass easily through its critical speed.

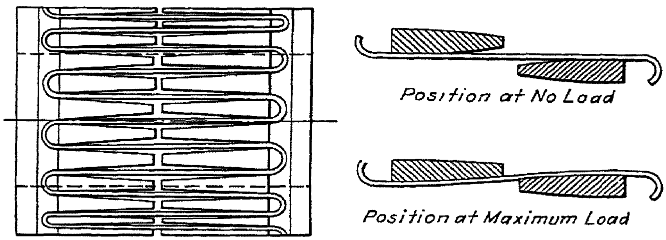


FIG. 16.—Falk-Bibby coupling.

224. Clutch Coupling.—The clutch shown in Fig. 17(*a*) will drive in either direction. The one shown in Fig. 17(*b*) will drive in one direction only, but may be readily engaged and disengaged while the shafts are in motion. When a positive clutch is thrown in under load, it should be done suddenly, to distribute the

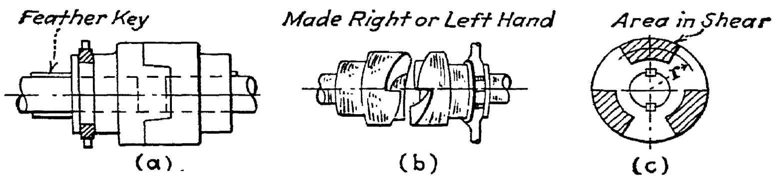


FIG. 17.—Solid-jaw clutches.

pressure over the entire working faces. When it is necessary to preserve relative angular position of the shafts, a one-tooth clutch is used. Clutches of this type should be designed to resist shearing through the jaw, and the bearing faces should be ample to resist crushing.

In Fig. 17(*c*), the torque of the shaft must be transmitted by the projecting jaws, therefore, for n number of jaws:

$$\frac{\pi d^3}{16} S_s = nArS_s \qquad (28)$$

in which d denotes the diameter of the shaft, in inches.

S_s denotes the permissible unit shearing stress of the shaft material, in pounds per square inch.

A denotes the shearing area of one jaw, in in.^{2}.

r denotes the mean radius of the jaw, in inches.

S_s' denotes the permissible shearing unit stress of the jaw material, in pounds per square inch.

For strength in bearing:

$$\frac{\pi d^3}{16} S_s = (r_1 - r_2) L r S_c n, \tag{29}$$

in which r_1 denotes the outside radius of the jaw, in inches.
r_2 denotes the inside radius of the jaw, in inches.
L denotes the length of the jaw, in inches.
S_c denotes the permissible unit bearing stress of the jaw material, in pounds per square inch.

Problems

1. A shaft 2 in. in diameter is rotating at 150 r.p.m. Assuming that the shaft is subjected to torsion only, what horsepower is the shaft transmitting for a maximum shearing unit stress of 7,000 lb. per square inch?
2. A line shaft rotating at 200 r.p.m. is to transmit 75 hp. The shaft is cold-rolled steel having an ultimate strength in tension and shear of 60,000 and 48,000 lb. per square inch, respectively. Assuming a factor of safety of 8, what will be the exact diameter of the shaft, and what is the nearest commercial size which would be used?
3. Show how the torsion formula for solid circular steel shafts is developed:

$$d = 68.4\sqrt[3]{\frac{\text{hp.}}{NS}}.$$

4. Show that for steel shafting, using a value of $E_s = 12{,}000{,}000$ lb. per square inch, the unit stress induced by a twist of 1 deg. in 20 diameters is 5,230 lb. per square inch.
5. A steel shaft $2\frac{1}{2}$ in. in diameter is 26 ft. long, and is subjected to a twisting moment which develops a unit shear stress of 30,000 lb. per square inch throughout the entire length. Determine the value of the angle of twist in degrees which is produced by this condition. $E_s =$ 12,000,000 lb. per square inch.
6. A shaft 6 in. in diameter, rotating at 150 r.p.m., is to be replaced by a hollow shaft rotating at the same speed. The outside diameter of the hollow shaft must not be greater than 7 in. Assuming that the same grade of steel is used in both the solid and hollow shafts:
   (a) Determine the bore of the hollow shaft.
   (b) Determine the percentage of weight saved by the use of the hollow shaft.
7. The distance between the bearings of a shaft 3 in. in diameter is 8 ft. 4 in. The shaft is deflected as shown in Fig. 4(b) so that $y = 0.25$ in.
   (a) Determine the load which caused the deflection of the shaft.
   (b) Determine the unit stress induced in the shaft.
   (c) Determine the factor of safety if the shaft is made of steel having an ultimate tensile strength of 63,000 lb. per square inch. $E = 30{,}000{,}000$ lb. per square inch.

8. A shaft is subjected to a maximum twisting moment of 63,000 in.-lb., and to a maximum bending moment of 94,000 in.-lb. Determine the twisting and bending moments which should be considered for design purposes if:
   (*a*) The shaft is stationary and the load is gradually applied.
   (*b*) The shaft is stationary and the load is suddenly applied.
   (*c*) The shaft is rotating and the load is steady.
   (*d*) The shaft is rotating and the load is gradually applied.
   (*e*) The shaft is rotating and the load is suddenly applied, and subjected to minimum shocks only.
   (*f*) The shaft is rotating and the load is suddenly applied and subjected to heavy shocks.
9. A shaft having a slenderness ratio of 60 is to be designed so that any column action will be accounted for in the unit stress. What unit stress should be used in design, if under ordinary conditions with no column action, a compressive unit stress of 8,000 lb. per square inch would be satisfactory?
10. A shaft which is 12 ft. long is subjected to column action and its slenderness ratio is 125. What factor α should be used to determine the allowable unit stress to be used in the design of the shaft? $C = 2.0$.
11. Determine the nearest machinery shafting size capable of transmitting 250 hp. at 200 r.p.m. with a unit shear stress of 6,000 lb. per square inch. The load may be considered steady.
   (*a*) Solve by formula (2).
   (*b*) Solve by formula (13).
12. Determine the outside and inside diameters of a hollow shaft which will transmit 250 hp. at 200 r.p.m. The maximum allowable unit shear stress is 8,000 lb. per square inch. The ratio of the inside diameter to the outside diameter is 0.70. The load is steady with no shocks.
13. A stationary shaft over a span of 4 ft. supports a center load of 5,000 lb. If a commercial cold-drawn steel shaft with a standard keyway is to be used, select the proper shaft diameter if the allowable unit shear stress is 6,000 lb. per square inch.
14. A machinery shaft is subjected to a maximum torsional moment of 6,900 in.-lb. and a maximum bending moment of 8,000 in.-lb. Determine the diameter of the shaft if the loads are steady and the allowable unit shear stress is 8,000 lb. per square inch.
15. Determine the horsepower which can be transmitted by a hollow shaft, rotating at 250 r.p.m., which has an outside and inside diameter of 5 and $4\frac{1}{2}$ in., respectively. The shaft is subjected to a maximum bending moment of 20,000 in.-lb., and all loads are suddenly applied with minimum shocks. The maximum unit shear stress is not to exceed 6,000 lb. per square inch.
16. Make a sketch showing three kinds of couplings, as follows: (*a*) clamp, (*b*) cone, and (*c*) flange.

   NOTE: Show a section if necessary for clearness.
17. The catalogue of a transmission machinery manufacturer shows the following dimensions for a flanged coupling:
   Outside diameter = $14\frac{1}{2}$ in.

Bolt circle diameter = $10\frac{3}{4}$ in.
Hub diameter = 8 in.
Bore = 4 in.
Keyways = 1 in. wide by $\frac{1}{2}$ in. deep.
Length of hub of each half = 6 in.
Thickness of web = $1\frac{1}{2}$ in.
Number and diameter of bolts = 6—$\frac{15}{16}$ in.
(*a*) Determine the horsepower capacity of the coupling for the following allowable unit stresses:
For the bolts and shaft—S_t = 8,000 lb. per square inch.
S_c = 8,000 lb. per square inch.
S_s = 4,500 lb. per square inch.
For the cast-iron flanges—S_t = 2,000 lb. per square inch.
S_c = 8,000 lb. per square inch.
S_s = 2,000 lb. per square inch.
(*b*) Make a pencil shop drawing showing all dimensions and other necessary information. Scale: 6 in. = 1 ft. – 0 in.

CHAPTER XII

TOOTHED GEARS

225. Toothed gears are used to transmit rotary motion between shafts when any or all of the following conditions prevail:

(*a*) When the distance between shaft centers is short.

(*b*) When the speed of the shafts is too slow for belt transmission.

(*c*) When a constant velocity ratio of shafts must be maintained.

(*d*) When the power to be transmitted is large.

The above requirements might be met by the use of two cylinders of proper size turning in contact with each other, if they could be squeezed together tightly enough to prevent surface slip, and at the same time have only moderate bearing reactions. By cutting grooves along the elements of two cylinders, teeth are formed, and positive rotary motion is transmitted by the interlocking teeth. In turning, the teeth of the driving cylinder push against the teeth of the driven cylinder, and contact between the teeth is continuous.

226. The Fundamental Law of Toothed Gears.—The fundamental law of toothed gears states that the normal to the common tangent of the tooth profiles must always pass through the instantaneous center or pitch point, if the pitch cylinders are to roll with a constant velocity ratio.

The following definition is taken from Chap. IV: An instant center is a point coincident in two bodies about which either body may tend to rotate with respect to the other, and being a common point it will have the same linear velocity in each body with respect to a third body.

In Fig. 1 are shown two gears, *A* and *B*, attached to a third body *C*. The gear *B* rotates about the center *bc*, and the gear *A* rotates about the center *ac*. According to Kennedy's theorem, the instant center of *A* with respect to *B* must lie on the line connecting *ac* and *bc*. This instant center *ab*, according to the

above statement, will have the same linear velocity in each body with respect to a third body C. The only point on the line connecting ac and bc which has the same linear velocity in the two gears A and B is the pitch point P. Therefore the instant center ab must lie at the pitch point P.

The point D is a common point for both gears A and B, and rotates about the center P. The linear velocity of D, as a point in A and B, is the same and is directed along the line normal to DP. For counterclockwise rotation of the gear A the movement of D as a point in A is to the right, and as a point in B the

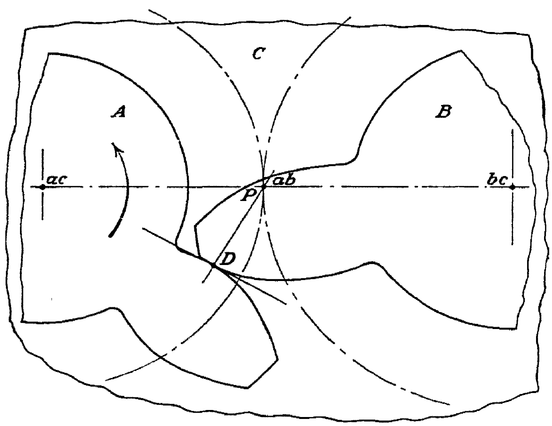


FIG. 1.

movement of D is to the left, both movements being normal to the radius DP. This movement cannot take place without interference unless the surfaces in contact at D are normal to the radius DP. The surfaces of the two teeth at the point of contact must therefore be such that the normal to the common tangent of the surfaces passes through the pitch point. It follows that the line Dab must intersect the line of centers at P.

There are a number of mathematical curves which may be used for developing gear tooth outlines, but the ones commonly employed are the *cycloidal* and *involute* curves.

227. Definitions of Tooth Parts.—If two cylinders or cones are in contact and roll together, the limiting circles of the solids in any plane normal to the axes are the *pitch circles*. The diameters of the pitch circles are called the *pitch diameters*, and the point of contact of the circles is called the *pitch point*. All circular

measurements of gear teeth are made on the pitch circle, and all radial dimensions are measured from it on a radial line.

The *pitch* of a gear tooth is a measure of its size, and it is expressed as *circular pitch* and *diametral pitch.* Circular pitch is the distance in inches, measured on the pitch circle, from any point on a tooth to a similar point on an adjacent tooth. Diametral pitch is the ratio of the number of teeth on a gear to the number of inches in the pitch diameter, or in other words, it is the number of teeth for each inch of pitch diameter. For example, if a gear has 30 teeth and its pitch diameter is 10 in., the diametral pitch is 3. It is more convenient to use the diametral pitch, rather than the circular pitch, in specifying the size of gear teeth, because its use will always result in a gear in which the number of teeth is a whole number. However, since the product

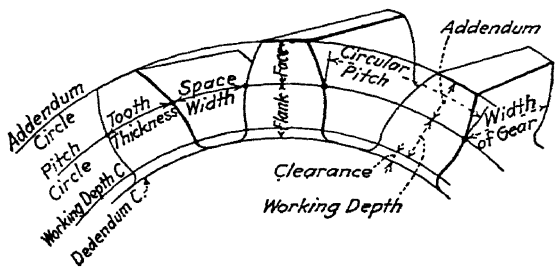


FIG. 2.—Gear-tooth nomenclature.

of diametral pitch and circular pitch is equal to π, conversion from one pitch to the other is readily made.

$$CP \times DP = \pi \tag{1}$$

$$CP = \frac{\pi}{DP} \tag{2}$$

$$DP = \frac{\pi}{CP} \tag{3}$$

in which CP denotes circular pitch,
DP denotes diametral pitch.

As shown in Fig. 2, the part of the tooth which extends outside of the pitch circle, or the part which has been "added to" the tooth for the engagement of the gear teeth, is called the *addendum* of the tooth, and its boundary circle is called the *addendum circle.* The distance which the tooth of one gear goes inside the addendum circle of a mating gear is called the *working depth* of the tooth,

and its limiting circle is called the *working depth circle.* It is evident that for gears having the same addendum length the addendum circle of one gear is tangent to the working depth circle of the mating gear, and that the addendum circles and the working-depth circles are equidistant but on opposite sides of the pitch circles. The distance from the top of one tooth to the bottom of the mating tooth is called *clearance,* and the boundary circle of the bottoms of the teeth is called the *dedendum* circle.

The *thickness of the tooth* is the width of the tooth measured on the pitch circle. The *width of the space* is the distance between teeth measured on the pitch circle. For gears with machined teeth the thickness of the tooth and the width of the space are each made equal to one-half of the circular pitch; but in cast teeth, because of the irregularities which might be present, the width of the space is greater than the thickness of the tooth, and this difference is called *backlash.*

The angle, measured on the pitch circle, which a gear turns through from the time contact between the teeth begins, until contact reaches the pitch point, is called the *angle of approach,* and the angle on the receding side is called the *angle of recess.* The angle which a gear turns through during the contact of any pair of teeth is called the *angle of action,* and it is the sum of the angles of approach and recess.

When the teeth of mating gears are engaged, the contact between the teeth will follow a definite line which is called the *line of contact* or the *pressure line.* The names of gear and tooth parts defined above apply to any system of gears.

228. The Involute System of Gear Teeth.—The involute system of gear teeth has grown in favor until it has almost supplanted the older cycloidal system. The contour of the involute tooth is the involute curve, which is generated by a point on a taut cord as it winds or unwinds on a cylinder. Also, as shown in Fig. 3(*a*), a point P on a straight rod will generate an involute curve, as the rod is rolled without slipping on the surface in a plane normal to the cylinder.

The development of the involute curve, shown in Fig. 3(*b*), may be carried out as follows: On a *base circle* of any radius $C0$, angles $0C1$, $1C2$, $2C3$ are laid off, and at the points 0, 1, 2, 3, tangent lines are erected as indicated. On the tangent lines the distances $1a$, $2b$, $3c$ are laid off, so that they are equal to the

arcs 01, 02, 03. The points a, b, c, so located, will be points on the involute curve.

The involute curve forms the tooth outline from the addendum circle to the base circle; a radial line from the base circle to the working-depth circle completes the outline; and a fillet is added at the bottom, extending from the working-depth circle to the dedendum circle, as shown in Fig. 4.

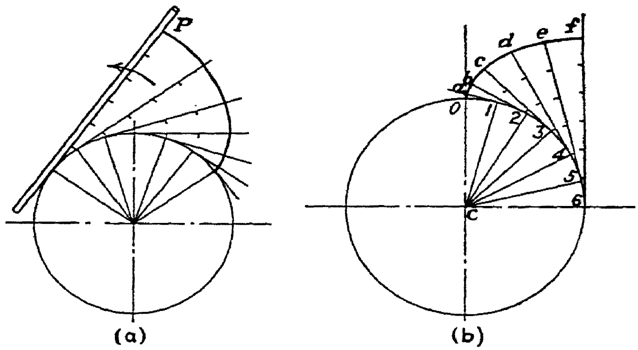


FIG. 3.

If the involute curve for one side of several successive teeth is drawn, the distance between these curves is constant for any line drawn tangent to the base circle. This distance is also equal to the length of the arc on the base circle between successive teeth, and is called the *normal pitch*.

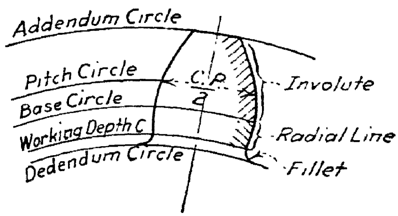


FIG. 4.—Involute-tooth contour.

229. Involute Tooth Curves.—Figure 3 shows how the involute form of tooth may be generated but it does not make clear the relations between base circles and pitch circles for two mating gears. In Fig. 5 suppose a taut cord is unwrapped from the cylinder A and wrapped upon the cylinder B. As the cord is unwrapped from A it will trace the involute curve aPb, and as it is wrapped upon B it will trace the curve cPd. Since the normal

to the involute curve is tangent to the base circle, the two involute curves have a common normal at the point of contact P and this normal must be tangent to both base circles. When the cylinders A and B revolve with the curves aPb and cPd in contact, it follows that the point of contact P must move along the straight line tangent to both base circles. This tangent line is called the line of action, or the pressure line.

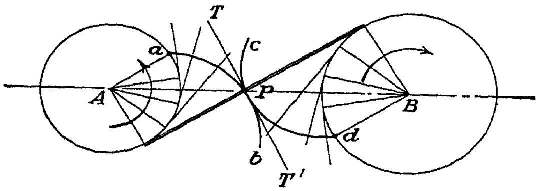


FIG. 5.

In Fig. 6, let a and b be the centers of two spur gears with pitch radii aP and bP. The line ab is drawn connecting the two centers of the pitch circles. To determine the base circles from which the involute curves are developed, the line ed is drawn through the pitch point P at any convenient angle α. The

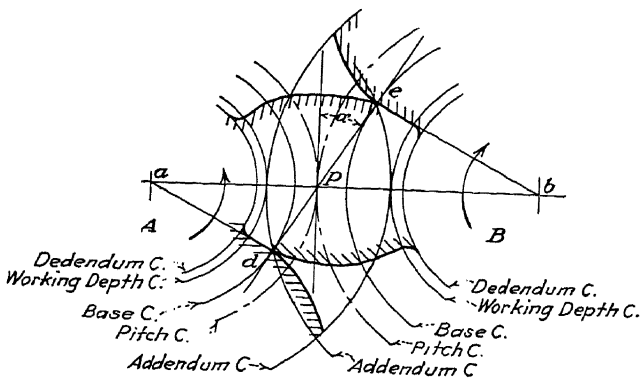


FIG. 6.—Involute spur-gear tooth outline.

normals to the line ed through the centers a and b determine the radii of the base circles ad and be.

The angle α is called the angle of obliquity or pressure angle, and it is evident that this is the angle which the line of action makes with a tangent to the pitch circles drawn through the pitch point. The pressure angle might have been given various values, and it is clear that this angle determines the diameters

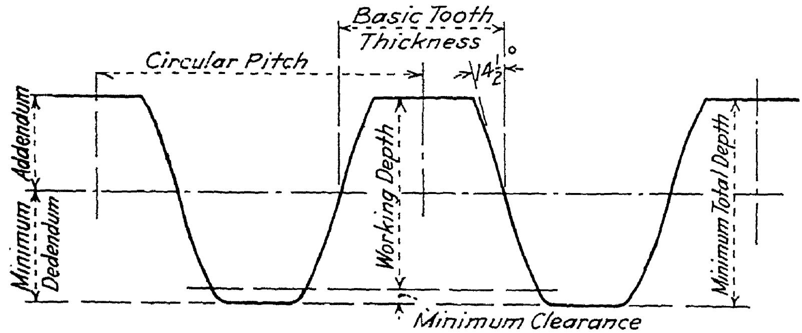
of the base circles, or conversely, the diameters of the base circles determine the pressure angle. Professor Willis suggested in 1838 that the pressure angle be made 14½ deg., since this value would give satisfactory results for the sizes of gears which might be wanted for any set. The sine of 14½ deg. is 0.25038 or practically one-fourth, which was easily laid out by the pattern maker when most gears were made with cast teeth. Some gear manufacturers have adopted an angle of 15 deg.

From Fig. 6 it is now clear that involute curves developed from the base circles satisfy the fundamental law of toothed gears, which requires that the normal to the curve of tooth contact passes through the pitch point.

230. Involute Tooth Contact.—If, in Fig. 6, the gear A is the driver and B is the driven, it is evident that tooth contact will begin at d, where the addendum circle of the driven gear cuts the pressure line. Contact will then follow the pressure line de, and contact will end at e where the addendum circle of the driver cuts the pressure line. According to the geometry of the problem the addendum circle of gear B should pass through point d, and the addendum circle of gear A should pass through point e, but this would result in a different length of tooth addendum for each gear. To simplify the problem the addendum lengths for all gears of one pitch are usually made the same. The tooth proportions on this basis will give addendums which in the majority of cases will fall inside the points e and d of Fig. 6. In Fig. 6, contact will begin at d on the driving tooth flank, and end at e on the driving tooth face. It should be noted that if the addendums of the teeth are not long enough, contact may not begin at d, and the pressure line will be shortened accordingly. If in Fig. 6, gear B is the driver, contact between the teeth will begin at e and end at d. If the direction of rotation of A is reversed, A being the driver, the pressure line dPe will have the opposite slope, the construction, however, remaining the same.

In Fig. 6, it will be assumed that the angle of approach is equal to the angle of recess and that each of these angles is called β, the angle of action being 2β. The angle β would be slightly larger than the angle α in the figure. If the second tooth is to come into contact at d when the first tooth is at e, the pitch arc must be 360 deg. divided by the number of teeth N in the smaller gear, or β must equal 180 deg. divided by N. If β is taken as 15 deg., evidently the least number of teeth which can be used is

TABLE I.—FULL-DEPTH TOOTH PROPORTIONS FOR 14½-DEG. COMPOSITE SYSTEM FOR SPUR GEARS



	Diametral pitch	Circular pitch
Addendum	$\frac{1 \text{ inch}}{DP}$	$0.3183 \times CP$
Minimum dedendum[1]	$\frac{1.157 \text{ inches}}{DP}$	$0.3683 \times CP$
Working depth	$\frac{2 \text{ inches}}{DP}$	$0.6366 \times CP$
Minimum total depth[1]	$\frac{2.157 \text{ inches}}{DP}$	$0.6866 \times CP$
Pitch diameter	$\frac{N}{DP}$	$0.3183 \times N \times CP$
Outside diameter	$\frac{N + 2}{DP}$	$0.3183 \times (N + 2) \times CP$
Basic tooth thickness on pitch line	$\frac{1.5708 \text{ inches}}{DP}$	$0.5 \times CP$
Minimum clearance[1,2]	$\frac{0.157 \text{ inch}}{DP}$	$0.05 \times CP$

N = Number of teeth.

Diametral pitch used up to 1 DP, inclusive. Circular pitch used for 3-in. CP and over.

[1] A suitable working tolerance should be considered in connection with all minimum recommendations.

[2] Minimum clearance refers to the clearance between the top of the gear tooth and the bottom of the mating gear space, and is specified as "minimum" so as to allow for necessary cutter clearance for all methods of producing gears. At the present time this value cannot be standardized.

12, and this is taken as the smallest standard interchangeable gear.

Since it is desirable to have more than one pair of teeth in contact at the same time, it is necessary to have the pitch arc less than the arc of contact. For instance, if the pitch arc is

one-half of the arc of contact, then two pairs of teeth will be in contact simultaneously.

At the origin of the involute tooth curve on the base circle the radius of curvature of the involute is small and changes rapidly, but becomes less and less sensitive as the distance from the base circle increases. The portion of the involute curve near the base circle is difficult to produce accurately, and for this reason it is not desirable to have the profile of the tooth extend close to the base circle.

231. Standard Tooth Proportions.—All gears of the same pitch should have teeth of uniform size, in order that any gear may work equally well with any other gear, independently of the number of teeth. When this condition is satisfied the manufacture of gears is also greatly simplified. Table I gives the empirical formulas for the proportions of tooth parts for the 14½ deg. American Standard[1] gear teeth.

The term "diametral pitch" is used for the smaller gear teeth up to a value of $1DP$, inclusive, while the term circular pitch is used for the larger sizes for circular pitches of 3 in. and over.

A suitable working tolerance should be considered in connection with all minimum recommendations. The term *minimum clearance* refers to the clearance between the top of one gear tooth and the bottom of the mating-gear space, and is specified as mimimum so as to allow for necessary cutter clearance for the various methods of producing gears. At present there is no standard value for this tolerance.

232. The Involute Rack.—A *rack*, shown in Fig. 7, is a gear having its center at infinity, hence its pitch circle is a straight line, and the involute curve which forms the tooth outline is a straight line making an angle with the base line equal to the pressure angle. With this arrangement it is seen that a tangent to the base circle at the point of contact of the pinion will be normal to the rack tooth outline, and pass through the pitch point, thus satisfying the fundamental law of toothed gears. Evidently a rack cannot have continuous motion in one direction.

Figure 7 shows a pinion, which is the name given to the smaller of a pair of gears, driving a rack to the left. The point b is the tangent point on the base circle of the pinion at the end of the

[1] The American Standard has been approved by the American Engineering Standards Committee, and is sponsored by the American Gear Manu facturers' Association and the American Society of Mechanical Engineers.

pressure line, and contact between the teeth must not occur before the point b is reached, if interference between the teeth is to be avoided. Using the standard dimensions given in Table I, the rack teeth have a length of addendum extending beyond b by the amount f. This condition would make contact begin at a

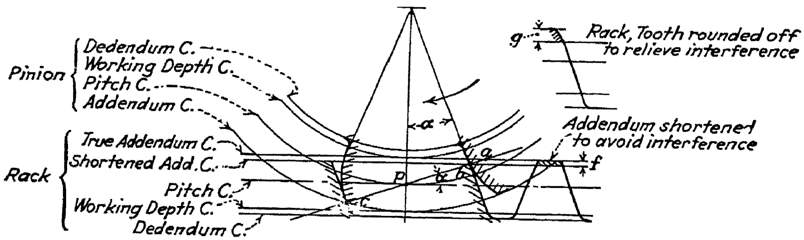


FIG. 7.—Involute rock and pinion.

point a, and would result in interference from a to b. This interference may be relieved by reducing the length of the addendum of the rack teeth by the amount f, and it may also be relieved by leaving the addendum of standard length but rounding the tops of the teeth as shown at g. It may be

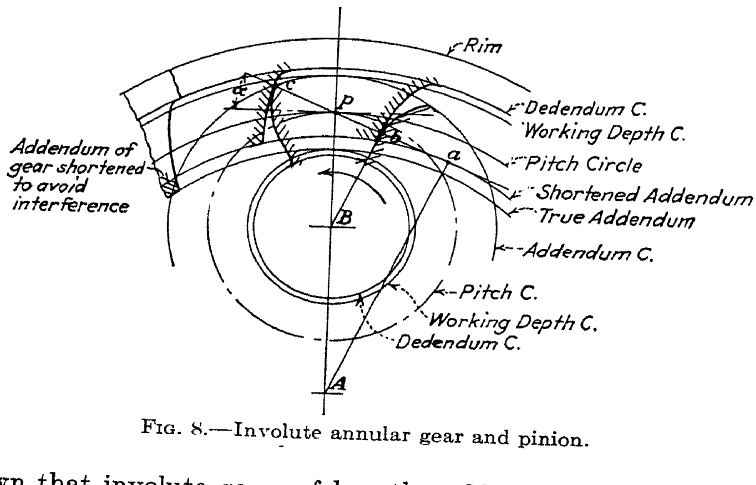


FIG. 8.—Involute annular gear and pinion.

shown that involute gears of less than 30 teeth will have interference to some extent if the standard addendum of $1/DP$ is used.

233. Annular Gear.—The teeth of an *annular* gear, as shown in Fig. 8, point toward the center of the gear, and the angular gear turns in the same direction as the pinion, because the centers of the two gears lie on the same side of the pitch point. The

teeth of the annular gear take the shape outlines of the space between the teeth of the pinion, which has externally formed teeth. The teeth of the annular gear are shortened to eliminate interference, just as was the case with rack teeth.

234. Cycloidal Gear Teeth.—The faces of cycloidal gear teeth are formed by the epicycloidal curve, and the flanks are formed by the hypocycloidal curve. In Fig. 9, *CD* is the pitch circle of a gear, *Bp* is the radius of the circle rolling on the outside of the pitch circle and generating the epicycloidal curve, and *Ap* is

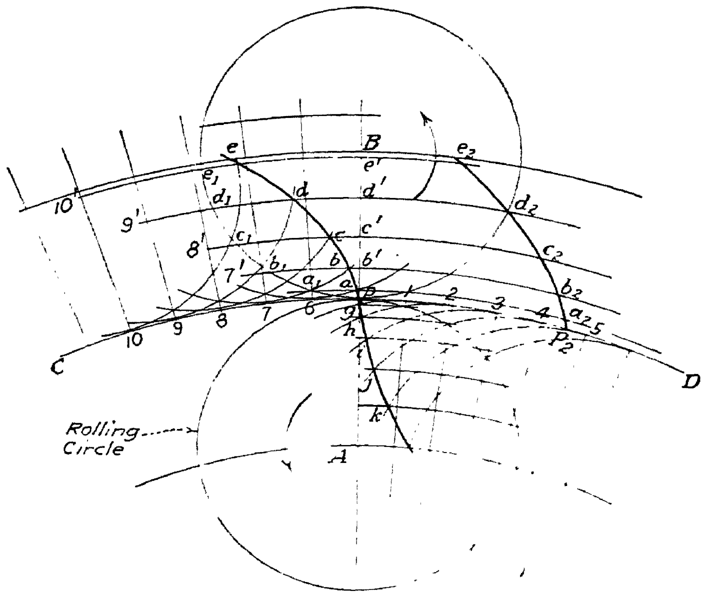


FIG. 9.

the radius of the circle rolling on the inside of the pitch circle and generating the hypocycloidal curve. The point p rolling to the left will trace the epicycloidal curve pe forming the face of the tooth, and the point p rolling to the right will trace the hypocycloidal curve pk forming the flank of the tooth.

The epicycloidal curve shown in Fig. 9 may be constructed by dividing the semi-circumference of the rolling circle into any convenient number of equal distances pa_1, a_1b_1, b_1c_1. On the pitch circle divisions of equal length are laid off from p, such as $p6$, 67, 78. Through the latter points radial lines are drawn to the center of the pitch circle at E (not shown). Using E as a center and radii equal to Ea_1, Eb_1, Ec_1, arcs are drawn cutting

the diameter of the rolling circle at a', b', c'. From points $6'$, $7'$, $8'$, distances $6'a$, $7'b$, $8'c$, are laid off equal to $a'a_1$, $b'b_1$, $c'c_1$. The smooth curve drawn through the points a, b, c, will be the epicycloidal curve. The hypocycloidal curve may be constructed in a similar manner.

The proportions of cycloidal tooth parts may be obtained from Table I, as was the case for involute gear teeth.

It is possible to form cycloidal tooth outlines by using two sizes of rolling circles, but for interchangeable gears the inside and outside circles should be of the same size. For simplicity and uniformity, all gears of the same pitch have tooth curves generated by the same size rolling circle.

235. Contact of Cycloidal Teeth.—For cycloidal toothed gears the line of contact between the teeth will be along the upper

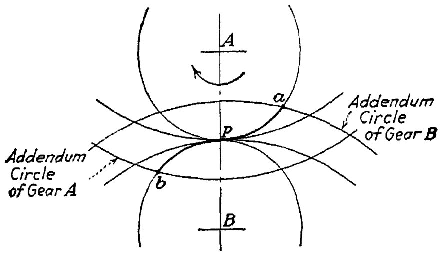


FIG. 10.—Line of action for cycloidal gears.

rolling circle from a to p, as shown in Fig. 10, and along the lower rolling circle from p to b, the gear A being the driver and turning clockwise. The point a will be where the addendum circle of gear B cuts the rolling circle, and the point b will be where the addendum circle of gear A cuts the rolling circle. The points a and b may be located as for involute gears, except that in this case the pressure line is along the arcs of the rolling circles, while for involute teeth it was along the line of obliquity.

In Fig. 9, if the portion of the tooth outline *edcb* is considered part of the gear with center at E, it may be rotated to the position $e_2d_2c_2b_2$, so that d_2 is on the line of contact for two mating gears. It will be found that a normal to the tooth outline at d_2 will pass through the pitch point p, thus satisfying the fundamental law of toothed gears. This must be true because at the instant when point d_2 is generated on the epicycloidal curve, the generating circle is rolling at the pitch point p, and p is the

instant center for the movement of the point d_2 with a radius from d_2 to p.

236. Standard Rolling Circle.—The standard rolling circle which has been adopted, is one which will generate a hypocycloidal curve for a 12-tooth gear. ~~This curve~~ which is coincident with the diameter of the pitch circle, thus giving the gear teeth radial flanks. This may be accomplished by making the diameter of the rolling circle equal to the radius of the pitch circle. Since engineering practice limits the least number of gear teeth to 12, the standard rolling circle for a given pitch is one whose diameter is equal to the pitch radius of a 12-tooth gear of that pitch. In terms of diametral pitch:

$$\text{Diameter of the rolling circle} = \frac{1}{2} \times \frac{12}{DP} = \frac{6}{DP}.$$

For example, a 32-tooth 4-pitch gear would have a pitch diameter of $^{32}/_{4}$ or 8 in. The pitch diameter of a 12-tooth 4-pitch gear would be $^{12}/_{4}$, or 3 in., and one-half of this, or $1\frac{1}{2}$ in., would be the diameter of the rolling circle.

237. Comparison of Involute and Cycloidal Teeth.—Involute gear teeth have a tendency to be undercut too much because the flanks of the teeth are inclined to radial lines, also, there is more interference of teeth than in the cycloidal system.

Some of the advantages of the involute system over the cycloidal system of gear teeth are:

1. The distance between gear centers may be increased without affecting the velocity ratio of the gears; in other words, the amount of backlash between the teeth is variable.

2. Since the pressure line is a straight line the pressure between the axes of the gears is constant and the wear on the tooth surfaces is uniform.

238. The American Standard Tooth Form.—In recent years there has been a tendency on the part of gear manufacturers to introduce a number of variations in tooth forms and proportions, and to meet a demand for uniformity, the American Standard Spur Gear-tooth Form has been developed, after a review of all standards now used. The sponsors[1] of this standard form have tried to incorporate the good features of the cycloidal and involute systems, and to eliminate as far as possible the disadvantages of both.

[1] The American Gear Manufacturers' Association and The American Society of Mechanical Engineers.

The American System has a tooth form which has cycloidal curves for the top and bottom portions, and the involute curve for the middle portion of the tooth outline. Figure 11 shows the application of these curves to the 14½-deg. composite system.

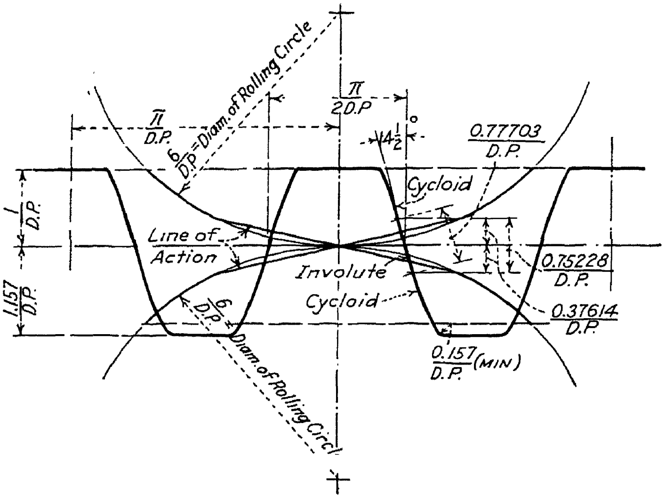


FIG. 11.—The American Standard tooth form. Composite system.

The cycloidal form of tooth is difficult to construct and its layout may be approximated as shown by Fig. 12, the cycloidal curve being drawn with the arc of a circle. The basic rack form of tooth shown by Fig. 12 is the one used in practice.

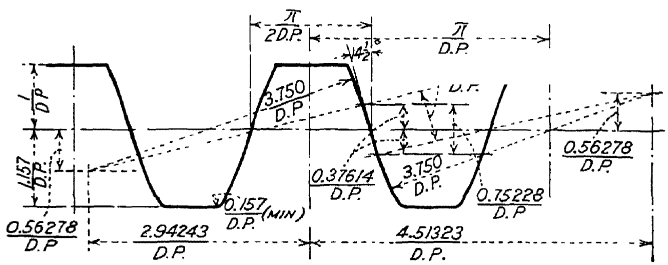


FIG. 12.—Approximation of the basic rack of the 14½-deg. Composite system.

The tooth proportions of the 14½-deg. composite tooth, as shown in Fig. 12, are given in Table I, and are identical with the proportions of the 14½ deg. Brown and Sharp Standard.

239. Strength of Gear Teeth.—It is assumed that as the load is being transmitted from one gear to another it is all given and

taken by one tooth, since it is not always safe to assume that the load is distributed among several teeth. When contact begins, it is assumed to be at the end of the driven tooth, and as contact ceases it is at the end of the driving tooth. This may not be true when the number of teeth in a pair of mating gears is large, because the load may be distributed among several teeth, but it is almost certain that at some time during the contact of the teeth, the proper distribution of load does not exist and that one tooth must transmit the full load.

For any pair of gears having an unlike number of teeth, the gear which has the fewer teeth will be the weaker, because the tendency towards undercutting of the teeth becomes more pronounced in gears as the number of teeth becomes smaller.

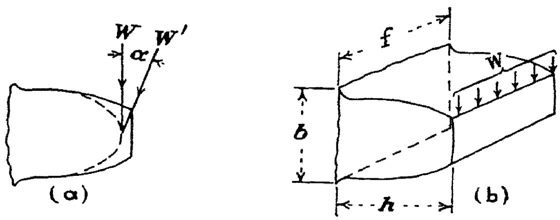


FIG. 13.

In Fig. 13(a) the load W' acting on the tooth makes an angle with W equal to the angle of obliquity. W is the tangential component of W', and is assumed to be uniformly distributed over the dimension f of the tooth. The driving component W produces a reaction at the bearing of the gear, which is increased by the horizontal component of W', but for the $14\frac{1}{2}$-deg. involute and cycloidal systems this component is small and is usually neglected. The increase in pressure due to this component is about 7 per cent for the 20-deg. involute system.

The gear tooth in Fig. 13(b) is treated as a cantilever beam, one tooth is assumed to take the full load, and the load W is assumed to be the same as the force transmitted at the pitch point. Hence:

$$Wh = \frac{SI}{c} = \frac{Sfb^2}{6}, \tag{4}$$

$$W = \frac{Sfb^2}{6h} \tag{5}$$

in which W denotes the load or tangential pressure at the pitch point, in pounds.

h denotes the length of the tooth, in inches.

S denotes the allowable tensile unit stress of the gear material, in pounds per square inch.

b denotes the thickness of the tooth, in inches.

f denotes the width of the gear face, in inches.

In formula (5), b and h are variables depending upon the circular pitch (CP) and the form of the tooth. A large number of gear-tooth forms were measured by Wilfred Lewis[1] from drawings, and he thus established a coefficient which he called y. In doing this the normal force W' was extended from the point of the tooth to the center line of the tooth, as shown in Fig. 13(a). By replacing $b^2/6h$ in formula (5) by $(CP)y$, the Lewis formula becomes:

$$W = Sf(CP)y. \tag{6}$$

Since the quantities b and h are proportional to the circular pitch, it was found that y was practically independent of the circular pitch, but was determined largely by the number of teeth in the gear.

The coefficient y, as determined for the gear-tooth systems most commonly used, are given by the following formulas.

For the 14½- or 15-deg. involute and cycloidal teeth:

$$y = 0.124 - \frac{0.684}{N} \tag{7}$$

For teeth with radial flanks:

$$y = 0.075 - \frac{0.276}{N}. \tag{8}$$

For the 20-deg. involute teeth.

$$y = 0.154 - \frac{0.912}{N}, \tag{9}$$

in which N denotes the number of teeth in a gear.

In proportioning a gear it is usual to make the width of the gear about three to four times the circular pitch for cut teeth, and from two to three times the circular pitch for cast teeth. These proportions are used to insure an even distribution of bearing pressure along the tooth for the entire width of the gear.

[1] *Proc.* Engineers' Club of Philadelphia, January, 1893.

240. Unit Stresses Allowed in Gear Teeth.—At the beginning of contact between two gear teeth, there is assumed to be more or less shock when the teeth strike together, and this tendency increases with higher speeds. For gear teeth that bear along the full width of the gear, C. G. Barth gives a formula for allowable unit stress which allows for a decrease in stress as the speed of the gear increases:

$$S_t \quad S_t'\left(\frac{600}{600 + V}\right), \tag{10}$$

in which S_t' denotes the allowable tensile unit stress at a low velocity or zero velocity, in pounds per square inch.

V denotes the velocity of a point in the pitch circles, in feet per minute.

S_t denotes the allowable tensile unit stress at the velocity V, in pounds per square inch.

The value of S_t' for the ordinary gear materials is given in Table II.

Table II.—Allowable Bending Stress in Gear Teeth and Ordinary Velocities of Pitch Point

Material in the teeth	Value of S_t' at low speed or 0 velocity, pounds per square inch	Speed, feet per minute
Wood	3,000	2,400
Rawhide (treated leather)[1]	8,000	1,800
Bakelite micarta (Westinghouse Co.)[1]	8,000	1,800
Fabroil (General Electric Co.)[1]	8,000	1,800
Cast iron, ordinary, cast teeth	8,000	1,800
Cast iron, good grade, cut teeth	10,000	3,000
Semi-steel	10,000	3,000
Bronze	12 000 to 15,000	3,000
Steel castings (cast teeth)	20,000	3,000
Mild steel, untreated (cut teeth)	25,000	3,500
Alloy steels, casehardened (cut teeth)	50,000	3,500
Chrome-nickel steel, hardened and ground	100,000	4,000
Chrome-vanadium steel, hardened and ground	100,000	4,000

[1] The American Gear Manufacturers' Association recommends the use of the following formula for computing the allowable unit stress for nonmetallic spur gears composed of laminated phenolic materials or rawhide:

$$S_t = \left\{\frac{150}{200 + V} + 0.25\right\} \times 6{,}000.$$

In this formula 6,000 lb. per square inch is the working stress for static loads.

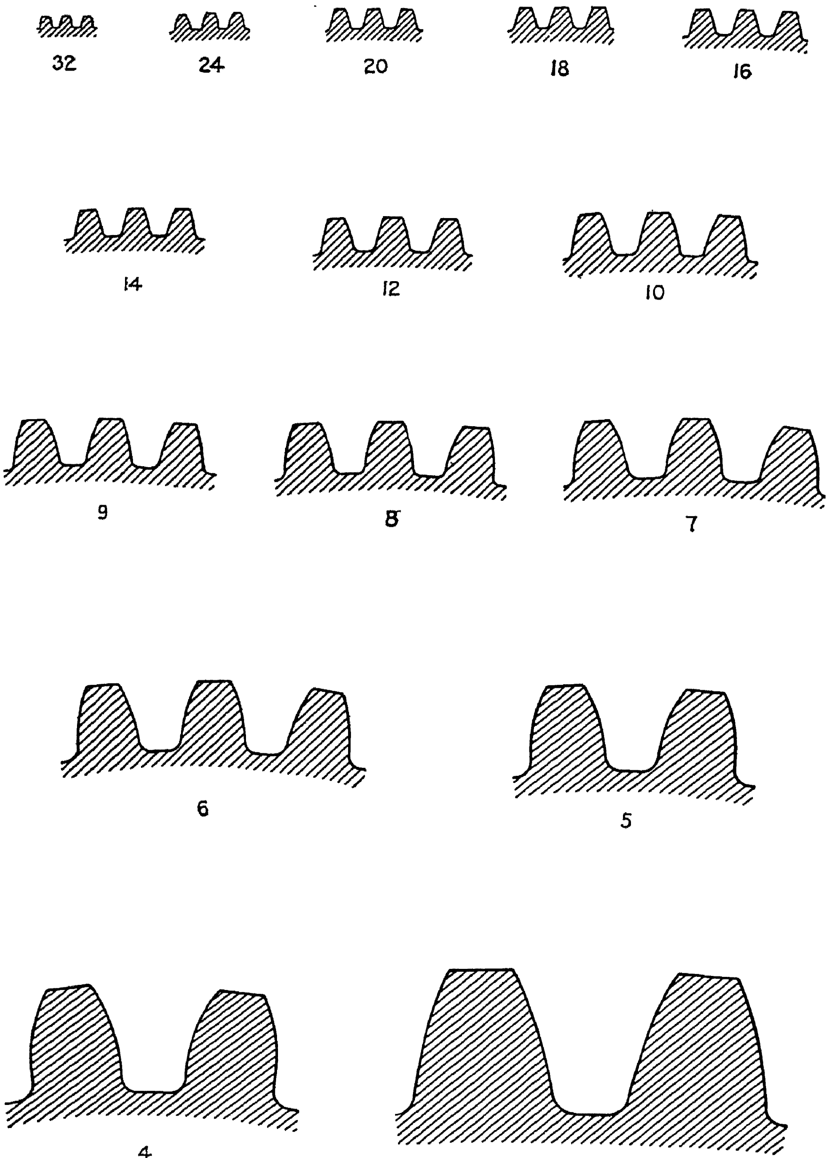


FIG. 14.—Gear teeth of different diametral pitch, full size.

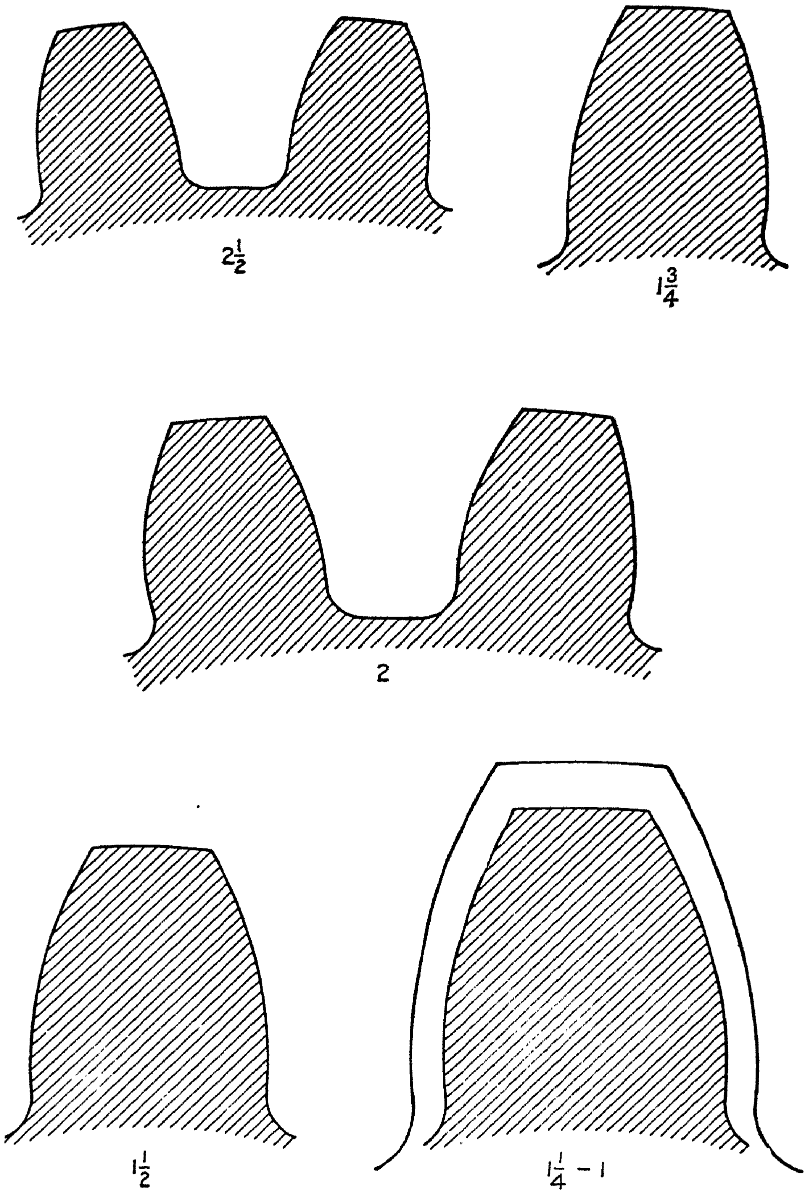


FIG. 14.—*(Continued)*.

It should be noted that the wear of gear teeth is as important as the strength, and the use of the high values of allowable unit stress given in Table II for chrome-nickel and chrome-vanadium steels should be governed by a consideration of their wearing qualities.

241. Procedure in Design.—In designing gear teeth for strength, the distance between gear-shaft centers is known or readily assumed, the velocity ratio of gear shafts is ordinarily fixed, and the pitch diameters of the two gears are inversely proportional to the number of revolutions per minute of the gears, hence:

$$\frac{\text{Pitch diameter of driver}}{\text{Pitch diameter of driven}} = \frac{\text{r.p.m. of driven}}{\text{r.p.m. of driver}}$$

For the different classes of gear application there are several sizes of teeth to choose from, and often any one of several sizes will be satisfactory. A tooth size is chosen from Fig. 14, using the diametral pitch because its use will give a whole number of teeth. The allowable unit stress to be employed is determined by the material used and the linear velocity at the pitch circle. By choosing reasonable values for the elements in formula (6), the force W is readily found. In any case, if the result for the number of teeth is not a whole number, a slight adjustment of gear-center distance will not affect the velocity ratio appreciably.

Example.—A cast-iron $14\frac{1}{2}$-deg. involute gear, having cut teeth, is to transmit 12 hp. at a constant rate. The distance between centers is 20 in., the velocity ratio of driver to driven is 1 to 4, and the speed of the driven shaft is to be 60 r.p.m.

The driving gear will turn four times as fast as the driven gear, or 240 r.p.m., and its pitch diameter is one-fourth the size of the driven gear. The distance between gear centers is the sum of the pitch radii, hence:

$$R + r = 20 \text{ and since } R = 4r.$$
$$R = 16 \text{ in., and } r = 4 \text{ in.}$$

$$\text{Horsepower} = \frac{2\pi r(\text{r.p.m.})W}{33{,}000 \times 12}.$$

Substituting:

$$12 = \frac{2 \times 3.14 \times 4 \times 240 \times W}{33{,}000 \times 12}.$$
$$W = 788 \text{ lb.}$$

Choosing a diametral pitch of 5 or a circular pitch of $\pi/5 = 0.628$ in., the driver will have $5 \times 4 = 20$ teeth and the driven will have $5 \times 16 = 80$ teeth.

The width of the gear may be determined from the Lewis formula (6):

$$W = Sf(CP)y \text{ or } f = \frac{W}{S \times (CP) \times y}.$$

Substituting:

$$f = \frac{788}{5,440 \times 0.628 \times 0.0858} = 2.73 \text{ in., call it } 2\tfrac{3}{4} \text{ in.}$$

In the above formula, y was determined from formula (7):

$$y = 0.124 - \frac{0.684}{20} = 0.0858,$$

and S was determined from formula (10):

$$S_t = 10,000\left(\frac{600}{600 + V}\right).$$

With a diameter of 8 in. and a speed of 240 r.p.m.:

$$V = \frac{\pi d \times 240}{12} = \frac{3.14 \times 8 \times 240}{12} = 502 \text{ ft. per minute.}$$

$$S_t = 10,000\left(\frac{600}{600 + 502}\right) = 5,440 \text{ lb. per square inch.}$$

242. Stub Teeth.—The stub tooth (see Fig. 15), so-called because it is shorter than a standard tooth, is intended to meet the demand for a stronger tooth. It is an involute tooth devel-

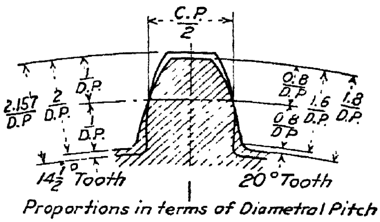


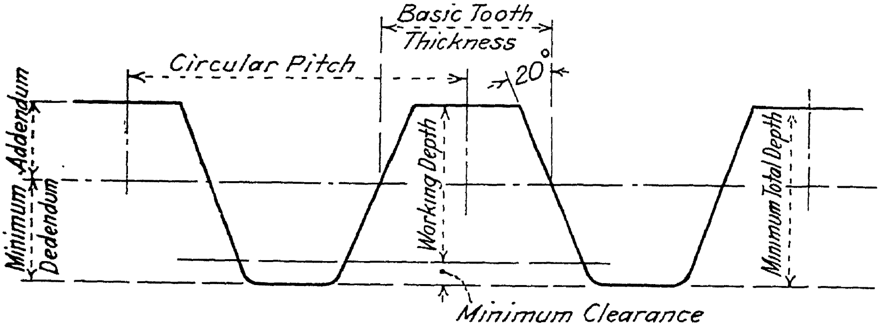
FIG. 15.—Comparison of standard 14½-deg. and 20-deg. tooth proportions.

oped from base circles found by using a 20-deg. angle of obliquity. The proportions recommended for the American Standard 20-deg. Involute System for Spur Gears are given in Table. III.

The Fellows Stub Tooth.—The Fellows Gear Shaper Company of Springfield, Vermont, developed a 20-deg. involute tooth for gears, and designated the tooth proportions by a fractional pitch. A Fellows $^3/_4$-pitch tooth has the width of a 3-pitch tooth and the height of a 4-pitch tooth, resulting in a shorter and thicker tooth, and consequently a stronger one. The usual diametral pitches for the Fellows stub-tooth gears are $^3/_4$, $^4/_5$, $^5/_7$, $^6/_8$, $^7/_9$, $^8/_{10}$, $^9/_{11}$, $^{10}/_{12}$, and $^{12}/_{14}$.

For example, to lay out a $^3/_4$-diametral-pitch tooth, the tooth thickness is taken from a table of standard-tooth parts according to the dimensions of a 3-diametral-pitch tooth; and the addendum, working depth, and clearance according to the dimensions for a 4-diametral-pitch tooth.

TABLE III.—PROPORTIONS FOR 20-DEG. STUB INVOLUTE SYSTEM FOR SPUR GEARS[1,2]



	Diametral pitch	Circular pitch
Addendum	$\frac{0.8 \text{ inch}}{DP}$	0.2546 × CP
Minimum dedendum	$\frac{1 \text{ inch}}{DP}$	0.3183 × CP
Working depth	$\frac{1.6 \text{ inches}}{DP}$	0.5092 × CP
Minimum total depth	$\frac{1.8 \text{ inches}}{DP}$	0.5729 × CP
Pitch diameter	$\frac{N}{DP}$	0.3183 × N × CP

Diametral pitch used up to 1 DP, inclusive. Circular pitch used for 3-in. CP and over.

[1] A suitable working tolerance should be considered in connection with all minimum recommendations.

[2] Minimum clearance refers to the clearance between the top of the gear tooth and the bottom of the mating gear space, and is specified as "minimum" so as to allow for necessary cutter clearance for all methods of producing gears. At the present time this value cannot be standardized.

243. Wearing of Gear Teeth.—The wearing quality of the gear-tooth surfaces is sometimes more important than the strength, this being true when gears are subjected to heavy constant loading and continuous service. For such service, alloy steels like chrome-nickel and chrome-vanadium are often used. Blanks for such gears are usually forgings, and after machining and cutting, the gears are heat treated, ground, and lapped. Lubrication of gear teeth to reduce friction and wear is of the greatest importance.

244. Cutting Gear Teeth.—The method of cutting gear teeth depends upon the number of gears to be cut. The production

of gears has assumed such large proportions that special automatic gear-cutting machines have been developed, and gears are manufactured in shops that make a specialty of gear cutting. For cutting gears in limited quantities the milling machine is employed, using a circular rotating tool shaped to conform to the space between the teeth of the gear that is to be cut. Figure 16(a) shows such a cutter, one of a set of eight for the involute system. Each cutter is marked, showing the system, pitch, and range of teeth that it will cut.

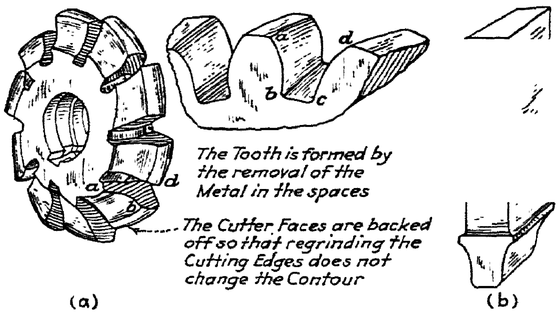


FIG. 16.—Gear-tooth cutters.

Theoretically, a different cutter is necessary for each gear having a different number of teeth, but for practical reasons only eight cutters are used for the $14\frac{1}{2}$-deg. system. These cutters are formed to the correct shape for gears of the smallest number of teeth in their range and are satisfactory for general work.

No. 1 cutter will cut from 135 teeth to a rack.
No. 2 cutter will cut from 55 to 134 teeth.
No. 3 cutter will cut from 35 to 54 teeth.
No. 4 cutter will cut from 26 to 34 teeth.
No. 5 cutter will cut from 21 to 25 teeth.
No. 6 cutter will cut from 17 to 20 teeth.
No. 7 cutter will cut from 14 to 16 teeth.
No. 8 cutter will cut from 12 to 13 teeth.

There is a second set of cutters which are listed by half numbers for cutting teeth when greater accuracy is required; for example, the No. $7\frac{1}{2}$ cutter will cut gears of 13 teeth only and the No. $6\frac{1}{2}$ cutter will cut the 15- and 16-tooth gears, the No. 7 having been designed for the exact contour of the 14-tooth gear.

The cycloidal system of gears requires a set of 24 cutters because the change in tooth curvature is more pronounced in using the two-curve tooth contour.

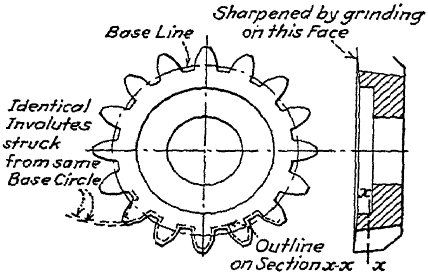


FIG. 17.—Fellows gear-shaping cutter.

Figure 16(b) shows a planer tool, shaped and ground for cutting gear teeth, which is used principally for cutting replacement teeth for broken gears.

The Fellows gear shaper[1] uses a cutter which is formed like a gear, as shown in Fig. 17. The diameter of these cutters is limited to 3 or 4 in. in pitch diameter, which means that coarse-toothed cutters cannot be formed with

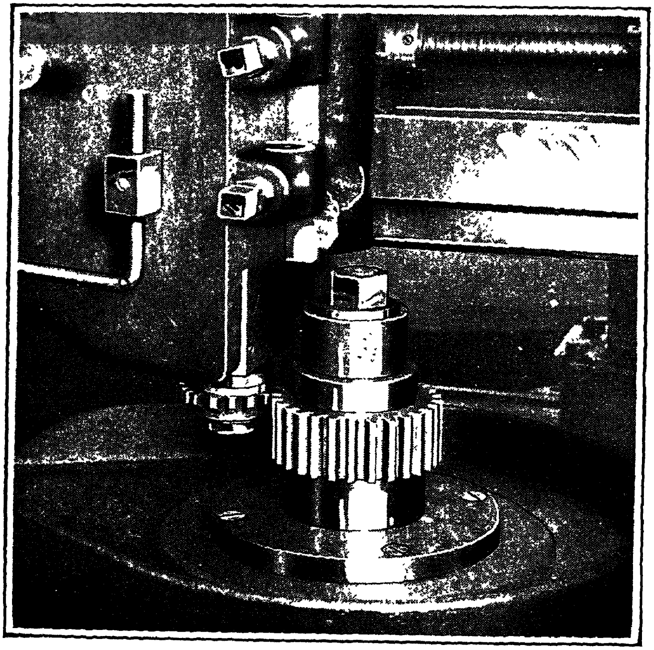


FIG. 18.—Cutter and gear blank. (*Fellows Gear Shaper.*)

teeth of the correct length. The cutter reciprocates, and both the cutter and the gear blank turn slightly between cuts, so that when the gear blank has made a complete turn the teeth have all been

[1] The Fellows Gear Shaper Company, Springfield, Vt.

cut. The addendum of the teeth on the cutters are made longer by an amount equal to the clearance to be cut at the bottoms of the grooves. Figure 18 shows the Fellows gear shaper with the gear-shaped cutter and gear blank in position for a cutting operation. Figure 19(a) shows the gear teeth being formed and the shape of the metal chips. There is no interference of teeth cut in this manner, because the cutter in "generating" the teeth relieves the tooth corners so that they have the correct shape for non-interference, and gears with as few as 12 teeth have the tooth outlines correctly formed.

Spur gears with coarse pitches have the teeth planed by cutters which are guided by a former which is an exact reproduction of the tooth outline required. These machines are

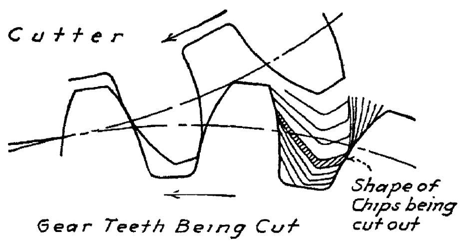


FIG. 19(a).

specially designed draw-cut shapers having an index wheel to space the teeth. A cutter that is the segment of a rack is used by some gear manufacturers to cut the teeth in gears with coarse pitches. The cutter reciprocates and the blank is turned so that the teeth are generated similar to the Fellows method described above. On large gears the teeth are sometimes rough cast in the blank, and on small gears a roughing cut is often made prior to the finishing cut.

When gears are to be produced in quantity, the "hobbing" process is often used, because it is one of the most rapid methods of forming gear teeth. Hobbing is a generating process, and one hob may be used to cut all gears of the same pitch. A hob is a worm-shaped cutter having cutting teeth formed by milling a number of grooves across the threads. When extreme accuracy and uniformity in results are demanded, the sides and tops of the teeth on the hob are ground true to form after hardening. The hobbing machine is a special type of milling machine, and the process consists of revolving and advancing the cutter

through a revolving blank. When cutting, the hob is set at an angle with the gear blank so that the helix at the middle of the hob tooth is tangent to the side of the gear tooth. Figure 19(*b*) shows a spur gear having the teeth formed by the hobbing process.

A machine of recent development finishes gear teeth by a "shave" cut down the edges of the teeth in the direction of the sliding action of the teeth.

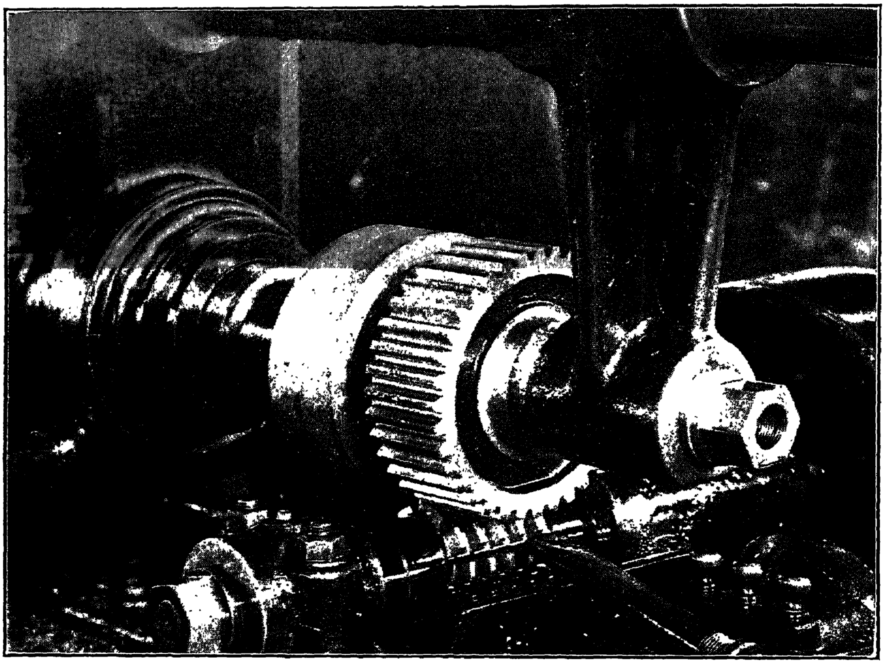


Fig. 19(*b*).—Hobbing process of forming gear teeth.

Heat-treating processes in gear producton tend to distort the gear teeth, and if extreme accuracy of tooth form is required, the teeth are finished by grinding on a gear grinder, after having been heat treated. The grinding is done by the flat side of a diamond-dressed carborundum-wheel point which follows a true tooth curve, and the teeth are finished with an accuracy of 0.0002 in.

245. Proportions for Gear Parts.—The gear rim is dependent for its proportions on the practicability of moulding and machining. The dimensions have a relation to the size of the tooth

and of the adjoining spokes, so that shrinkage stresses will not cause the misalignment of parts, and so that the rim will not spring due to the cutter pressure when the teeth are cut. Practice has determined that a rim thickness below the bottom of the teeth equal to 1.3 times the thickness of the teeth is satisfactory.

The hub should be long enough to prevent the gear from rocking on the shaft, and this means that the hub length should be at least 1.25 times the shaft diameter. For a gear having a face greater than this amount, the hub lengths should be the same as the face width. The diameter of the hub should be 1.75 to 2 times the shaft diameter. Since the keyway in the hub is a source of weakness because of the high localized stresses at the corners, it is sometimes necessary to reinforce the hub across the keyway. In such cases one-half the key thickness added to the diameter of the hub has proved satisfactory.

Small gears are made solid, or if size permits, with a web connection between the rim and the hub, located centrally when possible. Web sections are often cored with round holes to lighten the casting and give it the effect of having spokes.

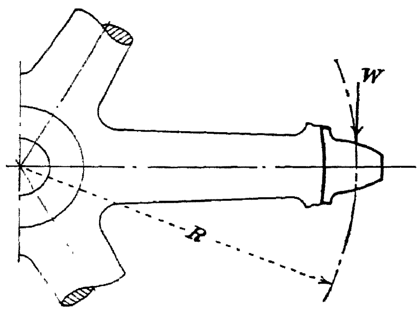


FIG. 20.

Large gears have spokes joining the rim to the hub, the number of spokes depending upon the size of the gear. Up to a diameter of 60 in., six spokes are used; for diameters from 60 to 80 in., eight spokes; and for diameters larger than 80 in., ten spokes.

The spokes are designed on the assumption that they act like cantilever beams fixed at the hub end, and that they are subjected to a load at the tooth end which is equal to the total load transmitted divided by the number of spokes. In Fig. 20 the bending moment M is equal to WR, and this is put equal to the resisting moment SZ, hence:

$$Z = \frac{WR}{NS} \tag{11}$$

In which W denotes the load in pounds, and is the same as found by the Lewis formula.

R denotes the pitch radius of the gear, in inches.

S denotes the allowable tensile unit stress of the spoke material, in pounds per square inch.
N denotes the number of spokes.
Z denotes the section modulus of the spoke at the hub end, in inches³.

If the gear teeth and the spokes are of the same material, formula (11) becomes:

$$Z = \frac{f(CP)yR}{N}, \tag{12}$$

in which the quantities f, (CP), and y are the quantities of the Lewis formula.

For common commercial gears the oval cross-section for spokes is best for small and medium-sized gears, because its appearance is good and it is easily molded. For large gears, subjected to heavy loads, other shapes of spoke section are used to obtain the most economical distribution of metal. The calculated size of the arms at the hub and rim are often modified considerably, because of the practical problems of casting and machining to produce the finished gear, but these modifications are always on the side of safety.

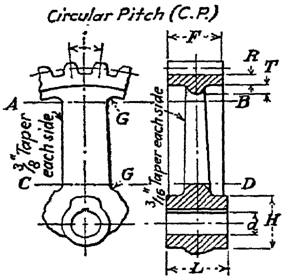
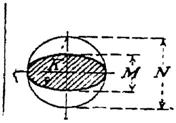
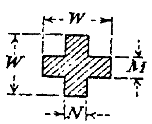
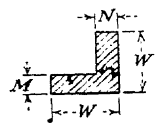
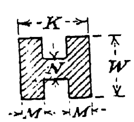


Fig. 21.

The following empirical shapes and sizes, shown in Fig. 21 and given in Table IV, will give gear proportions which are on the side of safety.

Table IV.—Empirical Formulas for Gear Proportions

Section of spoke at A-B	 Oval	 Cross	 Tee	 I or H
F	3CP	3CP	3CP	3CP
R	0.625CP	0.625CP	0.625CP	0.625CP
T	0.5CP	0.5CP	0.5CP	0.5CP
d	Diameter of shaft = bore in inches			
H	1.8d	1.8d	1.8d	1.8d
L	1.25d (Min)	1.25d (Min)	1.25d (Min)	1.25d (Min)
	$N = 2.15 \times$ CP	$N = 0.3 \times$ CP	$N = 0.5 \times$ CP	$N = 0.3 \times$ CP
	$M = 0.5 \times N$	$M = 0.5 \times$ CP	$M = 0.3 \times$ CP	$M = 0.5 \times$ CP
	$K = 0.75N$	$W = 2.3 \times$ CP	$W = 2.3 \times$ CP	$W = 2.4 \times$ CP
G	Make fillets (G) large as consistent			
				$K = F - (0.5 \times$ CP)

246. Non-metallic Gears.—To reduce the vibrations and noise which accompany the working of a pair of metal gears when operating above normal speeds, the pinion is sometimes made of a softer material, which will also reduce the wear on the gear.

Bakelite-micarta gears are made of canvas fabric impregnated with bakelite. A number of thicknesses are put together and subjected to heat and pressure, resulting in a product which is strong and resilient and well suited to the manufacture of gears. These gears do not require end plates, are unaffected by extremes

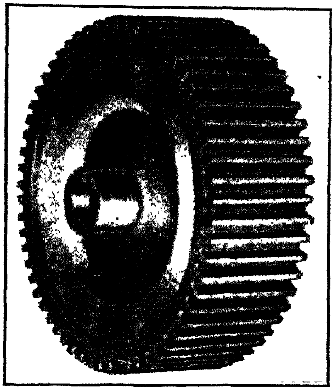


FIG. 22.—Micarta gear with steel center.

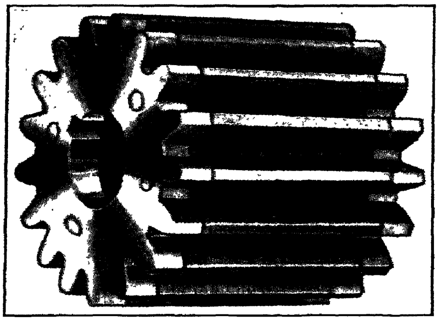


FIG. 23.—Brass flanged rawhide pinion. The effective width of the gear does not include the metal flanges.

in temperature, will operate in water or oil, and will not deteriorate with age. Figure 22 shows a cut micarta gear, the central portion of the gear being made of metal.

Rawhide gears are used for operating conditions similar to those described for micarta gears. The gears are made by cementing together disks of treated cowhide, forming a gear of the proper face width, the ends being reinforced with metal flanges securely riveted. Figure 23 shows a brass-flanged rawhide pinion. These gears should not come into contact with water, and are affected by heat and by extremely dry air conditions.

Wood is sometimes used for gear teeth, the teeth being mortised and wedged securely in a metal rim, as shown in Figs. 24(a) and 24(b). Wood for such teeth should be close grained and wear resisting, and for this reason maple is the wood most commonly used.

Other non-metallic materials having various trade names are also used, and for design purposes the strength may be estimated to be equal to that of cast iron.

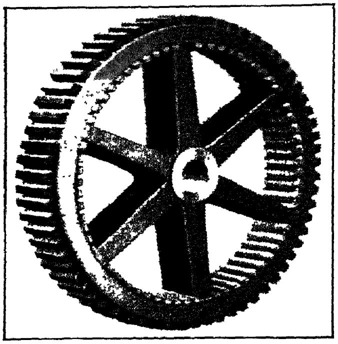


FIG. 24(a).—Mortise gear. Cast-iron wheel with maple teeth.

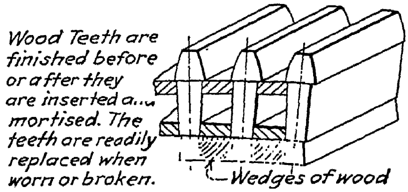


FIG. 24(b).

247. Bevel Gears.—Bevel gears are used for transmitting power at a constant-velocity ratio between shafts which are non-parallel but lying in the same plane. The teeth are formed by cutting grooves along the elements of a cone frustrum, the outside surface of the teeth being larger than the pitch cone by an amount equal to the addendum of the teeth. Figure 25 shows the method of finding the pitch cones of a pair of bevel gears, which are to transmit power at a given velocity ratio, by shafts which make a given angle with each other.

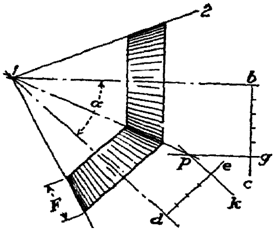


FIG. 25.

When bevel gears have their axes at an angle other than 90 deg. they are called *angular* gears. *Miter* gears are bevel gears which have axes making an angle of 90 deg. with each other and a velocity ratio of unity, hence they are of the same size and have the same number of teeth.

Example.—Find the pitch cones for a pair of bevel gears to transmit power at a velocity ratio of 5 to 4, between two shafts having an angle α between them.

The lines $1b$ and $1d$ are drawn making an angle α with each other. At any point on the line $1b$, such as b, the perpendicular bc is erected, and on it 5 unit divisions are stepped off to any scale. At any point on the line $1d$, such as d, the perpendicular de is erected, and on it 4 divisions are stepped

off to the same scale used before. Through the last division point on each line the lines *pg* and *pk* are drawn parallel to 1*b* and 1*d*, respectively, so that they will intersect at some point *p*. Through *p* and 1 the line *p*1 is drawn, which will be the common pitch-cone element for the gears. The pitch cones are completed by drawing the lines 12 and 13.

From Fig. 25 it can be seen that there are a number of pitch cones which might satisfy the requirements of such a problem. Usually, however, the amount of power to be transmitted will determine the diameter of the pitch circle and the width of the gear face.

248. Form of Bevel-gear Tooth.—Bevel-gear teeth have the 14½- or the 20-deg. involute form, modified to eliminate interference of teeth and to overcome the undercutting which the involute system develops on gears with a small number of teeth.

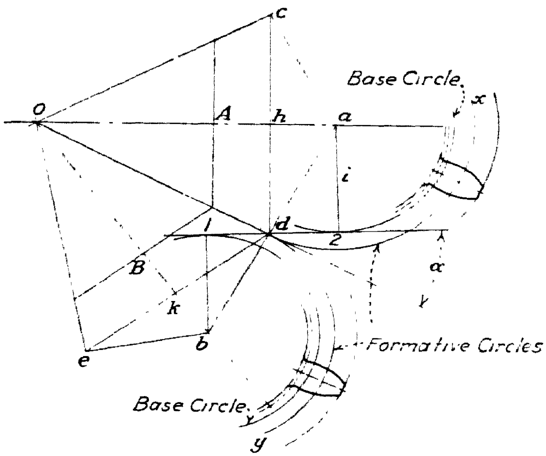


Fig. 26.

A point on a cone rotates about the apex and at a fixed distance, indicating that the point moves with spherical motion and that the teeth of bevel gears should be developed on the surface of a sphere. Since this cannot be done readily an approximate method is used. The pitch cones for the two gears are laid out as shown in Fig. 26. The lines *ca* and *da* are then drawn normal to *oc* and *od*, respectively, and form the *back cone* of the pitch cone *cod*. Similarly, the lines *db* and *eb* form the back cone of the pitch cone *doe*.

The gear teeth are developed on these back cones[1] as follows. With *ad* and *bd* as radii the arcs *dx* and *dy* are drawn. The line 12 is drawn, making an angle α with the line *od* which equals the

[1] Tredgold's approximate method.

angle of obliquity, 14½ or 20 deg. With a and b as centers normals $a2$ and $b1$ are drawn to the line 12, determining the base circles. The tooth curves may now be developed and the tooth parts proportioned as for a pair of spur gears. Points on the tooth curves are swung about the centers a and b into the back cones of the gears, and the teeth are outlined by the lines as they are projected to the apex o as shown in Fig. 27. The face length of a bevel gear, as shown in Fig. 27, should be from 1.5 to 2.5 times the circular pitch, and the maximum length should not exceed ⅓ of the cone distance for gears up to 3-in. pitch

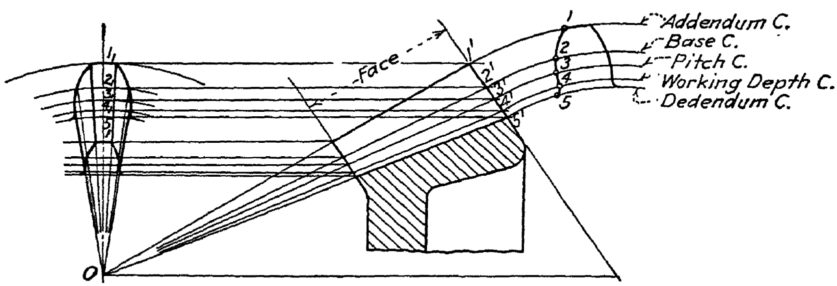


FIG. 27.

diameter, and one-fourth of the cone distance for gears from 3- to 20-in. pitch diameter.

249. Strength of Bevel-gear Teeth.—The strength of bevel-gear teeth is calculated by the formula:

$$W = S_t f(CP) y \frac{d}{D}, \tag{13}$$

in which W denotes the load or tangential pressure at the pitch point at the large end of the gear, in pounds.

S_t denotes the allowable tensile unit stress of the gear material, in pounds per square inch.

f denotes the width of the gear face, in inches.

CP denotes circular pitch of the gear at the large end.

y denotes the coefficient for form and number of teeth, and is found by formulas (7), (8), or (9).

d denotes the pitch diameter at the small end of the gear, in inches.

D denotes the pitch diameter at the large end of the gear, in inches.

In the above formula S_t is determined from Table II and the formula which takes velocity into account, just as was the case for spur gears.

The coefficient y in the above formula depends upon the number of teeth on the formative circle. In Fig. 26 the formative circle of the gear A has ad for its radius, but the pitch circle has hd for it radius. The relation of the formative number N to the actual number of teeth, from Fig. 26, is:

$$\frac{N}{\text{actual number}} = \frac{ad}{hd} \quad \sec \alpha.$$

Hence

$$N = \text{actual number} \times \sec \alpha. \tag{14}$$

For the same reasons which were given for spur gears, the smaller of a pair of mating gears is used in designing for tooth strength.

250. Axial Thrust of Bevel Gears.—Due to the form of bevel gears, they have a tendency to push away from each other.

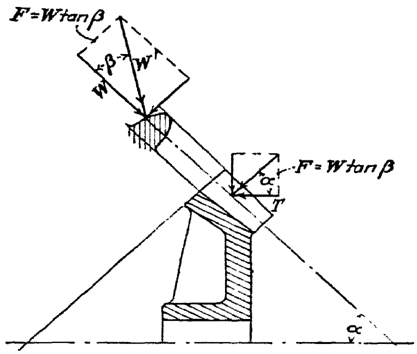


FIG. 28.

Referring to Fig. 28:

β denotes the pressure angle.

α denotes the pitch angle of the gear.

W denotes the pressure at the middle of the tooth face, in pounds.

F denotes the separating force $= W \tan \beta$, in pounds.

T denotes the thrust, in pounds.

Then,

$$T = W \tan \beta \sin \alpha \tag{15}$$

The thrust on the mating gear is:

$$T' = W \tan \beta \cos \alpha. \tag{16}$$

For the usual gear ratios of gear to pinion, the factors by which the tooth pressure is multiplied to determine the thrust may be taken from Table V.

TABLE V.—THRUST FACTORS FOR BEVEL GEARS

Gear ratio	Factor			
Gear to Pinion	Pressure angle = 14½ deg.		Pressure angle = 20 deg.	
	Gear	Pinion	Gear	Pinion
1 to 1	0.183	0.183	0.257	0.257
1½ to 1	0.215	0.143	0.303	0.202
2 to 1	0.232	0.116	0.325	0.163
2½ to 1	0.240	0.096	0.338	0.135
3 to 1	0.246	0.082	0.345	0.115
3½ to 1	0.249	0.071	0.350	0.100
3¾ to 1	0.250	0.067	0.352	0.094
4 to 1	0.251	0.062	0.353	0.088
4½ to 1	0.253	0.056	0.355	0.079
5 to 1	0.254	0.051	0.357	0.072
5½ to 1	0.255	0.046	0.358	0.065

251. Cutting Bevel Gears.—The generating principle is applied in the cutting of bevel-gear teeth. In Fig. 29(*a*)[1] the cutting tools cut alternately toward the cone center so that correct tapering-tooth profiles are produced along the gear face, the tools rotating with the work in the same manner as though they were finished teeth properly in mesh. Figures 29(*b*), (*c*), and (*d*) show the tools used with the bevel-gear generator. The ratio of roll motion between the cutting tools and the work is controlled by change gears. The machine is fully automatic, stopping when the last tooth has been cut. A bevel-gear planer operates on the same principle as a spur-gear planer, the cutting tools being guided by templates. Gear planers are often adjustable for the cutting of either spur- or bevel-gear teeth.

252. Bevel-gear Blanks.—Bevel-gear blanks are usually cast, and are made of iron, steel, or bronze, but when lighter weight as well as strength is desired, steel forgings are used for the blanks.

[1] Gleason Works, Rochester, N. Y.

The rim and web are usually stiffened by ribs running to the hub. A web gives a more uniform connection between rim and hub, and if the gear is large the web is reduced in weight by a

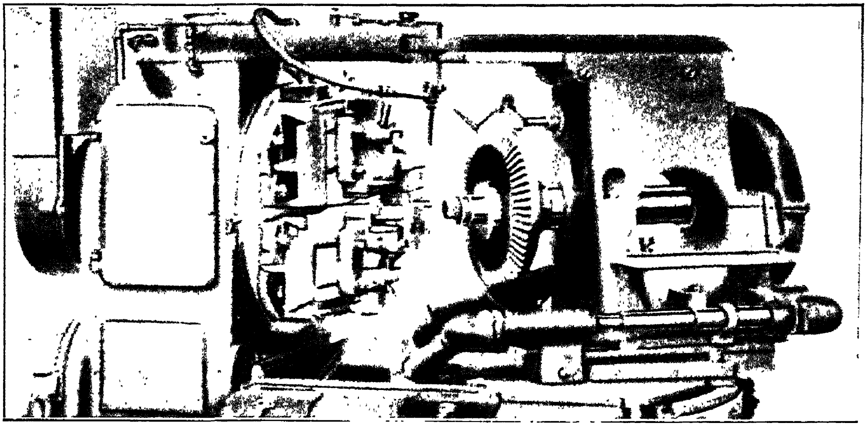
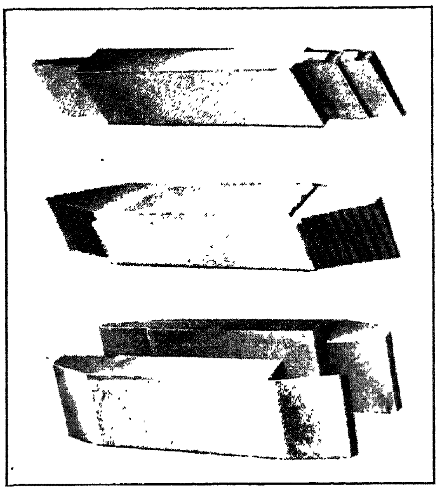


FIG. 29(*a*).—Gleason bevel gear generator.



(*b*)
(*c*)
(*d*)

FIGS. 29(*b*), (*c*), and (*d*).—Cutting tools used with the Gleason bevel gear generator.

series of holes located between the ribs, giving the effect of a number of wide flat spokes. The hubs should be made as long as practicable because of the end thrust. When great strength and light weight are wanted, a disk is cut from a round steel

billet, and forged into the required shape. The advantage gained by this method is that the fibers of the material tend to be parallel to the direction of the stress on the teeth, thus making the teeth stronger.

253. Spiral Bevel Gears.—Spiral bevel gears have the teeth cut along circular arcs on the gear face, each tooth advancing a distance greater than the circular pitch, as indicated by Fig. 30.

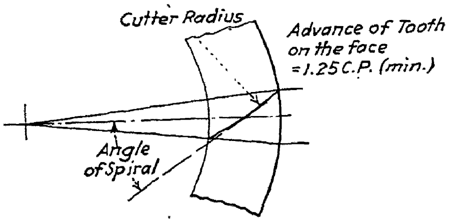


FIG. 30.

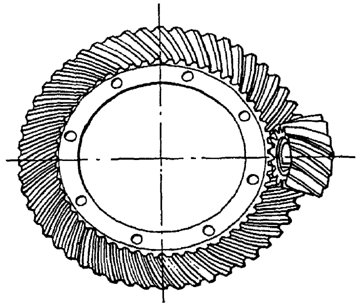


FIG. 31.—Spiral-bevel gear and pinion.

Figure 31 shows a spiral bevel gear and pinion. The cutting of the teeth on this form of bevel gear requires a roughing cut without the generating motion, and then a finishing cut with the generating motion. Figure 32[1] shows the cutter forming the teeth on this form of bevel gear. The spiral bevel gear has a larger number of teeth in contact than if a straight-cut bevel gear were used, it is less noisy, and the stress on each tooth is less because the line of contact between two teeth extends diagonally across the tooth, so that the average moment arm is reduced.

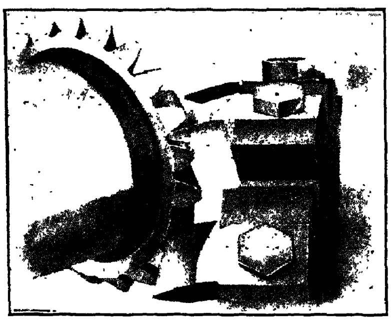


FIG. 32.—Cutter forming teeth on spiral bevel gear.

Spiral bevel teeth, if designed for strength as though they were straight teeth, will have an ample factor of safety. Due to the lengthwise curvature of the teeth, the gears are subjected to end thrust against the bearings, and if the motion of the gears is reversed the thrust is reversed. Bearings should be provided with thrust collars for light thrust

[1] Gleason Works, Rochester, New York.

reactions, and ball or roller bearings if the thrust is heavy. The thrust load depends upon the tooth load, spiral angle, pressure angle, and pitch angle.

The bearings should be as close as possible to the gear hub, and whenever practicable the bearings of both gears should be cast in one piece to maintain true alignment and promote bearing along the full length of the teeth of both gears.

254. Helical Gears.—Helical gears, sometimes wrongly called spiral gears, have teeth which are cut to follow a helical curve around the cylinder of the gear. They may be used to transmit power between shafts which are located as follows:

(*a*) Parallel.

(*b*) At right angles and not in the same plane.

(*c*) At any angle and not in the same plane.

The common application of helical gears is for connecting parallel shafts as shown in Fig. 33, and they are used instead of spur gears because of the following advantages:

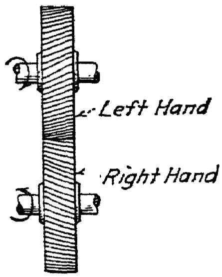


FIG. 33. -Helical gears for parallel shafts.

(*a*) The gears are stronger because more teeth are in contact at one time, and the load is more widely distributed.

(*b*) They are less noisy and have smoother action because the teeth are less liable to shock conditions.

The objection to helical gears is the end thrust, which is proportional to the helix angle, and as with spiral bevel gears, suitable thrust bearings must be provided. If two spiral gears with opposite helix angles are mounted on the same shaft, the end thrust is neutralized. It is this feature which leads to the employment of herring-bone gears, which are double helical gears.

255. Herring-bone Gears.—Herring-bone gears, shown in Fig. 34, are the most efficient type of toothed gear for the transmission of power between parallel shafts, and because of improved manufacturing methods in forming the continuous tooth gear, their use for the transmission of heavy loads is being extended. Accurately formed teeth will allow pitch-line velocities as high as 5,000 ft. per minute, and gear ratios as high as 15 to 1 are possible. Any form of tooth may be used, the 20-deg. involute being common, and the pitch for any condition is smaller than would be chosen for spur gears. The helix angle used varies

between 20 and 30 deg., the smaller angle causing less wear on the teeth. The minimum gear-face width is 6 times the circular pitch.

The teeth for herring-bone gears are designed according to the Lewis formula, in which y is given a value for this type of tooth. A speed factor modifying the allowable unit stress, and a wear factor which depends upon the type of lubrication, are also incorporated.

Herring-bone gears are especially adapted for the transmission of heavy loads at ordinary speeds, and for speed-reducing mechanisms.

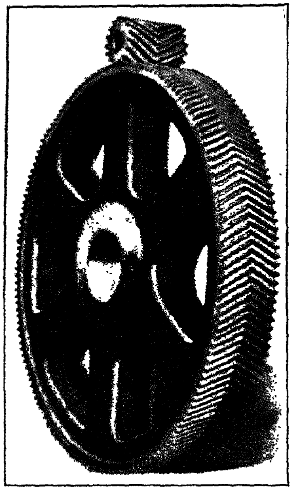


FIG. 34.—Continuous tooth herring-bone gear and pinion.

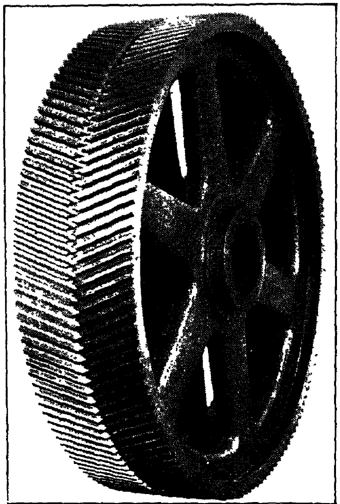


FIG. 35.—Staggered tooth herring-bone gear.

256. Cutting Herring-bone Gear Teeth.—The increasing use of herring-bone gears is due to the improvement in the art of forming the teeth in the cutting process, and gears with cast teeth are seldom used. Cut gears are usually made from steel of good quality and high wear resistance. Large gears are sometimes cast in halves which are partially machined, and then bolted together and machined further for tooth cutting, the parting being made so that it will finish at the bottom of the groove between two teeth.

One form of herring-bone gear has the teeth staggered as shown in Fig. 35.[1] The teeth are cut with a hob in a special double-

[1] The Wuest Gears, manufactured by the Falk Corporation, Milwaukee, Wis.

milling machine, two cuts being made simultaneously on opposite sides of the gear.

When the teeth are continuous across the gear face, the groove metal is sheared away in a gear planer with a double head, one head advancing the cutting tool while a second cutter backs away to a new stroke position. The teeth are cut according to the generating principle, and the machine is fully automatic.

257. Worm Gears.—A worm-gear combination, shown in Fig. 36, consists of a worm and a worm gear to transmit motion between two shafts, which are at right angles to each other but in different planes. The worm is a cylinder which has one or more threads cut on it in the form of a helix. When one thread is cut on a worm, the worm is a single-thread worm, and when turned through one revolution, the linear displacement of a point on the pitch circle of the worm wheel will equal the circular pitch of the teeth. In this case the *lead* is equal to the pitch. If the has two, three, or four threads cut on it, the linear displacement of a point on the pitch circle of the worm wheel will be increased proportionately; the lead will be two, three, or four times the circular pitch, and the worm is called double cut, triple cut, or quadruple cut.

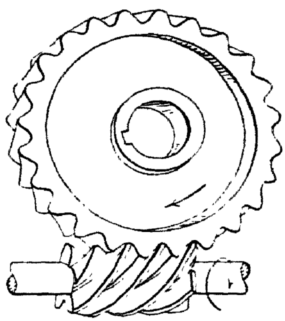


FIG. 36.—Worm and worm gear.

The velocity ratio between a worm and worm gear depends upon the lead of the worm thread and the number of teeth on the worm wheel, being independent of the pitch diameters of the gears. Since these diameters are arbitary, there are a number of variations which will meet the requirements for velocity ratio, but as a rule the pitch diameter of the worm should be as small as possible so as to reduce pitch-line velocities and yet allow for an efficient helix angle. The helix angle is the angle between the tangent of the helical curve and a normal to the axis of the worm.

There are two general purposes for which worm gearing is adapted: (*a*) when power is to be transmitted under conditions making smoothness of action and great reduction of velocity desirable, and (*b*) when a great increase in effective power or torque is required.

There are three general types of worm gears, classified according to the form of the gear face, as straight face, hobbed straight face, and concave face.

The straight-face worm gear is shown in Fig. 37(*a*), and is a helical gear used with a worm, to replace spiral gears. It is not as efficient as the concave face, and should be used only when the load to be transmitted is small.

(a) (b) (c)

Fig. 37.—Types of worm gears.

The hobbed straight-face gear is shown in Fig. 37(*b*), and is first generated on a hobbing machine, and the face is then turned straight. It is not as efficient as the concave face, and is used for indexing purposes and where the load to be transmitted is small.

The concave face, shown in Fig. 37(*c*), is more efficient than the other two types because there is more intimate tooth contact

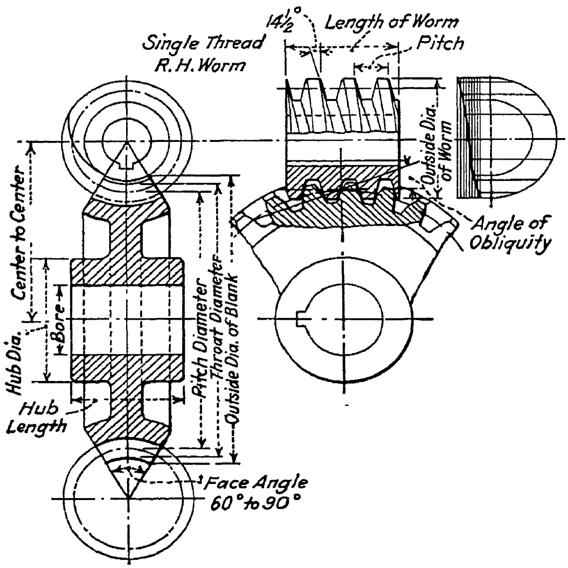


Fig 38.

between the worm and gear, the tooth contours conform closely to each other, and since the load is more evenly distributed, the wear on the teeth is probably more even. The concave face is considered the most acceptable form of gear and it is recom-

mended for worm-gear combinations. Figure 38 shows a shop drawing of a single-thread, right-hand worm and worm-gear combination. A section on the center line shows the teeth in contact as for a spur gear and rack.

258. Design of Worm Gearing.—Fine pitches offer advantages in the design and operation of worm gears, because the load is distributed over a greater number of teeth, reducing the pressure between tooth surfaces so that a good oil film may be maintained between them. Worm-gear proportions given in Table VI are recommended.[1]

TABLE VI.—HORSEPOWER, CIRCULAR PITCH, AND PITCH DIAMETER OF WORM GEARS*

Horsepower	Circular pitch, inches	Pitch diameter, inches
15	0.8125	$2\frac{1}{2}$
25	0.8750	$2\frac{3}{4}$
50	1.0000	$3\frac{1}{2}$
75	1.1250	$4\frac{1}{4}$
100	1.2500	5

* (Automotive and similar applications.)

After the pitch diameter of the worm and the pitch diameter of the gear have been determined, it is necessary to determine the number of teeth in the worm gear that will give the velocity ratio desired.

The load capacity of a worm and worm gear is controlled by the heating effect and the resulting abrasion, rather than by the strength of the teeth. When the load concentration on the teeth is too great, friction causes the temperature to rise to a point at which it is difficult to maintain proper lubrication between tooth surfaces.

The usual and recommended system of teeth for worm and worm gears is the involute system, because with involute curves for tooth outlines the threads on the worm become straight sided as for rack teeth. A worm is a continuous rack in principle, and its tooth sections are rack teeth. Both the $14\frac{1}{2}$-deg. and the 20-deg. form of tooth is employed, and the interference of teeth

[1] THOMAS, HUGH KERR, "Worm Gearing," McGraw-Hill Book Company, Inc.

which is characteristic of the involute system is eliminated by the methods used in forming the teeth.

It is safe to assume that the threads of the worm are stronger than the teeth on the worm wheel, and as a rule, the worm is made of steel and the worm wheel of cast iron or bronze. This is done because the worm threads are subjected to greater wear on account of more continuous contact, and because unlike metals are in general more satisfactory for sliding contact than like materials.

The teeth of the worm wheel are designed as for spur-gear teeth of like pitch, but the width of the gear face is limited by the diameter of the worm. The Lewis formula for spur gears (formula (6)) will give satisfactory results, using the values for y which are found from formulas (7) and (9) according to the pressure angle. The fiber stress, however, is modified somewhat, and in place of using the working stresses as given for spur and bevel gears, those given in Table VII are recommended.

TABLE VII.—WORKING UNIT STRESSES FOR WORM GEARS

Velocity of pitch point, in feet per minute	Working unit stress, pounds per square inch	
	Cast iron	Phosphor bronze
0	5,500	8,000
100	4,500	6,800
200	4,000	6,000
300	3,500	5,500
400	3,000	4,700
500	2,700	4,200
600	2,200	3,800

It is assumed in worm gearing that more than one tooth is in contact, and that the load is distributed over two or possibly more teeth; that there is an absence of shock loading as in spur gears; and that conditions in general are such that the Lewis formula is on the side of safety. Little is known about the true surface conditions of a worm and worm-wheel under load, so that reliance is placed upon the fact that thousands of worm-gear mechanisms have been manufactured and operated successfully, and the practice followed by the manufacturers is considered to be a safe guide for design.

259. Analysis of Forces.—In Fig. 39(a) a portion of one worm thread only is shown for the sake of clearness, and the worm wheel is assumed to be above the worm. For the rotation indicated for the worm, the worm wheel would turn clockwise. In Fig. 39(b) the forces are shown more clearly. The x axis is parallel to the axis of the worm, the y axis is the radial direction on the worm, and the z axis is the tangential direction for the worm. The angle α, the lead angle, is laid off in the tangential direction, and the angle ϕ, the friction angle, also in the same

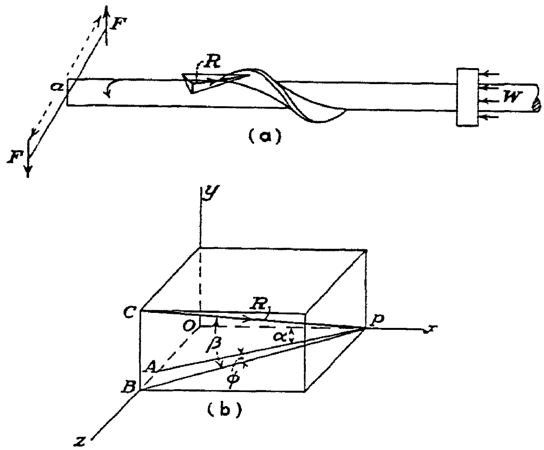


FIG. 39.

plane. The angle β, the thread angle of the screw, is laid off normal to the xz plane. The resultant force R would then act as indicated.

It would be more correct to lay off the angle β normal to the line Ap, and then the angle ϕ parallel to the xz plane. However, the coefficient of friction for modern designs is so small, 0.01 or less, that the approximation involved produces a negligible error.

As shown in the figure, the torque Fa which drives the screw is working against a thrust W, which is taken by a thrust bearing at the end of the worm shaft. The bearing reactions are not shown in the figure.

Summing up forces along the x axis:

$$\Sigma F_x \quad R \cos \beta \cos (\phi + \alpha) - W = 0. \tag{17}$$

Taking the moment sum with respect to the axis of the screw:

$$\Sigma M = Rr \cos \beta \sin (\phi + \alpha) - Fa = 0. \tag{18}$$

in which r denotes the pitch radius of the worm, in inches.

If T is the force which acts tangentially to the screw at the pitch point, evidently:

$$T = \frac{Fa}{r} = R \cos \beta \sin (\phi + \alpha). \tag{19}$$

The axial thrust on the worm is:

$$W = R \cos \beta \cos (\phi + \alpha). \tag{20}$$

The radial thrust on the worm, which tends to separate the worm and worm wheel, will be called H and is:

$$H = R \sin \beta. \tag{21}$$

It is convenient to have the components acting along the axes in Fig. 39(b) expressed in terms of T, the tangential force at the pitch point. When the horsepower transmitted by the worm is known, T may always be found as follows:

$$T = \frac{Fa}{r} = \frac{63{,}024 \text{ hp.}}{nr}, \tag{22}$$

in which hp. denotes horsepower.

n denotes speed of the worm, in revolutions per minute.

From formula (19):

$$R \cos \beta = \frac{T}{\sin (\phi + \alpha)}.$$

Then from formula (18):

$$\text{Axial thrust } W = \frac{T \cos (\phi + \alpha)}{\sin (\phi + \alpha)} = T \cot (\phi + \alpha). \tag{23}$$

From formula (19):

$$R = \frac{}{\cos \beta \sin (\phi + \alpha)}.$$

Then from formula (21):

$$\text{Radial thrust } H = \frac{T \sin \beta}{\cos \beta \sin (\phi + \alpha)} = \frac{T \tan \beta}{\sin (\phi + \alpha)}. \tag{24}$$

If the angle ϕ is negligible compared with α, formulas (23) and (24) become:

$$\text{Axial thrust } W = T \cot \alpha \tag{25}$$

$$\text{Radial thrust } H = \frac{T \tan \beta}{\sin \alpha} \tag{26}$$

260. Efficiency of Worm Gears.—Basically, the worm is a screw thread used for the transmission of power, and the efficiency of screw threads with inclined sides has been discussed in Sec. 187 of Chap. IX. In Fig. 40 Curve 1 shows the theoretical efficiency which might be expected based upon formula (14), Chap. IX, using 0.05 as the coefficient of friction and assuming a frictionless step. Curve 2 is based upon the same formula, using a coeffiiceint of friction of 0.05 for the screw and also for the step. The plotted points are taken from experiments performed by Wilfred Lewis in which the same coefficients of friction were developed as used in Curve 1. The agreement

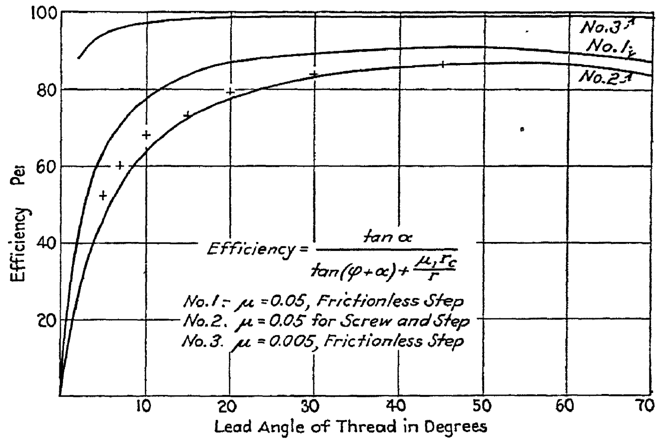


FIG. 40.—Efficiency curves for worm gears.

between experiment and theory is seen to be quite satisfactory. The curves indicate that reasonable efficiencies are obtainable with helix angles as small as 10 deg., and that there is little to be gained by using a helix angle greater than 30 deg.

Curve 3 in Fig. 40 is plotted from the above formula using a coefficient of friction of 0.005, and indicates that very high efficiencies may be obtained. Thomas[1] uses a coefficient of friction of 0.002 for automobile drives, and shows that such a value is quite reasonable.

Example.—Find the 14½-deg. worm and worm wheel that will transmit 15 hp. between shafts which are 10 in. apart, if the speed reduction is to be 10.5 to 1, and the driving shaft is making 1,200 r.p.m.

The student should keep in mind that there are a number of tentative solutions of this problem, because there are several possible combinations of

[1] THOMAS, H. K., "Worm Gearing," McGraw-Hill Book Company, Inc.

pitch radii which will total 10 in. A worm having a pitch diameter of 3.5 in. and a circular pitch of 1.25 in. will be chosen.

Choosing a quadruple thread for the worm, the lead is four times 1.25, or 5 in. The tangent of the helix angle is (see Fig. 41):

$$\tan \alpha = \frac{\text{lead}}{(\text{P.D.})} = \frac{5}{3.14 \times 3.5} = 0.445.$$

The helix angle is therefore 24 deg. 28 min., and the efficiency which may be expected, as taken from Fig. 40, is about 80 per cent.

For a quadruple-thread worm, the worm wheel will have 42 teeth and the pitch diameter of the worm wheel will be $\frac{42 \times 1.25}{3.14} = 16.7$ in., and the sum of the pitch radii should equal the distance between shafts.

$$\frac{16.7}{2} + \frac{3.5}{2} = 10.1 \text{ in.}$$

Assuming that the distance of 10.1 in. between shafts is satisfactory, the worm gear may be checked for strength.

$$\text{hp.} = \frac{2\pi rNW}{33{,}000 \times 12}$$

Substituting:

$$15 = \frac{2 \times 3.14 \times 8.35 \times 114 \times W}{33{,}000 \times 12}, \text{ or } W = 993 \text{ lb.}$$

From the formula of Lewis, formula (6):

$$W = Sf(CP)y \text{ or } S = \frac{W}{f(CP)y}.$$

$$S = \frac{993}{2.75 \times 1.25 \times 0.108} = 2{,}680 \text{ lb. per square inch.}$$

In the above calculation the face of the worm wheel was taken as $2\frac{3}{4}$ in., and the fiber stress in the worm-gear teeth is satisfactory for phosphor bronze and a pitch line velocity of 498 ft. per minute.

In order to transmit 15 hp. to the worm wheel with an efficiency of 80 per cent, there must be supplied a total of 18.8 hp. The torque corresponding to this is:

$$\text{torque} = \frac{63{,}024 \times 18.8}{1{,}200} = 988 \text{ in.-lb.}$$

With a circular pitch of 1.25 in., the dedendum according to Table I will be 0.46 in., or the worm will have a diameter at the root of the teeth of 2.58 in.

The torsional stress in the worm will be:

$$S_s = \frac{988 \times 16}{\pi \times 2.58^3} = 293 \text{ lb. per square inch.}$$

This is very low, and therefore safe.

When the worm is hollow, and bored to fit a 1.25 in. shaft, the unit stress will be 311 lb. per square inch.

261. Cutting of Worm Gears.—The worm may have its threads cut in a lathe, because it is a simple operation to form threads which have straight sides. However, when worms are manufactured in quantities, they are formed on thread millers. The worm-gear teeth are formed by a hob, shown in Fig. 42, which is a driving worm of the same form as the worm which is to run with the gear. The teeth on the gear are generated, the gear blank turning with a positive relative feed.

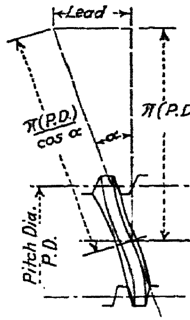


FIG. 41.

Problems

Show by a sketch the development of the involute curve and its application to the involute tooth form. The involute pinion *A* is driving the gear *B* clockwise. Assume all dimensions and show where contact begins and ends for maximum angles of approach and recess. The pinion is to the left on a horizontal center line.

3. An involute rack is moving to the right and is driving a pinion. Assume all dimensions and show where contact begins and ends so that there will be no interference of teeth.
4. A pinion drives an involute annular gear in a counterclockwise direction. Assume all circles and show the tooth curves at the point where contact begins and ends.

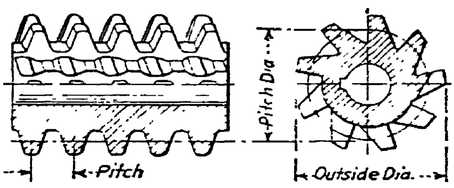


FIG. 42.—Gear hob.

5. The distance between two gear centers is 28 in., and the velocity ratio of the pinion to the gear is 3 to 1. The diametral pitch is 3. What is the outside diameter and the number of teeth for each gear?
6. Make a sketch and name all the tooth parts and circles for a tooth of the American standard $14\frac{1}{2}$-deg. composite system.
7. Assume all circle sizes and show the development of: (*a*) the hypocycloidal curve; (*b*) the cycloidal curve; (*c*) the epicycloidal curve.
8. Show the application of the cycloidal curves to the development of a gear tooth, assuming all circle sizes.
9. A cycloidal gear is to have: (*a*) four pitch teeth; (*b*) five pitch teeth; (*c*) eight pitch teeth. What standard size of rolling circle would be used to generate the cycloidal curves?

10. Show by sketches the line of contact for: (*a*) a pair of involute gears; (*b*) a pair of cycloidal gears.
11. Make a sketch of a rack and pinion, 14½-deg. composite system, with the pinion driving counterclockwise. There is to be no angle of recess but a maximum angle of approach. Assume all circles, and show by hatched lines where the teeth begin and leave contact.
12. Make a full-sized ink drawing of two spur gears which are in mesh. The teeth are:
    (*a*) Involute 14½-deg. standard system.
    (*b*) American Standard, 14½-deg. composite system.
    (*c*) Cycloidal system.
    (*d*) American Standard, 20 deg., composite system.

Data for Gear:
- 32 teeth.
- 2 D.P.
- 6 elliptical arms.
- 1¾-in. hub bore.
- 3¼-in. hub diameter.
- 7/16- by 7/16-in. key.
- Thickness of rim 1.3 times tooth thickness.

Data for Pinion:
- 20 teeth.
- 2 D.P.
- Core circular holes in web to give the appearance of arms.
- 1½-in. hub bore.
- 2⅞-in. hub diameter.
- ⅜- by ⅜-in. key.

The pinion is the driver and turns clockwise.

NOTE: Locate the gear centers ½ in. from the top- and bottom-border lines, and in the middle of the sheet from the right- and left-border lines. Place the gear at the top. Show the outlines of the teeth and other gear parts in black ink. Show the location of the teeth as contact begins and ends by hatched red lines. Show and indicate the values of the angles of approach and recess, and the angle of action. All construction lines to be light red lines. Lay out the tooth outlines by developing the curves. Make wood templates of pieces of soft pine about 1 in. wide and 1/16 in. thick, pin them at the gear centers, and shape them to conform to the tooth outlines. Check the templates by pushing one finished edge against the other, and note if contact follows the line of action. If it does, the curves are correct. Draw in the tooth outlines by using the templates. Print your name on the templates and hand them in with the completed drawings.

13. Make a full-sized ink drawing of a rack and pinion which are in mesh, the pinion to be according to the data of Problem 12.

NOTE: Locate the pinion in the center of the sheet, and draw the whole pinion. Draw the pitch line of the rack to the left of the pinion center and in a vertical position. Shorten the rack teeth to avoid interference. Show the outlines of the teeth and gear parts by black lines. Show the location of the teeth as contact begins and ends by hatched red lines. Show and indicate the values of the angles of approach and recess, and the angles of action. All construction lines to be light red lines.

14. Make a full-sized drawing of an annular gear and pinion which are in mesh. The data for the pinion are identical with those of Problem 12.

Data for annular gear:
32 teeth.
2 D.P.
1¾-in. hub bore.
3¼-in. hub diameter.
7⁄16- by 7⁄16-in. key.
Thickness of rim 1.3 times the tooth thickness.
Cored offset web construction.

NOTE: Locate the center of the annular gear on the horizontal center line of the pinion of Problem 13, and to the right of the pinion center. Follow the general directions as given for Problems 12 and 13.

15. A cast-iron gear with 14½-deg. involute cut teeth has the following proportions:
48 teeth.
32-in. pitch diameter.
4½-in. face width.
4-in. hub bore.
6 arms.
What horsepower will this gear transmit at 90 r.p.m.? Make a pencil drawing of this gear, one-fourth size, giving all dimensions and information necessary for the shop to make the gear.
16. A cast-iron pinion has the same pitch as the gear of Problem 15. (*a*) What horsepower will it transmit if it is turning twice as fast as the gear? (*b*) If this pinion is made of semi-steel what horsepower will it develop under like conditions?
17. If the gear of Problem 15 is made of mild steel, untreated cut teeth, what horsepower will it transmit under the same conditions?
18. A 15-tooth rawhide pinion is rotating at 600 r.p.m., and transmits 37 hp. to a cast-iron gear. The velocity ratio is 5 to 1. The teeth are of American Standard form and have a face width three times the circular pitch. Find: (*a*) the diametral pitch; (*b*) the pitch diameter; (*c*) the outside diameter of each gear.
19. The drum of a hoist is 12 in. in diameter and revolves at 6 r.p.m. The pitch diameter of the drum gear is 36 in. and the pitch diameter of its pinion is 6 in. Both gears are cast iron with American Standard cut teeth. (*a*) Calculate the pitch and number of teeth on each gear for a load of 1½ tons on the drum cable. (*b*) What horsepower is being developed?
20. Make a pencil drawing of the gears of Problem 19, giving all dimensions and data The drawing is to be made on standard 15 by 22 in. paper, using a scale of 3 in = 1 ft.
21. Make a full-size pencil drawing of a pair of cast-iron bevel gears with cut teeth, to transmit power between two shafts at right angles to each other (see Fig. 43).

Data for Gear:
- 24 teeth.
- 2 D.P.
- $1\frac{3}{4}$-in. shaft diameter.
- $\frac{7}{16}$- by $\frac{7}{16}$-in. key.
- $3\frac{1}{2}$-in. hub diameter.
- $4\frac{1}{2}$-in. hub length.
- $2\frac{1}{4}$-in. face width.
- Back of gear stiffened by four ribs.

Data for Pinion:
- 16 teeth.
- $1\frac{1}{2}$-in. shaft diameter.
- $\frac{3}{8}$- by $\frac{3}{8}$-in. key.
- $2\frac{3}{4}$-in. hub diameter.
- $3\frac{5}{8}$-in. hub length.
- Back of gear stiffened by four ribs.

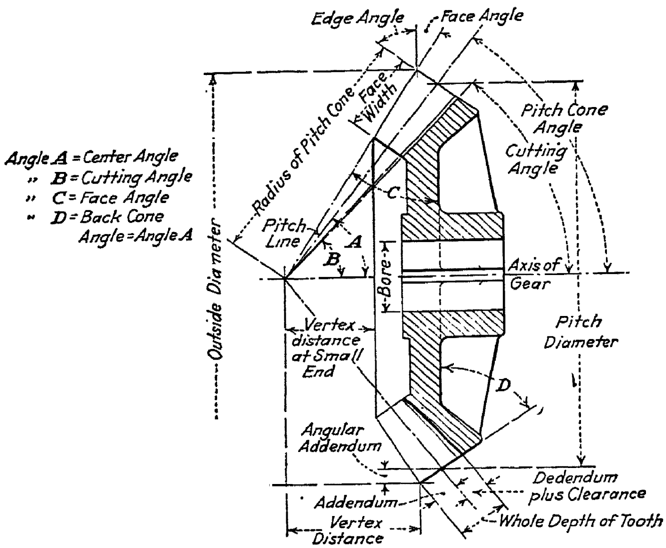


FIG. 43.

Show all important dimensions and finish marks, and tabulate the values of the center angle, cutting angle, face angle, and back-cone angle.

NOTE: Locate the gear on a vertical center line and in the upper area of the sheet. If any interference of gear teeth is noted, direct attention to it by cross-hatching.

22. Make a full-size pencil drawing of a pair of cast-iron bevel gears with cut teeth, which will have an angular velocity ratio of 2 to 3, the D.P. being 3. Show all important dimensions and tabulate any other information which will be required for the making of these gears. The proportions of all gear parts and other details are to be determined by the student, and should follow standard practice.

23. Make a full-sized pencil drawing of a $14\frac{1}{2}$-deg. involute mitre gear, showing two views of the complete gear. Develop the teeth on the back-cone circle, and show the teeth by projection similar to the tooth shown in Fig. 27.

Data: 24 teeth; 2 diametral pitch; face of gear is $2\frac{3}{4}$ in.

24. A pair of cast-iron bevel gears is to transmit 15 hp. The angular velocity ratio is 2 to 1, and the width of the face is two times the circular pitch. The larger gear has a pitch diameter of 20 in., and rotates at 125 r.p.m. Determine the diametral pitch for the Browne and Sharpe standard $14\frac{1}{2}$-deg. tooth.

25. A pair of cast-iron mitre gears have 18 teeth, 7.20 in. pitch diameter, and a 2-in. face width. Determine the horsepower which these gears will transmit at 125 r.p.m.

26. What would be the thrust against the hub collars of the gears of Problem 25?

27. Design a worm and worm-wheel combination which will meet the following requirements: 8 in. distance between shafts; 16 to 1 velocity ratio; 450 r.p.m. for the worm; and the horsepower transmitted is 16. The worm is made of high-carbon steel, and the worm wheel of bronze.

28. An elevator is lifted at the rate of 75 ft. per minute, the diameter of the elevator drum is 18 in., and the total load to be lifted is 4,950 lb. Assuming a speed of the driving motor of 600 r.p.m., and an efficiency of 93 per cent, determine: (*a*) the required horsepower of the motor; (*b*) the worm-and-wheel combination which should be installed.

29. Make a pencil drawing of a worm-and-wheel combination having $14\frac{1}{2}$-deg. involute teeth, for a velocity ratio of 18 to 1. The worm is to have a double-cut right-hand thread, $2\frac{1}{2}$ in. outside diameter, 4 in. long, and keyed to a $1\frac{1}{4}$-in. shaft with a $\frac{5}{16}$- by $\frac{5}{16}$-in. key. The worm is bronze, face angle 60 deg., circular pitch of 0.7854 in., and keyed to a $1\frac{1}{2}$-in. shaft with a $\frac{3}{8}$- by $\frac{3}{8}$-in. key. The drawing is to be similar to Fig. 38, full-size, and located centrally on the sheet. Show all important dimensions, using decimals if needed.

CHAPTER XIII

FRICTION AND LUBRICATION

262. Friction is the resistance which is offered to the sliding of one body upon another. Friction may be partly explained by the interlocking of rough or slightly irregular surfaces, and the deformation and tearing off of small particles as sliding occurs.

Lubrication is the introduction of any substance between the surfaces in contact so as to promote easy sliding. The lubricating agent causes the surfaces to slide on an intervening film instead of being directly in contact with each other.

Measure of Friction.—In Fig. 1 the weight of the body is W and force the F acts in a manner tending to slide the body to the left. Due to friction the total reaction R of the floor on the body must act at an angle ϕ with the normal to the surface of contact. When the body is just on the point of sliding, R makes a maximum angle with the normal, which is called the "angle of friction." The total reaction R may be resolved into two rectangular components, one, the normal pressure, perpendicular to the surface, and the other, the friction, parallel to the surface. The ratio of the maximum friction to the corresponding normal pressure when motion impends is called the coefficient of friction, and is denoted by μ. Hence:

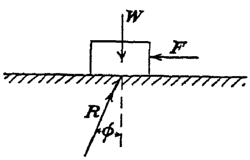


FIG. 1.

$$\frac{F_{\max}}{N} = \mu.$$

$$F_{\max} = \mu N \qquad (1)$$

It is evident from Fig. 1 that when motion impends:

$$\mu = \tan \phi, \qquad (2)$$

in which ϕ denotes the angle of friction.

When the body rests on an inclined plane, as shown in Fig. 2, and the angle θ is such that sliding impends, θ is called the angle of repose. For equilibrium, the total reaction R must be colinear

with W, and it is evident that the angle of friction ϕ is equal to the angle of repose. Hence:

$$\mu = \tan \phi = \tan \theta. \tag{3}$$

Numerous experiments have shown that the friction between the surfaces of bodies which are sliding, is less than the static friction between the bodies when they are at rest and just on the point of sliding. When sliding occurs the body particles do not have time to settle into place and interlock as they do when the bodies are at rest.

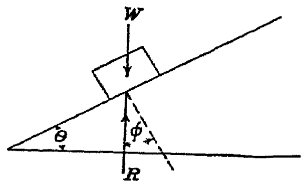


FIG. 2.

263. Laws of Friction.—There are three fundamental laws of friction:

1. The force of friction, when motion impends, is directly proportional to the normal pressure between the two surfaces in contact, when the pressure is low. As the pressure increases the friction increases proportionately, but with high pressures the friction increases rapidly until the two surfaces become interlocked. This interlocked condition is known as *seizing*.

2. For moderate pressures the coefficient of friction and the amount of friction are independent of the area of contact of the surfaces. For high pressures this law is modified as in Law 1.

3. When the velocity is low the coefficient of friction is independent of the velocity of the rubbing surfaces, but with high velocities the coefficient of friction decreases.

264. Efficiency of Machines.—The friction between the moving parts of a machine lowers its efficiency, and a fundamental knowledge of friction is necessary to the design engineer so that machine elements may be utilized to the best advantage. The ratio of the work done by a machine to the work required to keep the machine operating at a constant rate is called its *efficiency*. The maximum efficiency of a machine is attained when it develops the most work in proportion to the energy which it receives.

The efficiency of any machine is determined by testing it while it is operated under working conditions, and its capacity or rating is determined for the conditions under which its efficiency is a maximum. Friction losses are usually measured in percentage of the energy which a machine receives, and good

design implies the least friction losses. Efficiency may be expressed as follows:

$$e = \frac{W_o}{W_i} \times 100, \tag{4}$$

in which e denotes the mechanical efficiency in percentage.
W_o denotes the energy which the machine delivers.
W_i denotes the energy which the machine receives.

The time element is usually important in connection with machine energy, and when taken into account the machine receives and delivers work per unit time, and its rating is measured in horsepower. When the mechanical efficiency of a machine is given as 87 per cent, it is implied that 13 per cent of the energy which the machine receives is lost due to frictional resistance.

265. Average Efficiencies.—The efficiency of the ordinary pairs of machine elements is well known, and Table I lists the efficiencies which may be expected with good design and careful workmanship.

TABLE I.—EFFICIENCIES OF ELEMENTARY PAIRS OF MACHINE ELEMENTS

Kind of pair in contact	Efficiency in percentage
Bearings and journal, ordinary	93 to 97
Roller bearings and races	98
Ball bearings and races	99
Gears, spur with cast teeth, including bearings	93
Gears, spur with cut teeth, including bearings	97
Gears, bevel with cast teeth, including bearings	92
Gears, bevel with cut teeth, including bearings	96
Belting, flat	96 to 98
Belting, chain	95 to 98

A machine consists of a combination of fixed and movable elements, and the combined efficiency is the product of the efficiencies of the individual elements or pairs.

$$e = e_1 \times e_2 \times \qquad \times e_n. \tag{5}$$

266. Rolling Friction.—When there is a common surface in contact between two bodies, motion of one body over the other involves sliding friction, while if the bodies are so formed that

there is line contact between them, the relative motion is rolling, and involves rolling friction or a combination of rolling and sliding friction.

When two dry surfaces tend to roll, one over the other, without slipping, the force which opposes displacement, or frictional resistance, is less than if the two surfaces slide with respect to each other. In Fig. 3:

$$F = \frac{KW}{R}, \qquad (6)$$

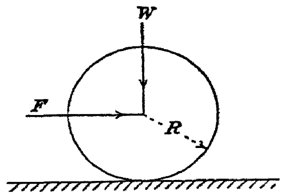


FIG. 3.

in which F denotes the horizontal force which rolls the body with uniform motion, in pounds.

W denotes the weight of the body, in pounds.

R denotes the radius of the body, in inches.

K denotes a coefficient which depends upon the materials of the two bodies and their surface conditions.

For steel on steel, iron on iron, or iron on steel, K equals 0.02.

267. Straight-line Bearings.—Straight-line bearings must have true plane or cylindrical surfaces for the best results. When great accuracy is required the surfaces should be scraped as

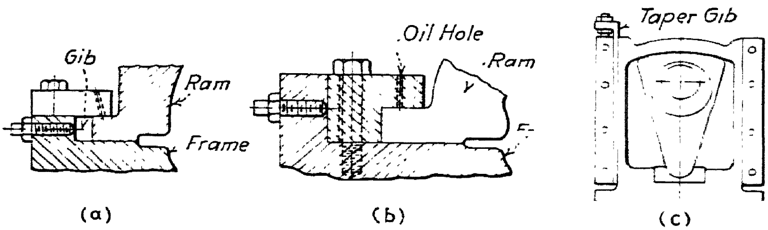


FIG. 4.—Flat slides and guides.

nearly true as possible. Slides should be stiff enough to prevent springing under the load, and of sufficient area to keep the intensity of pressure between the surfaces within established limits. The guide and slide should be of equal width, and the slide should overrun the guide to avoid the formation of a shoulder.

Flat slides like the ones shown by Fig. 4 have less friction than angular slides, shown by Fig. 5, because while the normal pressure on one side necessary to support the weight may be

less when it makes an angle with the vertical, the total corresponding friction is increased. Flat slides require an adjustment for wear as shown by the gibs in Figs. 4(*a*), 4(*b*), and 4(*c*). A gib, tapering lengthwise and adjusted at one end, as shown by Fig. 4(*c*), will prevent wearing in spots, as may be the case with thin gibs set up by screws on the side, as shown by Figs. 4(*a*) and 4(*b*).

Flat surfaces are lubricated by injecting oil through holes provided for that purpose, as shown in Fig. 4 .

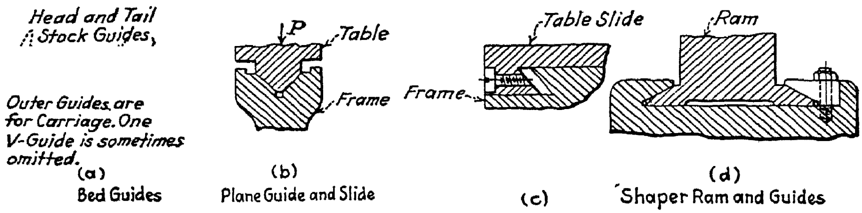


FIG. 5.—Angular guides and slides.

268. Angular Slides.—The V-form of slide is self-adjusting and readily oiled. Ways of lathes, Fig. 5(*a*), and of planers, Fig. 5(*b*), catch dirt and chips. The smaller the angle between the surfaces the less tendency there is for the table to lift from the guides when a side cut is taken, but there is a greater normal pressure between the surfaces. The included angle of the V is usually about 90 deg., but may be more on heavy

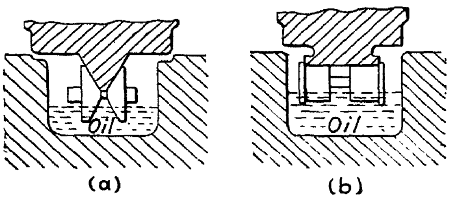


FIG. 6.—Lubrication of planer ways.

machines. Figures 5(*c*) and 5(*d*) show angular guides and slides with gibs which are adjustable by means of set screws.

Flat and angular sliding surfaces are easily lubricated because the oil film is swept along on the surfaces as the sliding member reverses its motion, and returns under reduced surface pressure for the next working stroke. Figures 6(*a*) and 6(*b*) indicate the lubricating scheme used for planer ways. The rollers are pushed

against the platen by springs, and they turn in a well of oil, thus carrying the lubricant against the surfaces.

269. Circular Guides.—Crosshead guides are often made circular because of the ease with which they may be machined. They may be centered as shown in Fig. 7(*a*), with one setting, or the centers may be shifted as indicated in Fig. 7(*b*), so that the sliding member is constrained from turning about the axis of the

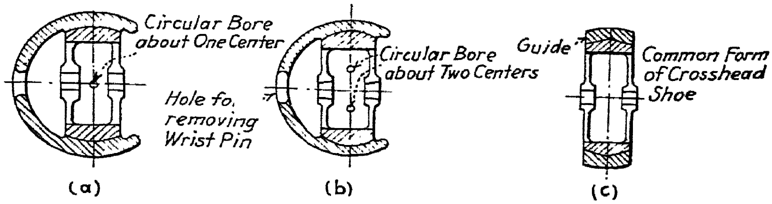


FIG. 7.—Circular guides and slides.

cylinder. The common form of crosshead shoe is shown in Fig. 7(*c*). In all cases in which the guides are fixed there must be some provision made for wear adjustment. This is usually done by providing the crosshead with adjustable wedges which will spread the bearing surfaces.

To distribute oil over the rubbing surfaces, grooves are sometimes cut into the guide member. The edges of the grooves should be well rounded to prevent the tendency to cut the oil

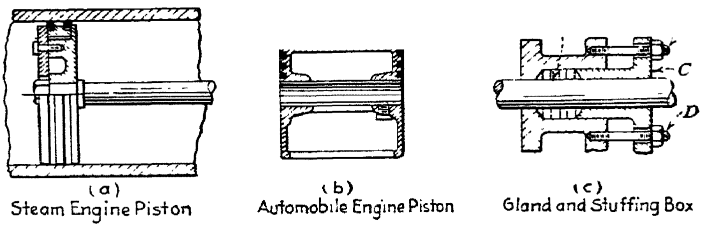


FIG. 8.—Circular guides.

film between the two rubbing surfaces. Oil is injected through holes made easily accessible.

Pistons are circular slides guided by the cylinder walls, and they should overtravel to avoid forming ridges. Figure 8(*a*) shows a steam engine piston and the cylinder walls. The rubbing surfaces are lubricated by oil which is injected into the steam by means of a forced-feed lubricator. Figure 8(*b*) is a

typical automobile engine piston, which is lubricated by oil splashed against the crank end.

Stuffing boxes and glands for engines and pumps belong to this class of guides, and possibly cause more friction and its ensuing troubles than any other form of sliding bearings. Figure 8(*c*) shows a guide of this kind. The packing may be metallic or one of the many commercial rubberized fabrics with graphite, and it is placed in the circular recess of the box at *B*, the gland *C* then being forced against the packing by the adjusting nuts *DD*. This form of joint is used for rotary shafts also, but with indifferent success. Stuffing boxes are imperfectly lubricated by means of a swab which rubs against the piston rod and allows some oil to become attached to it.

270. Pressures between Straight Line Surfaces.—On account of the wiping action between flat sliding surfaces, there is a tendency to expel the lubricating oil, and to insure the best results, light pressures between surfaces are recommended. Table II shows pressure intensities which have been used with success.

TABLE II.—PRESSURES BETWEEN STRAIGHT-LINE BEARINGS

Class of work	Location of surfaces	Pressure, pounds per square inch
Stationary engine, high speed	Crosshead and guide	10 to 30
Stationary engine, low speed	Crosshead and guide	30 to 50
Locomotive	Crosshead and guide	Up to 85
Marine engines:		
Slow vessels	Crosshead and guide	55 to 65
Large vessels	Crosshead and guide	65 to 80
Small cruisers Torpedo boats	Crosshead and guide	85 to 120

271. Circular Bearings.—A bearing is a support for a revolving shaft or axle. A journal is a rotating machine part which is supported in a bearing. The journal and the bearing are the elements which guide the motion of rotation or vibration in machines, and they form one of the most important machine parts.

Journal and Bearing Requirements.—The following fundamental items are considered in the design of journals and bearings:

(*a*) A journal must be a true cylinder or cone of revolution.

(*b*) A journal must be strong enough to resist deformation beyond assigned limits, to prevent excess pressure at the edges of the bearing box.

(*c*) A bearing should be large enough to avoid excessive pressure per square inch of projected area, and to guard against squeezing out of the lubricant.

TABLE III.—ALLOWABLE BEARING PRESSURES

Class of work, where used	Location of bearing	Pressure in pounds per square inch of projected area	
	Main bearings, total load	500 to 700	
Gas engine	Crankpin	1,500 to 1,800	
	Crosshead pin	1,500 to 2,000	
	Driving-wheel journal	Up to 560	
Railroad locomotive	Crosshead pins	3,000 to 4,000	
	Crankpins	1,500 to 1,700	
Locomotive tender	Axle bearings	Up to 425	
Car	Axle bearings	300 to 325	
Stationary engines (slow speed)	Main bearings:		
	dead load	80 to 140	
	live load	200 to 400	
	Crankpins	800 to 1,300	
	Wristpins	1,000 to 1,500	
	Main bearings:		
	dead load	60 to 120	
(high speed)	live load	150 to 250	
	Crank pins:		
	overhung	900 to 1,500	
	center	400 to 600	
	Wristpins	1,000 to 1,800	
		Navy	Merchant
	Main	275 to 400	400 to 500
Marine engine	Crankpins	400 to 500	400 to 500
	Steam-turbine bearings	Up to 85	
	Horizontal steam-turbines, bearings	40 to 60	
Electrical machines	Vertical step turbines, step	200 to 1,000	
	Generator and motor bearings	30 to 80	
	Punch presses and other slow-speed intermittent loads	3,000 to 4,000	
	Turn tables, bridges, slow-speed intermittent service	7,000 to 9,000	
Miscellaneous	Heavy slow-speed, step	Up to 2,000	
	Light line shafting (cast iron)	15 to 25	
	Heavy line shafting (bronze or babbitt)	100 to 150	
	Hoisting-machinery shafting	70 to 90	
	Main bearings	350 to 400	
Automobile engines	Crankpins	350 to 400	
	Crosshead pins	800 to 1,000	
Machine tools	Drill-press thrust collars	Up to 325	

(*d*) A bearing should be rigid and self-aligning for the same reason as *b*, above.

(*e*) A bearing must be as small as is consistent with the above considerations, in order to reduce the work lost by friction.

(*f*) Provision must be made for taking up wear, and the device should be easily adjustable and removable.

(*g*) Bearings should be designed so that most of the wear will be confined to the bearing, since it is more easily replaced than the shaft.

272. Bearing Pressures.—It is not definitely known how the pressure is distributed on an ordinary circular bearing. For design purposes it is assumed that the pressure due to the loading is uniformly distributed over the projected area of the journal, which is the product of the diameter and length of the journal. The pressures which design engineers use are those which have been used with success for the different classes of bearings, and these values are given in Table III.

273. Clearance.—The bearing is always finished slightly larger than the journal, and the difference between the two diameters is the clearance. The amount of clearance depends upon the speed of the shaft and the class of work where it is used, and is approximately 0.002 in. per inch of shaft diameter for small and medium-sized bearings, and slightly less for large ones. Small journals are those less than 1 in. in diameter, medium-sized journals are those from 1 to 3½ in. in diameter, and large journals are those which have a diameter larger than 3½ in.

274. The Relation of Length to Diameter of Journals.—Engineering practice has fixed the approximate length L of a journal in relation to its diameter d according to the values given in Table IV.

TABLE IV.—RELATION OF LENGTH TO DIAMETER OF JOURNALS

Type of bearing	Value of L/d
Marine-engine main bearings	1 to 1.5
Marine-engine crankpins	1 to 1.5
Stationary-engine main bearings	.5 to 2.5
Stationary-engine crankpins	1
Stationary-engine crosshead pins	1 to 1.5
Heavy shafting with fixed bearings	2 to 3
Ordinary shafting with self-adjusting bearings	3 to 4
Generator main bearings	2 to 3

275. High-speed Bearings.—For bearings which are well lubricated, the values given in Table V represent the practice of a large manufacturer of electrical machines made in small units.

TABLE V.—ALLOWABLE PRESSURES FOR HIGH-SPEED JOURNALS

Velocity in feet per second of rubbing surface	20	30	40	69	75
Allowable pressure in pounds per square inch of projected bearing surface........	165	190	205	225	230

The demarkation between the speeds of shafting is not closely drawn, but the student may use the following classification for a guide.

Low speeds are those below 80 r.p.m.

Medium speeds are those from 80 to 300 r.p.m.

High speeds are those above 300 r.p.m.

276. Materials of Journals and Bearings.—The journal is usually made of steel and is sometimes hardened. All journal surfaces should be accurately finished and highly polished. The

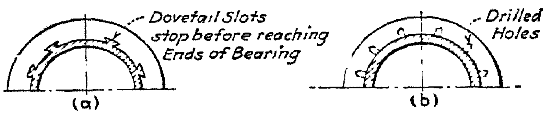


FIG. 9.—Method employed to make bearing metal adhere to bearing casting.

bearing material is generally cast iron, lined with a bearing metal, which is softer than the journal so that the wear will occur in the bearing. There are a number of bearing metals which are used for lining bearings, among them being cast iron, brass, bronze, babbitt metal, and various alloys which are known under trade names. The babbitt or anti-friction metal lining is the most common.

Figures 9(*a*) and 9(*b*) show the usual method employed to make the bearing metal adhere to the bearing casting. Ordinarily a mandrel of the exact size of the journal is placed inside the bearing support, and the babbitt is poured around the mandrel to form the bearing. For better-grade work a smaller mandrel is used, the babbitt is peened after pouring, and the bearing is bored to size. The bearing may or may not be scraped in fitting.

277. Forms of Bearings.—Bearings are classified as follows:

(*a*) One-piece or solid bearings.
(*b*) Two-piece or split bearings.
(*c*) Four-piece bearings.
(*d*) Thrust bearings.
(*e*) Ball and roller bearings.

278. One-piece Bearings.—One-piece bearings are made of cast iron, and may or may not be lined with a bearing metal such as babbitt or brass. Unlined bearings are used for ordinary work and slow speeds. The journal must be assembled and removed endwise, and there is no adjustment for wear. The bearing is provided with an oil hole or grease cup for lubrication. When a one-piece bearing is lined or provided with a bushing, the bearing metal or bushing may be replaced when worn.

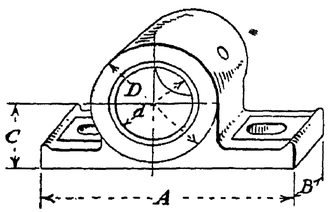


FIG. 10.—Solid bearing.

The solid bearing shown in Fig. 10 is a standard stock design, lined with babbitt, and is used on conveying and hoisting machinery. Other applications of solid bearings are piston-pin bearings of gas engines, loose-pulley bearings, and the bearings at the pin joints of linkages such as locomotive side rods. Empirical dimensions used for the bearing in Fig. 10 are as follows:

d denotes the diameter of the journal, in inches.
K denotes a constant equal to $d + \frac{1}{2}$ in.

Then:

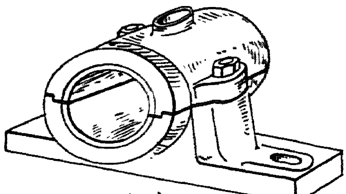
$$A = 2.5K,\ B = 1.25K,\ C = 0.8K, \text{ and } D = 1.5K.$$

The length of the bearing is $1.5K$.

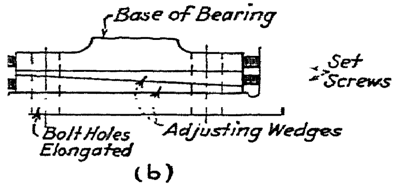
279. Two-piece Bearings.—Two-piece bearings are used universally on all classes of machines. They are of cast iron made in two parts, the parting line being located in the line of least pressure. Shims are sometimes provided to allow for easy adjustment, the bearing being closed in by reducing the thickness of the shims. Shims made up of thin sheets of brass or copper, sweated together, form a means of ready and accurate adjustment, the thickness being reduced by peeling off a layer of the laminated shim.

Figure 11(a) shows a common form of the two-piece bearing. Elongated bolt holes allow for alignment of the bearing and a

reservoir provides oil for lubrication. Vertical adjustment of this type of bearing is sometimes provided by the arrangement shown in Fig. 11(*b*). Another type of two-piece bearing is the connecting rod end shown by Fig. 12(*a*)



Two piece bearing.



Vertical adjustment for bearing.

FIG. 11.

This form of bearing may be lubricated in various ways depending upon the class of service in which it is employed, the speed of the journal, and its accessibility for frequent inspection. A wick-oiling scheme is shown in Fig. 12(*b*). The wick draws the

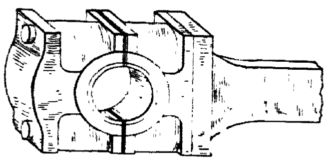


FIG. 12(*a*).—Marine type connecting rod end.

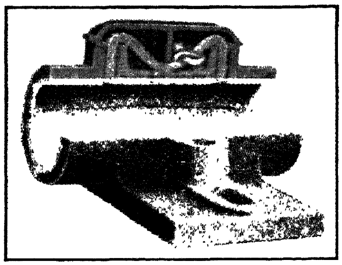


FIG. 12(*b*).—Wick oiling bearing.

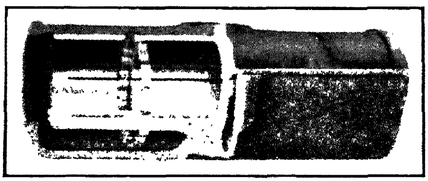


FIG. 12(*c*).—Chain oiling bearing.

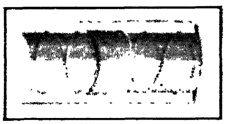


FIG. 12(*d*).—Bearing-metal liner showing oil grooves.

oil from the reservoir at the top and conveys it to the journal by capillary action. Chain- and ring-oiling systems are commonly used for the bearings of line shafting, electric motors and generators, small steam engines, and other machines. A chain-oiling bearing is shown in Fig. 12(*c*), oil being carried by an endless

chain from the reservoir at the bottom to the journal. This oiling system is not adaptable to small journals running at high speed, because of the tendency of the chain or ring to slip, and thereby fail to provide an adequate oil supply for proper lubrication. A bearing-metal liner is shown in Fig. 12(*d*). Grooves cut in the bearing surface lead the oil from its point of entrance at the top, and distribute it in the direction of the rotation of the shaft. If the shaft rotates in either direction the oil grooves lead both ways. The grooves of the bearing liner shown in Fig. 12(*d*) receive the oil from the counter bore at the middle, which in turn gets its supply from an oil cup at the top of the

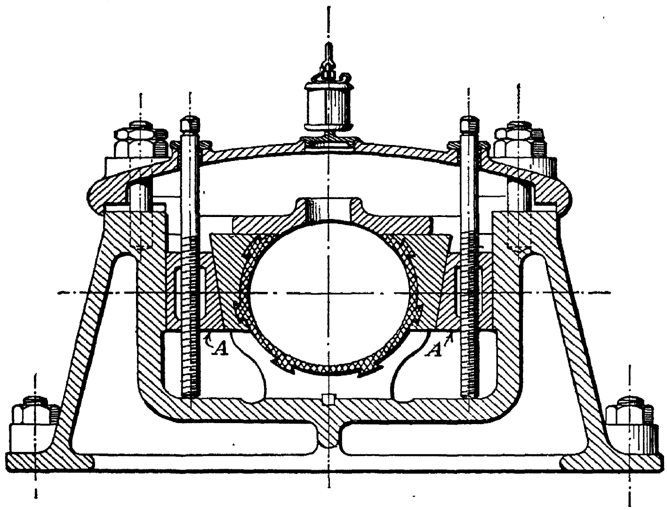


FIG. 13.—Main bearing, showing how adjustment for wear is made.

bearing. The lower half of a bearing is not grooved, because of the tendency of the grooves to prevent the formation of the oil film between the journal and its supporting bearing.

280. Four-piece Bearings.—Four-piece bearings are used for the bearings of large steam and gas engines. They consist of a bottom bearing, two side bearings called knee bearings, and the top bearing or cap. Side wear is compensated for by the adjustment of the wedges *A*, in Fig. 13, and wear on the bottom brass is adjusted in a similar manner by a horizontal wedge. This type of bearing is sometimes cooled by the circulation of water around the bearing in a cored cavity provided for that purpose. Lubrication is provided by multiple-feed drop oilers or non-pressure oil-circulating system, the oil being pumped to the

highest point and flowing by gravity to the bearings, passing through the bearings and into a reservoir, where it is filtered and recirculated.

The bearings of high-speed turbines and line shafting should be designed to allow for misalignment as the shaft passes through the critical speed, because the vibration due to the whirling of the unbalanced parts would cause high localized stresses, which might cause failure at the bearings and serious damage to adjacent parts. The principle of the self-aligning bearing is shown by Fig. 14. The point C is a point on the axis of the center of rotation, but the bearing is free to move with spherical motion in any direction about its support. Line shafting is equipped with self-adjusting ring- or chain-oiling bearings because the bearings are relatively far apart, and the shafting is readily misaligned because of unbalanced conditions and variable belt pulls.

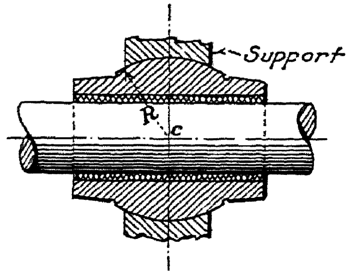


Fig. 14.—Principle of self-aligning bearing.

281. Thrust Bearings.—Thrust bearings are designed to carry pressures in the direction of the axis of the shaft, maintaining the shaft in its correct position while supporting the load.

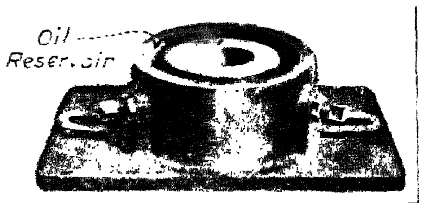


Fig. 15.—Foot-step bearing.

Thrust bearings carry the load in vertical steam turbines, water wheels, motors, and pumps; and provide for horizontal-load reactions in machine tools and marine drive shafts.

Thrust bearings which support a vertical load are called foot-step bearings, and a simple type is shown in Fig. 15. The bearing is concentric with an oil reservoir which provides the bearing surfaces with an oil bath, and the action of the turning shaft

carries the oil up the sides of the bearing, where it overflows to the reservoir.

In all types of thrust bearings the surface rubbing velocity is greatest at the outside of the bearing, and since the wear due to friction increases with the rubbing velocity, the best results are obtained when the difference in surface rubbing velocities is a minimum. When the load is relatively large, a collar thrust bearing is used because the bearing pressure may be distributed over a number of collars. Figure 16 shows a multiple collar thrust bearing. The difference between the outside diameter D and the inside diameter d of the collars should be as small as practicable without resulting in too many collars.

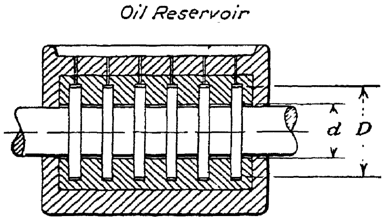


Fig. 16.—Multiple collar thrust bearing.

The allowable pressures permitted on thrust bearings are given in Table VI.

Table VI.—Allowable Pressures on Thrust Bearings

Mean velocity of rubbing surface in feet per minute $= \dfrac{\pi\left(\frac{D}{2}+\frac{d}{2}\right) \times \text{r.p.m.}}{12}$	Allowable pressure in pounds per square inch
Slow and intermittent	1,500
50	200
50 to 100	100
100 to 150	75
150 to 200	60
200 and above	50

When the bearings are of the collar type and water cooled, as in the bearings of ship-propeller shafts, the pressure may be increased to 800 lb. per square inch, but ordinary collar bearings are limited to pressures from 60 to 100 lb. per square inch.

282. Ball Bearings.—Ball bearings were first introduced into this country from Germany by the bicycle manufacturers, and at the present time they are used for all forms of bearings and for all excepting the more severe service conditions.

The contact between the balls and the stationary and revolving members is point contact. The balls are placed in rows, and are held in position by suitable retainers between an inner and outer raceway. The balls are made of high grade steel, machined accurately, hardened, ground, and sized by selection to 0.0001 in. A ball which is oversize by more than that amount will carry more than its portion of the load and may be overloaded, and this in turn will overload the ball races.

Annular and thrust bearings are manufactured in all sizes and for practically any load, speed, or loading conditions. Ball bearings are especially adaptable for use where the starting friction is great, and for unfavorable lubricating conditions.

283. Capacity of Ball Bearings.—Modern ball-bearing design is based upon the investigations of Stribeck,[1] who found that the load capacity of a single ball is:

$$P = Kd^2, \qquad (7)^2$$

in which P denotes the load on one ball, in pounds.
d denotes the diameter of the ball, in inches.
K denotes a constant which is 1,500 for hardened steel balls and races with two-point contact, and 750 for hardened steel balls and races with three- and four-point contact.

For radial bearings the load is distributed unevenly, the load being greater on one side of the bearing than on the other. Stribeck found that the greatest load carried by a single row of balls was one-fifth of the load on one ball multiplied by the number of balls:

$$P = Kd^2N \qquad (8)$$

in which P denotes the total load carried by the bearing, in pounds.
N denotes the number of balls.

There are two general types of ball bearings: (*a*) radial bearings, designed for vertical loads; and (*b*) thrust bearings, designed for horizontal loads. A combination bearing which may be loaded radially and endwise is made for light loads. Figure

[1] *Proc. A.S.M.E.*, Vol. XXIX, 1907, p. 420.

[2] Formulas (7), (8), and (9) are not used in actual design, but reliance is placed upon tables of load capacities obtained from manufacturers of ball-and-roller bearings.

17(*a*) shows a radial bearing with a single row of balls; Fig. 17(*b*) shows a radial bearing with a double row of balls; and Fig. 17(*c*) shows a radial bearing which is self-aligning. The thrust type of bearing is shown in Fig. 17(*d*), a one-direction bearing; Fig. 17(*e*), a one-direction spherical seat bearing; and Fig. 17(*f*), a two-direction bearing.

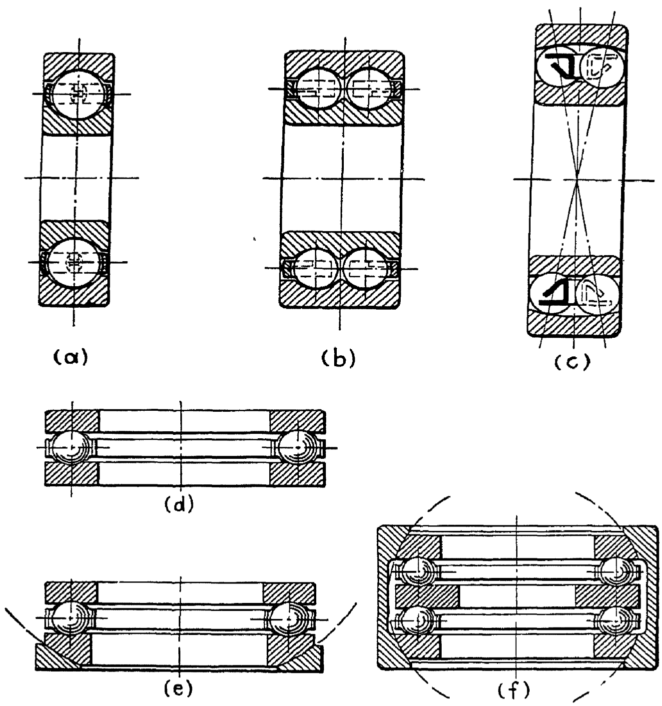


FIG. 17.—Ball bearings, radial and thrust type.

The design of ball bearings requires engineering experience which is highly specialized, and it is best to consult freely with the bearing manufacturers before the final selection is made.

284. Roller Bearings.—Roller bearings are used where the load conditions are too severe for the use of ball bearings. They are sections of true rolling cylinders or cones, and may be subjected to end-thrust reactions which are not present in ball bearings. Experiments have shown that there are certain principles which apply to all roller bearings, and which are independent of their form.

285. Capacity of Roller Bearings.—The load capacity of roller bearings is based upon the formula:[1]

$$P = Kld\frac{N}{5}. \tag{9}$$

in which P denotes the load, in pounds.

l denotes the length of the cylindrical roller, in inches.

d denotes the diameter of the roller, in inches.

N denotes the number of rollers.

K denotes a constant, determined by experiment as 1,000 for hardened surfaces.

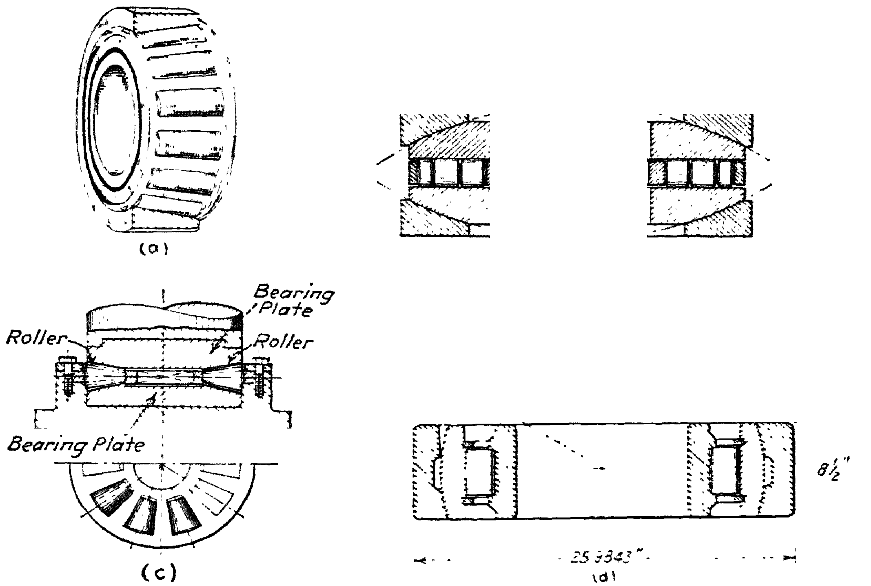


FIG. 18.—Roller bearings.

For rollers of conical form, d may be taken as the diameter at the mid-section of the roller.

The roller bearing shown in Fig. 18(a) is commonly used for automobile wheels and power-transmission machinery. Figure 18(b) shows a self-aligning roller thrust bearing designed for heavy loads. Figure 18(c) shows a step bearing with conical rollers. If the end thrust is excessive, balls are employed at the large end of the rolls to reduce the friction.

[1] Experiments of Prof. Stribeck, *Proc. A.S.M.E.*, Vol. XXVII, p. 444, 1906.

Ball and roller bearings are made in large sizes to carry very heavy loads. The roller bearing shown in Fig. 18(*d*) is a product of the Rollway Bearing Company, Incorporated, of Syracuse, New York, and is designed to carry loads as follows:

Revolutions per minute	10	25	50	150	250	500
Rated load capacity in pounds.........	1,040,000	766,000	609,000	422,000	356,000	283,000

Ball and roller bearings are lubricated with a heavy oil or grease because the lighter oils leave the bearings too quickly. Bearings of this type which are well protected from dust and dirt have been known to give six months' or a year's satisfactory service without attention. However, it is a safe rule to inspect, clean, and oil all moving parts of any machine at regular intervals. The function of the lubricant is to keep the polished surfaces of the balls and races clean, bright, and free from corrosion; to reduce the slight friction present; and to eliminate noise.

286. Lubricants.—A lubricant is any substance which reduces friction by preventing a moving machine member from coming into direct contact with its supporting part.

Solid lubricants are made of flaked graphite or powdered soapstone, which do not liquefy with heat. They are used for heavy bearing pressures and slow speeds, and they are usually mixed with oil or fat.

Liquid lubricants are animal, vegetable, or mineral oils, and the best lubricant is one which has the greatest adhesion to metallic surfaces and the least internal cohesion.

Lard oil has more body than any other oil, and its principal use is to lubricate metal-cutting tools. Its function is to carry off the heat developed in separating the chip from the work, to lubricate the chip as it slides off the cutting edge, to give a finish to the turned surface, and to wash away the small particles of metal produced by the cutting process.

Neats-foot oil is made from the feet of cattle, and it is adaptable to the lubrication of moving parts exposed to outdoor conditions.

Sperm oil is made from the fat of the sperm whale. It has little body, is used for light surface pressures, and gums rapidly when exposed to the air.

Porpoise oil is used for the lubrication of small surfaces and very light loads, such as the bearings of watches and clocks. It will not congeal at zero degrees Fahrenheit.

The principal vegetable oils are *olive oil, palm oil,* and *castor oil,* the latter being suitable for a wide range of temperature change.

Mineral oils are made from selected crude petroleum, which is refined and treated to give uniform characteristics. A high grade of mineral oil has a low viscosity, and will maintain a good lubricating film between sliding surfaces.

In general the desirable qualities for any lubricating oil are:

(*a*) It should have sufficient body to keep the metal surfaces separated.

(*b*) It should have a low coefficient of friction.

(*c*) It should vaporize only at a high temperature, and solidify only at a low temperature.

(*d*) It should be free from grit, acid, or any other chemical which might be injurious to the metals in contact.

(*e*) It should have the property of not gumming and should withstand decomposition.

There is a lubricating oil for each class of service, and the proper selection of a lubricant involves training and experience.

287. Lubricating Greases.—A grease is a mixture of lubricating oil and a fat, and it is compounded in various grades, such as light-bodied grease, which lubricates at a relatively low temperature, and dense-bodied grease, which will not lubricate until the bearing temperature becomes sufficiently high to soften the grease. Grease is particularly adapted for the lubrication of machines which operate in dusty surroundings, such as are encountered in cement mills, railways, coal hauling, and conveying equipment. The grease closes off the access of dirt and dust to the surfaces, by forming a grease collar around the bearing.

Soft greases are recommended for high-speed ball and roller bearings, because the grease will form fillets which close off the bearings and prevent the entrance of dust, and also because the grease will not be thrown out of the bearings as readily as will the lighter oils.

Grease should always be used to lubricate bearings which are located in inaccessible places. Such bearings should be provided with grease caps or cups and the grease forced to the bearing by

pressure. The screw grease gun with a flexible metal hose, as shown in Fig. 19, is a lubricating device which forces the lubricant under pressure to the bearing. The bearing is provided with a self-closing cap, to which the hose nozzle may be attached.

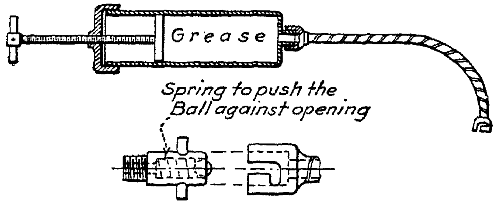


FIG. 19.—Simple grease gun and connections.

288. Lubricating Systems.—The oldest and simplest system of lubrication applies the lubricant by dropping or squirting oil through holes which lead to the bearing. Oil cups, which hold a supply of oil and apply it to the bearing drop by drop, are used on all forms and classes of bearings. Oil cups are often provided with a window for the observance and inspection of the oil flow, the rate of flow being regulated by means of a valve. Figure 20(*a*) shows a simple sight-feed lubricator of this type. A single oil reservoir may supply more than one oil lead, and such a device is called a multiple-feed lubricator.

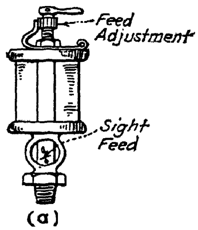
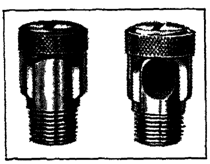
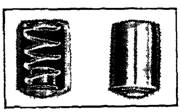


FIG. 20(*a*).—Simple sight-feed lubricator.

Open oil holes should be provided with one of the many types of caps (oilers) which are available. Figure 20(*b*) shows an oiler which will close off the entrance to the oil hole by turning



(*b*)



(*c*)

FIGS. 20(*b*) and (*c*).—Dust-proof oilers.

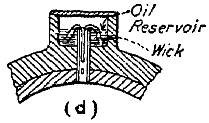


FIG. 20(*d*).—Drop-feed oiler.

the cap, thereby preventing dust or grit from getting into the oil lead. A self-closing oiler is shown in Fig. 20(*c*). Figure 20(*d*) shows a drop-feeding system which provides oil, drop by drop, by the action of a wick partly submerged in an oil bath. Figure 21

shows a scheme employed for oiling the crankpin bearing, the main bearing, and the eccentric of a steam engine having an overhung crank.

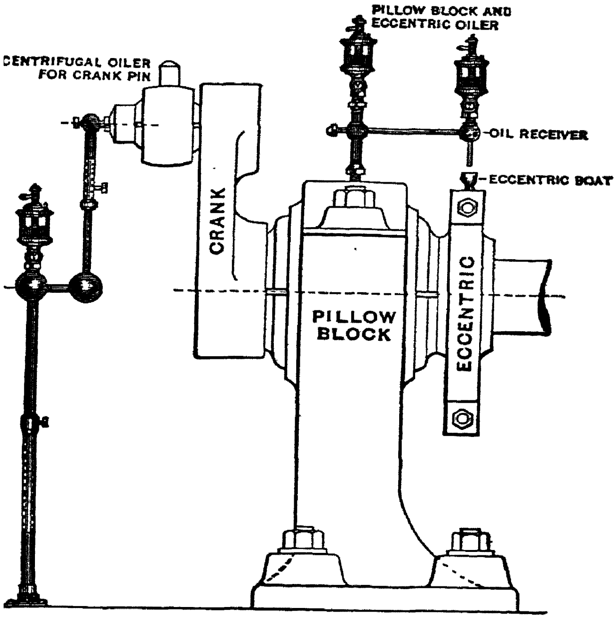


FIG. 21.—Oiling system for main bearing and eccentric of a steam engine.

289. Bath and Splash Systems.—When rubbing surfaces are located so that they may be surrounded by a closed reservior, the

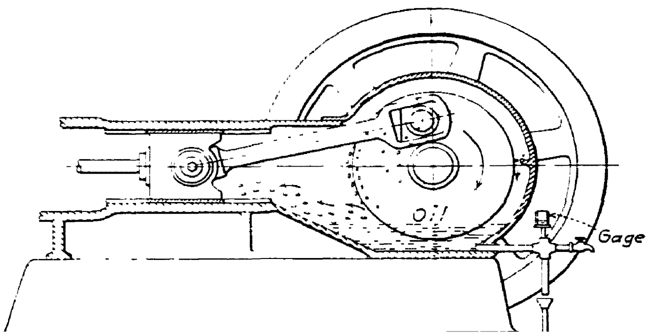


FIG. 22.—Splash system of lubrication.

bath system of lubrication may be used. Dippers are attached to rotating machine parts, which dip up a quantity of oil and

spill it over the parts to be lubricated. Small steam engines and gas engines are provided with chambers enclosing the crank, so that the crank disk will splash into the oil which fills the chamber to a fixed level, as indicated in Fig. 22. Pockets are cast inside the crank casing to gather oil for distribution to various channels leading to parts which require lubrication.

290. Circulation Oiling Systems.—There are two oiling systems which embody the principal of circulation:

(*a*) Non-pressure or gravity systems.

(*b*) Pressure or forced feed systems.

The gravity system of lubrication is used to lubricate engine bearings, crossheads, and large shaft bearings. A supply tank

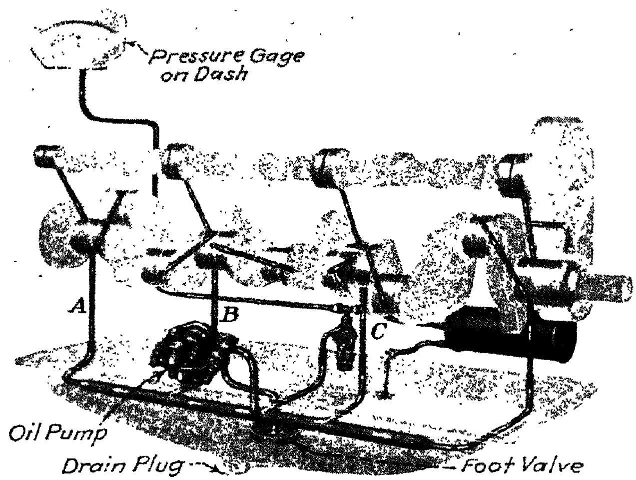


FIG. 23.—Pressure system of lubrication applied to an automobile engine.

filled with oil is located at a high point, so that pipes or leads will deliver oil to the points of application. After lubricating the bearings, the oil drains through return pipes to a sump tank, from which it flows to a circulating pump, which in turn pumps the oil through a closed cooler and filter to the supply tank. For small installations the cooler and filter are often omitted.

The pressure system delivers the oil to the bearings under pressure. This system is commonly employed for lubricating the bearings of steam turbines, Diesel engines, enclosed type of steam engines, and automobile engines. Figure 23 shows the application of the pressure system to the main bearings, crankpin

bearings, and the camshaft bearings of a modern automobile engine. The reservoir forms the bottom of the crankcase. The oil pump is designed to circulate a generous supply of oil at a low engine speed. The oil is distributed through the leads *A*, *B*, *C*, and *D* to the crankshaft bearings, which are connected to the crankpins by leads through drilled holes in the crankshaft, to supply the oil under pressure to the crankpin bearings. Leads from the main bearings feed oil to the camshaft bearings and timing-chain drive. After lubricating the rubbing surfaces the oil is returned to the reservoir in the crankcase.

291. Temperature of Bearings.—Under normal working conditions there is a rise in temperature of the bearing, and moderate temperatures may be considered as those under 140° F. Extreme temperatures are those above 140° F., and they may be caused by deficient radiation, friction, or the effects of high surrounding temperatures. When high temperatures occur, attention must be given to the bearings at once, because mechanical conditions may be wrong, improper oil may be the cause, or the oiling system may not be working as it should. When bearing temperatures exceed 170° F. it is an indication that the condition is serious. Induced heat may cause high temperatures, but this is overcome by using a positive lubricating system. If extreme temperatures are due to climatic conditions, heavy oils may overcome the trouble.

Frictional temperature should remain constant for constant room temperature. If the surrounding temperature is 70° F. and the bearing temperature is 90° F., then the frictional temperature is 20° F.

The lubrication of bearings in immediate proximity to steam-heated parts, such as are present in turbines, requires a lubrication system with a continuous flow of oil, to carry off the heat from the bearings, so that they will operate at safe temperatures. Steam-turbine oiling systems are provided with a circulating pump, oil filters, and oil coolers. Large bearings are sometimes water cooled by the mechanical circulation of water, to carry away the excess heat of friction.

Problems

1. A line shaft $2\frac{1}{2}$ in. in diameter rotates at 150 r.p.m. and the total reaction of the belt pulls and weight of pulleys and shafting is 8,600 lb. Assuming that these reactions are all acting in a vertical line, determine the friction horsepower.

2. A crane trolley weighing 300 lb. has four cast-iron wheels, each 10 in. in diameter, which travel on steel beams. The maximum capacity of the hoist is 5 tons. When the load is suspended, what force is required to move the trolley along the beams, assuming that the beams are level.
3. A horizontal pull of 30 lb. is required to cause a body to slide on a level horizontal surface, the coefficient of friction being 0.30. (*a*) What is the weight of the body? (*b*) What pull will be required to cause motion if the pull is inclined 30 deg. upward?
4. A drill press has an overall efficiency of 92 per cent. What horsepower is available at the work if the machine receives 1½ hp.?
5. A gear train consists of three spur gears with cut teeth. What is the combined efficiency of the transmission if each bearing has an efficiency of 97 per cent?
6. The crosshead of a high-speed stationary engine has 62 sq. in. of bearing surface on each of the top and bottom sides. The length of the connecting rod is 42 in., and the length of the crank is 8 in. What force is acting on the crank pin when the pressure between the crosshead and guides is a maximum?
7. The weight of a locomotive is equally distributed over eight journals, except the 14 per cent which is carried by the truck wheels. The journals are 8 in. in diameter and 9 in. long. What is the weight of the locomotive if the pressure between the driving-box brasses and the journals is limited to 175 lb. per square inch?
8. The maximum load which is taken by the bearings of a punch press as the punch enters a steel plate is 156,000 lb. Assuming that half of this load is taken by the bearing on either side of the eccentric, what should be the minimum diameter and length of the bearings?
9. The crosshead pin of a steam engine must be 2½ in. in diameter to withstand the shearing strain. If the maximum pressure is 10,000 lb., determine the length of the pin.
10. Sketch a one-piece bearing for a shaft 2½ in. in diameter, similar to the one shown in Fig. 10, and give all the dimensions.
11. A multiple collar bearing, like the one shown in Fig. 16, has five collars which have an outside diameter of 4 in., and an inside diameter of 3¼ in.

    (*a*) For a mean velocity of rubbing surfaces of 180 ft. per minute, what thrust load should the collar carry?

    (*b*) For slow and intermittent service what load should the collar carry?

    (*c*) Why should there be so great a difference in these loads?
12. Outline the steps which you would take in choosing a ball bearing for a loose pulley on a line shaft.
13. How would you decide when to specify roller bearings in preference to ball bearings?
14. How would you choose a lubricant? Cite a specific problem and give your reasons for specifying a certain kind of lubricant.

CHAPTER XIV

FRICTION AS A USEFUL AGENT

292. The force of friction is utilized in design engineering in a number of ways. Fluid-tight joints and structural joints are dependent upon frictional resistance for holding power to a more or less extent, although American practice makes no attempt to determine its magnitude. The amount of friction is indeterminate, but its presence adds to the strength of the joints and is on the side of safety.

Friction as a useful agent is prominent in the design of such power-transmission equipment as flat belting and clutches, and for the absorption of energy by brakes. When the velocity ratio between a driving and driven machine member need not be constant, and when it is advantageous to throw the driven member in and out of action without stopping the driver, friction devices are employed to utilize the law that "the frictional force is directly proportional to the normal pressure between two surfaces."

293. Friction Gears.—Friction gears are machine members which transmit motion or power by the force of friction developed between the surfaces in contact. A simple application of the force of friction as a power medium, is the arrangement of two rolling cylinders or friction wheels in contact, as shown in Fig. 1.

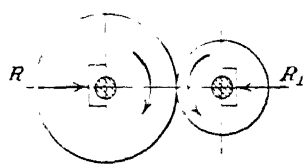


FIG. 1.—Friction cylinders.

The amount of power which may be transmitted by this means is limited by the bearing reactions. The surface of the driving member of a pair of friction wheels should be of more yielding material than the surface of he driven member, so as to avoid the tendency to wear soft spots on the driven wheel. The driver is usually covered with a material which will wear faster than the driven surface. The various materials used for surface coverings, and the allowable pressures which are used, are given in Table I.

TABLE I.—ALLOWABLE PRESSURES FOR FRICTION DRIVING MATERIALS

Material	Straw fiber	Leather fiber	Leather	Wood	Paper
Working pressure per inch of contact....................	150	240	150	150	150

The power which may be transmitted is dependent upon the coefficient of friction of the materials in contact, and when the velocity of the driver is great, materials with a low coefficient of friction should be used. Average values of coefficients of friction for the usual materials used for friction drives, are given in Table II.

TABLE II.—AVERAGE VALUES OF COEFFICIENTS OF FRICTION

Material	Coefficient of friction	Material	Coefficient of friction
Straw fiber on cast iron....	0.26	Paper on cast iron........	0.20
Straw fiber on aluminum...	0.27	Leather on cast iron......	0.14
Leather fiber on cast iron...	0.31	Leather on aluminum....	0.22
Leather fiber on aluminum.	0.30	Wood on metal..........	0.25

The friction wheels shown in Fig. 2 are a common form of drive on machine tools for feeds used in cutting, and may be used when the load is light and when a variable or reversible velocity ratio is desired. When there is no slippage, the velocity is inversely proportional to the radii:

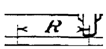


FIG. 2.—Brush-plate and wheel.

$$\frac{\omega_A}{\omega_B} \qquad R \tag{1}$$

in which ω_A denotes the angular velocity of A.
ω_B denotes the angular velocity of B.
R denotes the radius of B.
r denotes the radius of A.

A friction gear, in addition to driving the feed mechanism on machine tools or in other applications, functions as a safety device. If sticking or jamming occurs, due to hard spots or

lack of rigidity, the driver will slip, thus doing little if any damage to the driving or driven members.

294. Friction Cones.—Two cone sections, arranged as shown in Fig. 3, will transmit a variable velocity between the cones A and B, by shifting the short endless belt C to the desired position, and holding it in place by the belt shifter D. This form of driving device is in common use for transmitting power to automatic grinders.

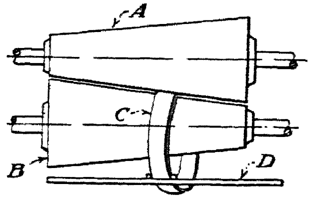


Fig. 3.—Friction cones.

295. Wedge-surface Gears.—The purpose of wedge-surface friction gears is to increase the power which may be transmitted without using excessively wide wheel surfaces. Figure 4(a) shows the arrangement of a pair of grooved wheels which offer wedge-shaped surfaces for contact with each other. The angle of the groove should not be less than 20 deg. to avoid wedging, and not more than 40 deg. to avoid losing the effect of the grooves. The number of wedges has no other effect than to distribute the wear, due to slippage, over a greater surface. The power which may be transmitted depends upon the normal pressure and the coefficient of friction. In Fig. 4(b) the radial force P is held in equilibrium by the normal pressures and frictions as indicated. Summing up along a vertical axis:

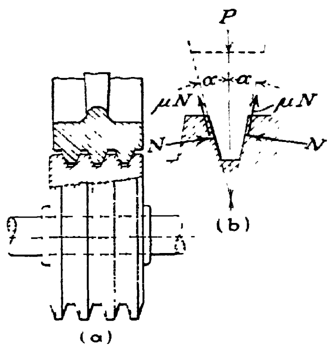


Fig. 4.—Grooved friction wheels.

$$P = 2N \sin \alpha + 2\mu N \cos \alpha.$$

$$N = \frac{P}{2 \sin \alpha + 2\mu \cos \alpha}. \tag{2}$$

Since the tangential friction force F is equal to $2\mu N$:

$$F = \frac{\mu P}{\sin \alpha + \mu \cos \alpha}. \tag{3}$$

The horsepower which may be transmitted is:

$$\text{hp.} = \frac{FV}{33{,}000}, \tag{4}$$

in which F denotes the tangential friction, in pounds.
V denotes the linear velocity of the mean radius of the wedge surfaces in contact, in feet per minute.

The rubbing velocity is greatest at the outside ends of the grooves, and there can be only one point at which the driver and the driven have the same linear velocity; so that in order to obtain the best results the depth of the grooves should not be greater than ½ in.

Block brakes

296. Friction Brakes.—Brakes are used for regulating speed and for stopping machines, by means of a frictional force applied to some moving part. The simplest form of friction brake is a block pressing against a wheel as shown in Fig. 5. The frictional force is μN, and from a moment equation:

$$Na = W(a + b),$$
$$F = \frac{\mu W(a + b)}{}, \tag{5}$$

in which F denotes the frictional force, in pounds.
μ denotes the coefficient of friction.
W denotes the force applied to the brake, in pounds.
a and b denote the lever arms, in inches.

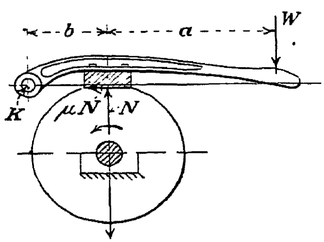


FIG. 5.—Simple block brake.

In Fig. 5 the fulcrum K may be placed a distance c above or below the action line of the friction force F. For the rotation shown, if K is a distance c above the action line of F:

$$\Sigma M = \mu N \times c - N \times b + W(a + b) = 0.$$

$$N = \frac{W(a + b)}{b - \mu c} \tag{6}$$

This indicates that under such conditions the brake is self-acting when μc equals b, and the normal pressure continually increases. Such proportions should of course be avoided.

Two blocks placed on opposite sides of a wheel may be controlled by levers and rods so that they are pulled together against the wheel to form a brake. This arrangement avoids the heavy bearing reactions which would occur in Fig. 5 when the load W is large.

The brake shown in Fig. 6 is a common type of brake used for heavy braking loads. The toggle joints are used with this

brake to form a locking device, which is important for the handling of braking loads in mine work.

The normal pressure on the brake wheel of Fig. 6 is due to the tension in the brake-shoe rods. If T is the tension in each rod, then the normal pressure N equals $2T$, and the total friction on both brake shoes is:

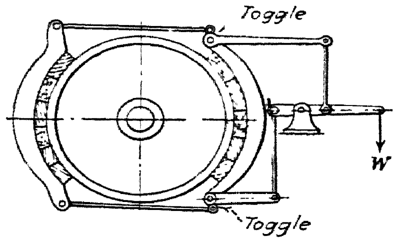


FIG. 6.—Band-block brake for heavy loads.

$$F = 4\mu T. \quad (7)$$

297. Band Brakes.—The ordinary band brake consists of a cast-iron or steel drum with a steel band wrapped around it, which may be tightened against the drum by applying force at the end of a lever. The brake band is usually lined with a woven fibrous material which should have the following qualities:

(a) It should be more yielding than the material of the drum.
(b) It should resist the heat of friction.
(c) It should resist changes of humidity.
(d) It should render noiseless service.

Commercial brake linings have these qualities to a greater or less extent.

The braking effort in a band brake is dependent upon the location of the band connectors with respect to the fulcrum.

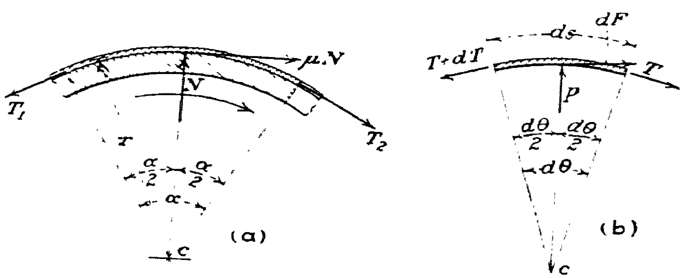


FIG. 7.

The student will understand what forces tend to retard the motion of a brake drum, if he will place a taut rubber band around the index finger of each hand, and observe what takes place when one finger is rotated through an angle.

A drum turning clockwise, as shown in Fig. 7(a), is retarded by the friction μN. In Fig. 7(b) ds is a short length of brake band subtended by the angle $d\theta$, p is the pressure per unit length of

the band, dF is the friction, and the band tensions are T and $T + dT$. Summing up the vertical forces:

$$\Sigma Fy = pds - T \sin \frac{d\theta}{2} - (T + dT) \sin \frac{d\theta}{2} = 0.$$

$$pds = 2T \sin \frac{d\theta}{2} + dT \sin \frac{d\theta}{2}. \tag{8}$$

Since the angle $d\theta/2$ is very small, the sin $d\theta/2$ may be replaced by $d\theta/2$, and the last term in the formula may be neglected. Hence:

$$pds = Td\theta,$$

$$p = \frac{T}{r}. \tag{9}$$

Taking moments about the center c:

$$\Sigma M = (T + dT) \times r - T \times r - dF \times r = 0.$$

$$dT = dF = \mu pds.$$

Substituting the value of p from formula (9):

$$dT = \mu \frac{T}{r} ds = \mu T d\theta.$$

$$\frac{dT}{T} = \mu d\theta. \tag{10}$$

Integrating between the limits T_2 and T_1, and between 0 and α:

$$\log_e T_1 - \log_e T_2 = \mu\alpha,$$

$$\frac{T_1}{T_2} = e^{\mu\alpha}, \tag{11}$$

in which T_1 denotes the tension on the tight side, in pounds.

T_2 denotes the tension on the loose side, in pounds.

e denotes the base of the natural logarithms, which is 2.718.

μ denotes the coefficient of friction between the band and the drum.

α denotes the angle on the drum subtended by the brake band, in radians.

For the simple band brake shown in Fig. 8, the band is attached so that the greater pull will be on the fixed pin. If the friction between the band and drum is called F, then, taking moments about the axis of the drum:

$$(T_1 - T_2)r = Fr,$$

$$T_1 - T_2 = F. \tag{12}$$

Taking moments of the forces on the lever:

$$P \times a = T_2 \times b \tag{13}$$

Substituting the values of T_1 and T_2 from formulas (12) and (13) in formula (11):

$$\frac{F + \frac{Pa}{b}}{\frac{Pa}{b}} = e^{\mu\alpha}, \quad \frac{Fb}{Pa} = e^{\mu\alpha} - 1, \quad \frac{Pa}{Fb} = \frac{1}{e^{\mu\alpha} - 1},$$

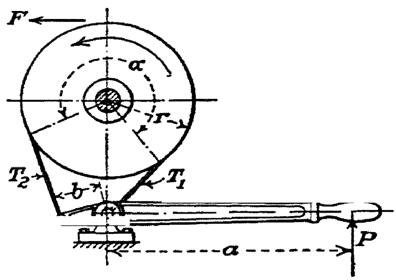


FIG 8.

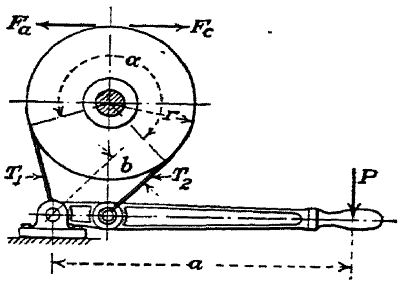


FIG. 9.

and

$$P = \frac{Fb}{a}\left(\frac{1}{e^{\mu\alpha} - 1}\right). \tag{14}$$

When the rotation in Fig. 8 is clockwise it can be shown that:

$$P = \frac{Fb}{a}\left(\frac{e^{\mu\alpha}}{e^{\mu\alpha} - 1}\right). \tag{15}$$

Example.—The band brake shown in Fig. 8 is acting against a frictional torque of 1,600 in.-lb. The diameter of the drum is 16 in., $a = 20$ in., $b = 4$ in., $\alpha = 250$ deg., and $\mu = 0.28$. What force will be required at the brake lever for counterclockwise rotation of the drum?

$$F = \frac{1{,}600}{8} = 200 \text{ lb.}, \quad \alpha = \frac{250}{180}\pi = 4.36 \text{ radians.}$$

Substituting in formula (14):

$$P = \frac{200 \times 4}{20}\left(\frac{1}{2.718^{0.28 \times 4.36} - 1}\right).$$

$$P = \frac{200 \times 4}{20}\left(\frac{1}{3.39 - 1}\right) = 16.7 \text{ lb.}$$

For clockwise rotation of the drum P equals 56.7 lb.

For the band-brake arrangement as shown in Fig. 9, for clockwise rotation:

$$= \frac{Fb}{a}\left(\frac{1}{e^{\mu\alpha} - 1}\right). \tag{16}$$

For counterclockwise rotation:

$$P = \frac{Fb}{a}\left(\frac{e^{\mu\alpha}}{e^{\mu\alpha} - 1}\right). \tag{17}$$

The band-brake arrangement shown in Fig. 10 is called a differential band brake, and for clockwise rotation:

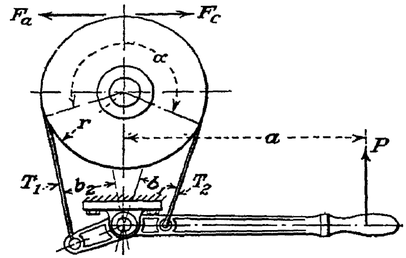


Fig. 10.

$$P = \frac{F}{a}\left(\frac{b_2 e^{\mu\alpha} - b_1}{e^{\mu\alpha} - 1}\right). \tag{18}$$

For counterclockwise rotation:

$$P = \frac{F}{a}\left(\frac{b_1 e^{\mu\alpha} - b_2}{e^{\mu\alpha} - 1}\right). \tag{19}$$

The values of $e\mu\alpha$ for various arcs of contact and for various coefficients of friction used in practice are given in Table III

Table III.—Value of $e^{\mu\alpha}$ for Various Contact Arcs and Friction. Coefficients

Arc of contact, α, degrees	Coefficient of friction (μ)									
	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
80	1.15	1.23	1.32	1.42	1.52	1.63	1.74	1.79	2.01	2.15
90	1.17	1.27	1.36	1.48	1.60	1.73	1.87	2.03	2.19	2.37
100	1.19	1.30	1.41	1.55	1.68	1.84	2.01	2.19	2.39	2.62
110	1.21	1.33	1.47	1.62	1.78	1.96	2.16	2.38	2.61	2.88
120	1.23	1.37	1.52	1.69	1.87	2.08	2.31	2.56	2.85	3.16
130	1.25	1.41	1.57	1.76	1.98	2.21	2.48	2.78	3.11	3.48
140	1.27	1.44	1.63	1.84	2.08	2.35	2.65	3.00	3.39	3.84
150	1.30	1.48	1.69	1.93	2.19	2.50	2.85	3.25	3.70	4.23
160	1.32	1.52	1.75	2.01	2.31	2.65	3.05	3.50	4.04	4.63
170	1.34	1.56	1.81	2.10	2.43	2.82	3.27	3.87	4.39	5.12
180	1.36	1.60	1.88	2.19	2.56	3.00	3.51	4.12	4.80	5.61
190	1.38	1.64	1.94	2.29	2.70	3.18	3.76	4.44	5.22	6.15
200	1.42	1.69	2.01	2.39	2.85	3.39	4.04	4.79	5.71	6.78
210	1.44	1.73	2.08	2.50	3.00	3.60	4.33	5.18	6.23	7.48
220	1.47	1.77	2.15	2.61	3.16	3.83	4.64	5.60	6.80	8.24
230	1.49	1.83	2.23	2.72	3.33	4.07	4.99	6.08	7.41	9.08
240	1.52	1.87	2.31	2.85	3.51	4.33	5.34	6.59	8.11	10.00
250	1.55	1.92	2.39	2.98	3.70	4.60	5.74	7.14	8.85	11.00
260	1.58	1.98	2.48	3.11	3.90	4.89	6.14	7.69	9.66	12.06
270	1.60	2.03	2.56	3.24	4.10	5.19	6.55	8.30	10.50	13.25
280	1.63	2.08	2.65	3.38	4.32	5.51	7.03	8.95	11.49	14.56
290	1.66	2.13	2.75	3.54	4.55	5.86	7.55	9.68	12.52	16.12
300	1.69	2.19	2.85	3.70	4.81	6.25	8.13	10.52	13.74	17.79

Table IV gives average values of the coefficient of friction which may be expected for various materials, dry and unlubricated, used for brakes.

TABLE IV.—COEFFICIENTS OF FRICTION OF BRAKES

Material	Coefficient of friction μ
Iron on iron	0.25 to 0.30
Steel on iron	0.20
Leather on iron	0.30
Oak on iron	0.30
Poplar on iron	0.35

298. Prony Brake.—The Prony brake is a common type of absorption brake which is used for measuring the power of engines, motors, and shafting. A band brake lined with wood blocks is wrapped around a cast-iron pulley, and the frictional force

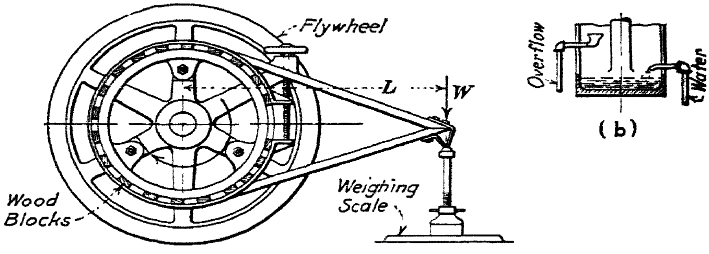


FIG. 11.—Typical Prony brake.

which prevents the brake from rotating with the pulley is determined on a platform scale. In Fig. 11(*a*) the force W acting at the distance L is the torque which the brake applies to the pulley, a correction being made for the torque due to the weights of the levers. Hence:

$$Fr = WL.$$

The engineering unit for power is the horsepower:

$$\text{Horsepower} = \frac{2\pi FrN}{33{,}000} = \frac{2\pi NLW}{33{,}000}, \tag{20}$$

in which N denotes revolutions per minute.

L denotes the length of the brake arm from the center of the pulley to the weighing point on the scale, in feet.

W denotes the corrected reading on the scale, in pounds.

In formula (20) the only variables are N and W, and it is common practice to make the length L such that the formula is simplified and easily used. If, for example, L is made 5 ft. 3 in. long, the horsepower formula becomes:

$$\text{hp.} = \frac{NW}{1{,}000}.$$

The brake wheel may be water-cooled by having a stream of water flowing into the flanged rim of the pulley, as shown in Fig. 11(*b*). If the block surface is sufficient, the heat will be carried away by radiation. One square inch of block surface for each 200 ft.-lb. of energy absorbed per minute is satisfactory.

299. Friction Clutch.—A friction brake and a friction clutch are identical in principle, and differ only in their application. A brake is fixed to a stationary part of a machine, and by the action of frictional forces absorbs energy from a moving part until it gradually comes to rest. A clutch, on the other hand, gradually picks up motion from a moving part, and after slipping to some extent, the surfaces in contact finally cease to have motion relative to each other, and the clutch then functions as a power-transmission device.

The friction clutch has its principal application in automobile transmissions, where it connects the engine to the drive shaft. The force of friction is employed to bring the driven shaft up to the proper speed, allowing slippage during the process. A clutch should engage without excessive slip, and should bring the driven member up to speed gradually.

Among the various types of friction clutches, the following ones are common:

(*a*) Disk clutch.
(*b*) Cone clutch.
(*c*) Expanding clutch.
(*d*) Magnetic clutch.

300. Disk Clutch.—A disk clutch has one or more pairs of flat rings which engage as shown in Fig. 12, this clutch being known as the Weston disk clutch. The inner disks are keyed to the sleeve, which is constrained to rotate with the shaft, but is permitted to move axially in order to make and break contact with the outer disks. These are bolted to the casing which is keyed to its section of the shaft. The two sets of disks are held in contact by a spring, and operate in a bath of oil to prevent seizing.

If the pressure due to the force of the spring is uniformly distributed, and if the coefficient of friction is constant, the frictional force developed by the disks is:

$$F = \mu n P,$$

in which F denotes the frictional force, in pounds.
μ denotes the coefficient of friction.
n denotes the number of disks.
P denotes the pressure between the disks, in pounds

The torque is:

$$T = Fr = \mu n P r. \tag{21}$$

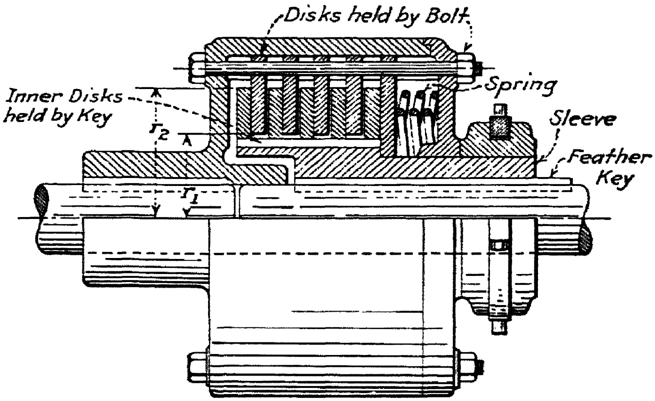


FIG. 12.—Weston disk clutch.

The mean radius r in formula (21) may be determined as follows:

$$r = \frac{2(r_2^3 - r_1^3)}{3(r_2^2 - r_1^2)}, \tag{22}$$

in which r_2 denotes the outside radius of the inner disk.
r_1 denotes the inside radius of the outer disk.

A more approximate method of determining the mean radius is to take one-half the sum of the inner and outer radii.

When a clutch has metal-to-metal contact the surfaces should be lubricated.

Dry-plate disk clutches have the plate surfaces faced with asbestos fiber, which is riveted to the steel disks. The dry-plate surfaces wear more rapidly than the lubricated metal surfaces, so that some provision is made for adjustment, and the linings may be replaced when badly worn. The popularity of the dry-

plate clutch for the automobile is due to its ability to function properly in the absence of lubrication.

The coefficient of friction between the disks is a variable quantity, depending largely upon the velocity and condition of the surfaces. Safe average values for pressures and coefficients of friction for friction clutches are given in Table V.

TABLE V.—MEAN PRESSURE AND COEFFICIENT OF FRICTION FOR CLUTCHES

Surface materials	Mean pressure, pounds per square inch	Coefficient of friction μ
Steel on asbestos fabric...	For single-plate clutch, 50 for ordinary service, 30 for severe service	Dry 0.20
Steel on asbestos fabric...	Same as above	Lubricated, 0.10
Cast iron on cast iron....	25	Lubricated, 0.10
Wood on cast iron.......	10 lb. at 3,600 ft. per minute 100 lb. at 200 ft. per minute	0.20
Cork inserts on cast iron..	25 to 50	Dry 0.35 Oily 0.30
Leather on cast iron......	25 to 50	Dry 0.30 Oily 0.20

For cone clutches it is recommended that the pressure be 8 lb. per square inch for leather on metal and 45 lb. per square inch for metal on metal.

There are various forms of disk and plate clutches manufactured to fit the common sizes of cold-rolled steel shafting. The clutch shown in Fig. 13 is known as the Lemley clutch. The driving and friction plates are forced into contact with the friction ring by the toggle levers, maintaining contact with a high pressure. The clutch sleeve may be replaced by a loose pulley which has the friction ring bolted to the spokes, but when so used the shaft would be continuous. The clutch arranged in this manner would transmit power to a second shaft by means of a belt or gear.

Reasonable care should be exercised when operating a clutch so that the friction surfaces may engage easily, and gradually bring the driven shaft up to speed. Proper alignment must be maintained and bearings should be located as close to the clutch as possible.

When speeds are high, manufacturers advise that clutches be chosen with 25 to 100 per cent greater capacity than the normal

size. When the starting load is very heavy, of a variable nature, or subjected to shock conditions, the clutch should have a capacity of 100 per cent greater than normal. Stock sizes and capacities are given in Table VI.

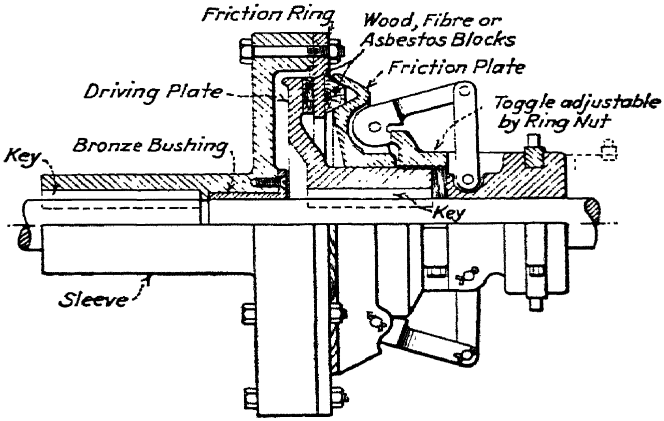


FIG. 13.—Lemley disk clutch.

TABLE VI.—STOCK SIZES FOR FRICTION CLUTCHES

Largest shaft size recommended, inches	Horsepower at 100 r.p.m.	Speed r.p.m.
$1\frac{1}{4}$	$1\frac{1}{4}$	600
$1\frac{1}{2}$	2	580
$1\frac{3}{4}$	3	560
2	4	540
$2\frac{1}{4}$	5	520
$2\frac{1}{2}$	6	500
3	10	480
$3\frac{1}{2}$	30	400
$4\frac{1}{2}$	50	360

The pressure per square inch of friction surface which may be used in the design of the type of clutch shown in Fig. 13 is:

(*a*) 100 lb. per square inch when the clutch is operated often.

(*b*) 200 lb. per square inch when the clutch is operated seldom.

301. Cone Clutch.—The cone type of clutch is adaptable to loads which are connected and disconnected frequently. The operating lever arrangement is similar to that for a disk clutch. The friction surfaces are held in contact by a spring, and for a

given spring pressure the force of friction produced is greater than it is with other types of clutches.

The application of a cone clutch to an automobile transmission is shown in Fig. 14(*a*). The fly wheel is of cast iron and the cone surface which engages with it is lined with leather or cork. The clutch is disengaged by foot pressure, which is multiplied by the lever ratio as shown in the figure, so that quite a powerful spring may be employed. On the other hand, the axial movement of the cone surface is less than the distance moved through by the pedal.

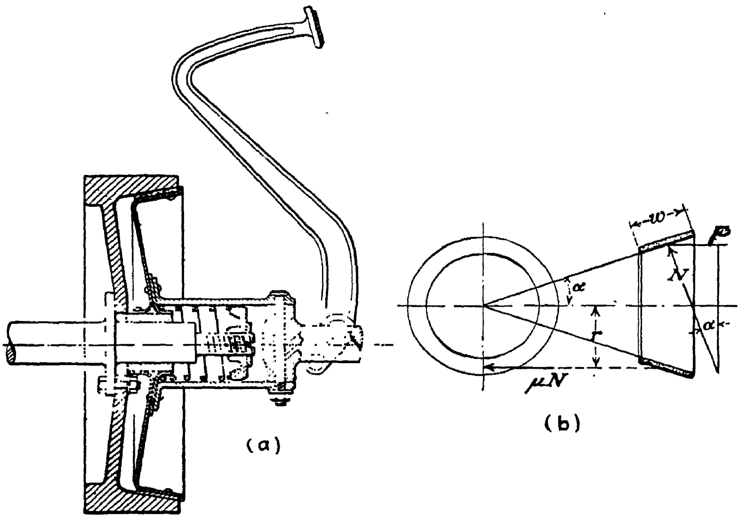


FIG. 14.—Cone clutch.

The axial force P which is required to compress the spring is from Fig. 14(*b*):

$$P = N \sin \alpha, \tag{23}$$

in which P denotes the axial force of the spring, in pounds.
N denotes the normal pressure at the friction surfaces, in pounds.
α denotes the angle between the cone surface and the axis of the shaft. (2α is the cone angle.)

The frictional force F is equal to μN, and substituting for N from formula (23):

$$F = \frac{\mu P}{\sin \alpha}.$$

The torque due to the friction is:

$$T = Fr = \frac{\mu Pr}{\sin \alpha}, \tag{24}$$

in which T denotes the torque due to friction, in inch-pounds.
r denotes the mean radius of the frictional force at the cone surface, in inches.
μ denotes the coefficient of friction at the cone surface.

The usual angle which is used by designers for cone clutches, and which is recommended by the Society of Automotive Engineers, is 12½ deg.

Example.—A cone clutch faced with leather is to transmit 20 hp. at 1,000 r.p.m. The cone angle is 25 deg., the mean radius is 6½ in., the coefficient of friction is 0.20, and the friction surface is 3 in. wide. What axial force will be required in the spring, and what will be the normal pressure per square inch at the friction surfaces (see Fig. 14(*b*)).

From the horsepower formula, formula 3, Chap. XI:

$$\text{Torque} = \frac{63{,}024 \text{ hp.}}{\text{r.p.m.}} = \frac{63{,}024 \times 20}{1{,}000} \quad 1{,}260 \text{ in.-lb.}$$

From formula (24):

$$1{,}260 = \frac{\mu Pr}{\sin \alpha} = \frac{0.20 \times P \times 6.5}{0.217}.$$

$$P = 210 \text{ lb.}$$

From formula (23):

$$210 = N \sin \alpha, \; N = \frac{210}{0.217} = 960 \text{ lb.}$$

The friction area is:

$$A = 2\pi rw = 2 \times 3.14 \times 6.5 \times 3 = 122.5 \text{ in.}^2.$$

The normal pressure per square inch is:

$$p = 960 \qquad 7.83.$$

This value is satisfactory since 8 lb. per square inch is considered good practice.

Example.—The data in the previous problem will be applied to a disk clutch. The outer diameter of the disks is 10 in., the inner diameter is 6 in., the normal pressure is 16 lb. per square inch, and steel plates and asbestos fabric will be used. The number of pairs of disks is required.

$$\text{Area of each disk} = \pi(5^2 - 3^2) = 50.2 \text{ in.}^2.$$

$$\text{Mean radius} = \frac{2}{3}\left(\frac{5^3 - 3^3}{5^2 - 3^2}\right) = 4.08 \text{ in.}$$

$$\text{Axial pressure } P = 16 \times 50.2 = 802 \text{ lb.}$$

From formula (21):

$$n = \frac{T}{\mu Pr} \quad \frac{1{,}260}{0.20 \times 802 \times 4.08} = 1.92. \quad \text{Use 2 pairs of disks.}$$

302. Expanding Clutches.—The expanding type of clutch has two or more shoes which are pushed in a radial direction to engage with the inner surface of a rotating drum or pulley. The shoes may be lined with wood or asbestos blocks, or the surfaces in contact may be cast iron on cast iron if the clutch is operated only occasionally. To overcome the internal pressure which is produced by the radial push of the inner shoes, a second set of shoes may be provided as shown in Fig. 15(a). The wood-lined jaws grip the friction ring with a force which is produced by the toggle joints. The toggle arrangement is very powerful in its

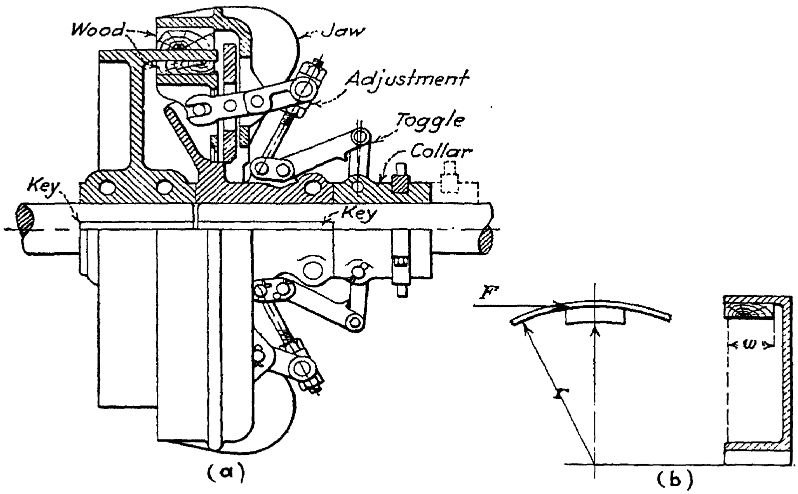


FIG. 15.— Radial expanding clutch.

action, and when full engagement is made the collar lever leans slightly toward the clutch, locking the operating levers in position.

The torque transmitted by a radially expanding clutch, with friction acting on the inside surface of the pulley only, is, from Fig. 15(b):

$$\text{Torque} = F \times r,$$

in which F denotes the frictional force, in pounds.
r denotes the radius at which the frictional force acts, in inches.

If the width of the friction lining is w, the normal pressure per square inch is p, and the coefficient of friction is μ:

$$\text{Torque} = 2\pi r^2 p\mu w, \qquad (25)$$

and

$$\text{hp.} = \frac{r^2 p \mu w (\text{r.p.m.})}{10{,}000}. \tag{26}$$

303. Magnetic Clutch.—Magnetic clutches are operated by an electromagnetic coil, which forces the friction surfaces together when the coil is energized by an electric current. Magnetic clutches are used where service and load conditions are severe, especially at high speeds. This type of clutch is constructed in stock sizes to transmit power for shaft sizes from $1\frac{1}{2}$ to 9 in. in diameter, and for a wide range of horsepower capacities. One of the advantages of magnetic clutches is that of remote control.

BELTING

304. General Considerations.—Belts are used to transmit power from one shaft to another by means of pulleys which rotate at the same or at different speeds, when the velocity ratio of the driver to the driven shaft may have a slight variation. The amount of power transmitted depends upon:

(*a*) The velocity of the belt.
(*b*) The tension under which the belt is placed on the pulleys.
(*c*) The arc of contact between the belt and the smaller pulley.
(*d*) The conditions under which the belt is used.

The shafting should be properly in line to insure uniform tension across the belt section. The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible. The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafting, thus increasing the friction load on the bearings. A long belt tends to swing from side to side, causing the belt to run out of line on the pulleys, which in turn develops crooked spots in the belt. The tight side of a belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys. To obtain good results with flat belts, the greatest distance between shafts should not exceed 25 ft., and the least distance should be three and one-half times the diameter of the larger pulley.

305. Leather Belts.—Leather is the most important material for flat belts. Various substitutes for leather belting are available, and they have been used with varying success. Leather belts are manufactured from the best quality of beef hides,

which reach the market either in the green or dried form. Green hides are pliable and have been salted to preserve them, while dried hides have been dried in the sun. After reaching the tannery, hides are soaked, and the adhering flesh parts and hair are removed by a mechanical or a chemical process. Hides are then soaked in tan liquid, an infusion of hemlock or oak bark containing tannin. The hide consists of bundles of fibers with a gelatinous binder, which is changed by the tannin into an insoluable compound. The hides are softened by fulling with hot grease, which lubricates the fibers and converts them from a hard and dry to a strong and pliable condition. The hides are then stretched and cut into strips of the required width.

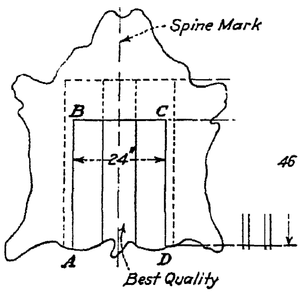


FIG. 16.

Belt leather requires the qualities of strength, firmness, and uniformity, so that they will maintain reasonable straightness. Only the back leather of a hide is used for belting, as shown in Fig. 16. The leather in the area outside the dotted lines is soft and flabby, and is unfitted to be used for belts. The section $ABCD$ in Fig. 16 contains the strongest and most uniform leather, and this portion is used for the higher grades of belts.

The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibers on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. For these reasons the hair side of a belt should be in contact with the pulley surface, thus giving a more intimate contact between belt and pulley, and placing the greatest tensile strength of the belt section on the outside, where the tension is greatest as the belt passes over the pulley.

306. Weight, Thickness, and Width of Leather Belts.—Leather is graded according to its weight and corresponding thickness, as given in Table VII.

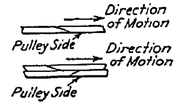


FIG. 17.

Because of the small hide area from which leather is obtained, belts are made up of lengths of about 46 in., which are cemented together. The joint is made by lapping the edges as shown in Fig. 17, the lap being from 3½ to 5½ in. long. Double belts are sometimes

stitched in addition to cementing. When possible, belts should be ordered the exact length and cemented by the belt manufacturer to form an endless belt.

TABLE VII.—WEIGHT, THICKNESS, AND WIDTH OF LEATHER BELTS

Grade	Weight per square foot, ounces	Thickness, inches			Width, inches
		Single ply	2 ply	3 ply	
					$\frac{1}{8}$ to 1, increment $\frac{1}{8}$
					1 to 4, increment $\frac{1}{4}$
Heavy (standard)	16 to 18	0.18	0.33	0.45	4 to $6\frac{1}{2}$, increment $\frac{1}{2}$
Medium	14 to 16	0.16	0.28		7 to 30, increment 1
Light	12 to 14	0.13	0.24		30 to 56, increment 2
					60 to 84, increment 4

307. Slip and Creep.—The linear velocity of a point on the pulley surface of the driving pulley is higher than the linear velocity of the belt, because the initial tension and working tension in a belt are not large in comparison with the ultimate strength of the leather. This indicates that slipping occurs between the pulley surface and the belt surface, called belt slip. Slip should not be in excess of 2 per cent, because larger amounts may cause damage to the belt.

Creep is caused by the elasticity of the belt material. The tension is greater on the pull or receiving side of the pulley, and as the tension decreases around the arc of contact, the particles of the belt creep back on the pulley as the belt approaches the delivery side. The driving pulley delivers a smaller length of belt than it receives by an amount equal to the creep. W. W. Bird[1] found that with ordinary belt transmission the creep was about 1 per cent. The loss in power due to slip and creep should not exceed 3 per cent for good engineering practice.

308. Velocity of Belts.—The power which a belt will transmit varies directly with its velocity up to the point at which centrifugal force begins to manifest itself. The velocity of the belt is intimately associated with the size of the pulleys over which it travels. F. W. Taylor[2] found that the most economical speed of belts was between 4,000 and 4,500 ft. per minute. To take advantage of the increased efficiency at high velocities, it is suggested that diameters of pulleys be chosen as large as is consistent with the amount of power to be transmitted.

[1] *Trans. A.S.M.E.*, Vol. XV, p. 204.

[2] *Trans. A.S.M.E.*, Vol. XXVI, p. 584.

The notation given below is used in the discussion which follows:

$T_1 = At_1$, denotes the total tension in the belt on the tight side, in pounds.

t_1 denotes the unit tension on the tight side, in pounds per square inch.

$T_2 = At_2$, denotes the total tension in the belt on the loose side, in pounds.

t_2 denotes the unit tension on the loose side, in pounds per square inch.

P denotes the effective tension or pull which transmits power, in pounds. $P = T_1 - T_2$.

p denotes the effective unit tension or pull, in pounds per square inch. $p = t_1 - t_2$.

e denotes the base of the natural system of logarithms, $e = 2.718$.

μ denotes coefficient of friction.

α denotes the arc of contact between the pulley and belt.

V denotes the velocity of the belt, in feet per minute.

A denotes the area of the belt section, in in.^{2}.

c denotes the centrifugal force which develops in a belt running at high velocity, in pounds.

309. Arc of Contact.—The power transmitted by a belt is dependent upon the arc of contact between the belt and the smaller of the two pulleys. The arc of contact is often increased by turning the belt over an idler pulley, so located that the belt will enclose a large portion of the power pulley. The arc of contact may be measured from a diagram, drawn to scale, of the pulley arrangement.

310. Coefficient of Friction.—The coefficient of friction varies with the velocity of the belt, according to a study made by Carl Barth,[1] and the values given in Table VIII have been used with excellent results. They are based upon the following formula:

$$\mu = 0.54 - \frac{140}{500 + V}, \tag{27}$$

in which μ denotes the coefficient of friction.

V denotes the linear velocity of the belt, in feet per minute.

[1] *Trans. A.S.M.E.*, Vol. XXXI, p. 29.

TABLE VIII.—COEFFICIENTS OF FRICTION OF LEATHER BELTS ON IRON PULLEYS

Velocity of belt (V) feet per minute	Coefficient of friction (μ)	Velocity of belt (V) feet per minute	Coefficient of friction (μ)	Velocity of belt (V) feet per minute	Coefficient of friction (μ)
0	0.260	800	0.432	3,000	0.500
50	0.285	900	0.440	3,500	0.505
100	0.307	1,000	0.446	4,000	0.509
200	0.340	1,200	0.458	4,500	0.512
300	0.365	1,400	0.466	5,000	0.514
400	0.384	1,600	0.473	5,500	0.517
500	0.400	1,800	0.479	6,000	0.519
600	0.413	2,000	0.484	6,500	0.520
700	0.423	2,500	0.493		

311. Strength and Working Tension.—The ultimate strength of leather belts may vary between 3,500 and 6,000 lb. per square inch. The service which a belt is expected to give is the factor which determines the working tension, and this is not directly related to the ultimate strength of the material. The working tension in a belt has been determined by experiment and from observed data of satisfactory belt transmissions.

Defining the effective tension in a belt as the difference in tensions between the tight and loose sides, or the force which causes the driven pulley to rotate:

$$P = T_1 - T_2.$$

Practice has fixed the value of P according to the values given in Table IX.

TABLE IX.—ALLOWABLE VALUES OF EFFECTIVE TENSION IN BELTS
Pounds per Inch of Width

	Ply or number of thicknesses of material											
Material in belt.	1	2	3	4	5	6	7	8	9	10	11	12
Leather	55	88	110									
Rubber	..	20	30	40	50	60	70	80	90	100	110	120
Canvas (stitched)	...	...	..	40	50	60	...	70	...	80	..	90
Balata	...	...	30	40	50	60	70	80				

The values given in Table IX for single-, double-, and triple-ply leather belts correspond to the following values of effective unit tension p per square inch of leather:

$$p = \frac{55}{0.18} = 250$$

$$p = \frac{88}{0.33} = 267 \qquad \text{Average} = 254 \text{ lb. per square inch.}$$

$$p \quad \frac{110}{0.45} \quad 245$$

312. Horsepower of Belts. Low Velocities.—The relation between the pull on the tight and loose sides of a belt is identical with that which was found for band brakes in Sec. 297, formula (11):

$$\frac{T_1}{T_2} = e^{\mu\alpha}.$$

Since the areas are equal for T_1 and T_2:

$$\frac{t_1}{t_2} = e^{\mu\alpha}. \tag{28}$$

The force which transmits power in a belt is:

$$P = T_1 - T_2,$$

and the force per unit area of belt is:

$$p = t_1 - t_2.$$

From the last equation $t_2 = t_1 - p$, which substituted in formula (28) gives:

$$t_1 = t_1 e^{\mu\alpha} - p e^{\mu\alpha}.$$

Solving for p:

$$p = \frac{t_1(e^{\mu\alpha} - 1)}{e^{\mu\alpha}}. \tag{29}$$

For convenience let

$$\frac{e^{\mu\alpha} - 1}{e^{\mu\alpha}} = K.$$

Then

$$p = t_1 K. \tag{30}$$

In engineering units, p is the force transmitted by 1 sq. in. of belt, hence the horsepower which 1 sq in. of belt will transmit is:

$$\text{hp.} \quad \frac{t_1KV}{33{,}000}, \tag{31}$$

in which V is the linear velocity of the belt in feet per minute.

313. Horsepower of Belts. High Velocities.—To determine the horsepower which a belt may deliver at high velocities, the centrifugal force acting when the belt passes over the pulley must be taken into account. The forces acting on the belt will be the same as shown in Fig. 7(b), except for the additional centrifugal force. This force is given by the formula:

$$\frac{12wv^2}{gR}. \tag{32}$$

in which w denotes the weight of the body, in pounds.
- v denotes the linear velocity of the body, in feet per second.
- g denotes the acceleration of gravity, 32.2 ft. per second per second.
- R denotes the radius of the pulley, in inches.
- c denotes the force due to centrifugal action, in pounds

Formula (32) gives the centrifugal force for 1 cu. in. of belt, if w is taken as the weight of 1 cu. in.; or this may be looked upon as the weight of 1 linear in. of belt having a cross-section of 1 sq. in. The centrifugal force, which acts upward, and which must be added to Fig. 7(b), equals cds. Summing up forces along the vertical:

$$\Sigma F_y = pds + cds - T\sin\frac{d\theta}{2} - (T + dT)\sin\frac{d\theta}{2} = 0.$$

As before, this reduces to:

$$pds + cds = Td\theta. \tag{33}$$

As before:

$$dT = \mu pds.$$

Also

$$cds = \frac{12wv^2}{gR}ds = \frac{12wv^2}{g}d\theta = c'd\theta.$$

Substituting these values in formula (33), and since for 1 sq. in. of belt area $T = t$:

$$pds = td\theta - c'd\theta = (t - c')d\theta.$$

$$\frac{dt}{\mu} = (t - c')d\theta, \frac{dt}{(t - c')} \quad \mu d\theta.$$

Integrating between the limits t_1 and t_2 for t, and between the limits 0 and α for θ:

$$\log_e \frac{t_1 - c'}{t_2 - c'} = \mu\alpha, \text{ or } \frac{t_1 - c'}{t_2 - c'} = e^{\mu\alpha}. \tag{34}$$

As before, the force transmitted by 1 sq. in. of belt section is:

$$p = t_1 - t_2.$$

Substituting for t_2 in formula (34) and solving for p:

$$p = (t_1 - c')\left(\frac{e^{\mu\alpha} - 1}{e^{\mu\alpha}}\right). \tag{35}$$

Letting

$$\frac{e^{\mu\alpha} - 1}{e^{\mu\alpha}} \quad K,$$

$$p = (t_1 - c')K. \tag{36}$$

In the same way as before:

$$\text{hp.} = \frac{(t_1 - c')KV}{33{,}000}. \tag{37}$$

This is the horsepower transmitted by 1 sq. in. of belt.

The effect of centrifugal force is usually neglected for linear speeds less than 1,500 ft. per minute, and Table X gives the values of c' in formula (37) for speeds from 1,500 to 5,000 ft. per minute, based upon a weight of 0.035 lb. per cubic inch for leather.

TABLE X.—VALUES OF COEFFICIENTS c' FOR LEATHER BELTS

Belt velocity, feet per minute	1,500	2,000	2,500	3 000	3,500	4,000	4 500	5,000
Coefficient c'	8.16	14.9	23.2	33.5	44.3	57.9	73.2	90.6

Table XI gives the values of K to be used in formulas (31) and (37) for various combinations of arc of contact and coefficient of friction.

TABLE XI.[1]—VALUES OF $\frac{e^{\mu\alpha} - 1}{e^{\mu\alpha}}$ K, FOR VARIOUS COEFFICIENTS OF FRICTION AND ARCS OF CONTACT

Value of μ	Arc of contact between the belt and pulley (α in degrees)										
	90	100	110	120	130	140	150	160	170	180	200
0.15	210	0.230	0.250	0.270	0.288	0.307	0.325	0.342	0.359	0.376	0.408
0.18	246	0.270	0.292	0.314	0.335	0.356	0.376	0.395	0.414	0.432	0.466
0.20	270	0.295	0.319	0.342	0.364	0.386	0.408	0.428	0.448	0.467	0.582
0.23	303	0.331	0.357	0.382	0.406	0.430	0.452	0.474	0.495	0.514	0.624
0.25	325	0.354	0.381	0.407	0.432	0.457	0.480	0.503	0.524	0.544	0.649
0.28	356	0.387	0.416	0.444	0.470	0.496	0.520	0.542	0.564	0.585	0.502
0.30	376	0.408	0.438	0.467	0.494	0.520	0.544	0.567	0.590	0.610	0.553
0.33	404	0.438	0.469	0.499	0.527	0.554	0.579	0.602	0.624	0.645	0.684
0.35	423	0.457	0.489	0.520	0.548	0.575	0.600	0.624	0.646	0.667	0.705
0.38	449	0.485	0.518	0.549	0.578	0.605	0.630	0.654	0.676	0.697	0.735
0.40	467	0.502	0.536	0.567	0.597	0.624	0.649	0.673	0.695	0.715	0.753
0.43	491	528	0.562	0.593	0.623	0.650	0.676	0.699	0.721	0.741	0.777
0.45	.507	0.544	0.579	0.610	0.640	0.667	0.692	0.715	0.737	0.757	0.792
0.48	.529	0.567	0.602	0.634	0.663	0.690	0.715	0.738	0.759	0.779	0.813
0.50	.544	0.582	0.617	0.649	0.678	0.705	0.730	0.752	0.773	0.792	0.825
0.53	.565	0.603	0.638	0.670	0.700	0.726	0.750	0.772	0.793	0.811	0.843
0.55	.578	0.617	0.652	0.684	0.713	0.739	0.763	0.785	0.805	0.822	0.853
0.58	.598	0.637	0.672	0.703	0.732	0.758	0.781	0.802	0.821	0.838	0.868
0.60	.610	0.649	0.684	0.715	0.744	0.769	0.792	0.813	0.832	0.848	0.877
1.00	.792	0.825	0.853	0.877	0.897	0.913	0.927	0.937	0.947	0.956	0.969

[1] Extracted from the paper, by W. LEWIS "Experiments on the Transmission of Power by Belting," Vol. VII, p. 579, *Trans.*, *A.S.M.E.*

314. Maximum Working Unit Stress.—The maximum working unit stress t_1 for leather belts, which may be assumed in design, is dependent upon the material in the belt and the type of joint which is used to make the belt connection. The following values for the unit tensile stress in the tight side of belts have proven satisfactory:

Leather belts, endless or factory jointed, t_1 = 400 to 425 lb. per square inch.

Laced or metal joints, t_1 = 200 to 215 lb. per square inch.

Example.—A machine requiring 18 hp. for operation, with a receiving pulley 14 in. in diameter and with 8-in. face, is driven from a pulley 18 in. in diameter and 8-in. face, on a line shaft which is running at 225 r.p.m. Find the width of the required open leather belt.

The pulleys are nearly of equal size, so that 180 deg. of arc may be assumed.

$$V = \frac{\pi d(\text{r.p.m.})}{12} = 3.14 \times \frac{14}{12} \times 225 = 823 \text{ ft. per minute.}$$

From Table VIII, $\mu = 0.432$ for this velocity.
Assume $t_1 = 400$ lb. per square inch for a cemented joint.
From Table XI, $K = 0.741$.
For this speed the effect of centrifugal force is neglected. Using formula (31):

$$\text{hp.} = \frac{t_1 K V}{33{,}000} = \frac{400 \times 0.741 \times 823}{33{,}000} = 7.4 \text{ per square inch.}$$

$$\text{Belt area} = \frac{18}{7.4} = 2.43 \text{ in.}^2.$$

A single-ply belt is 0.18 in. thick and would require a width of 13.5 in., which is too wide for a pulley with an 8 in. face. A double-ply belt would require a width of 7.36 in., and this is satisfactory. The total tension T_1 on the tight side is:

$$t_1 A = 400 \times 2.43 = 972 \text{ lb.}$$

The total effective tension P which transmits the power is:

$$P = \frac{\text{hp.} \times 33{,}000}{V} = \frac{18 \times 33{,}000}{823} \quad 722 \text{ lb.}$$

The total tension T_2 in the loose side is:

$$T_2 = T_1 - P = 972 - 722 = 250 \text{ lb.}$$

315. Rubber Belts.—Rubber belting is manufactured by using layers of cotton duck for plys, cementing them together with rubber, and vulcanizing under pressure. Rubber belts are not affected by ordinary heat or cold, and are especially adapted to moist conditions. Rubber belts are used where the service and surroundings make leather belts impracticable, and they are especially adapted for the transmission of power in saw mills, creameries, chemical plants, and paper mills. Such belts are also used for conveyors to carry ore, coal, sand, and gravel, and when so used are reinforced with a heavy layer of rubber.

316. Canvas Belts.—Stitched canvas belts are made of cotton duck sewed with three or four rows of stitching to the inch. The belt material is water-proofed and the surfaces are usually painted for added protection. The material is adapted to variable temperatures and rough usage, such as is experienced in saw mill, quarry, and farm work.

Solid-woven cotton belts are made by weaving throughout the full cross-section of the belt. The material is very strong and flexible, and is not affected by heat, cold, water, or grease.

The solid form of cotton belting is adapted for conveying purposes and is employed for handling light-weight materials.

Balata belts are canvas belts impregnated with the gum of the balata tree. Balata gum does not oxidize readily, and its presence gives a good coefficient of friction to the belt surface, and durability to the cotton belts.

317. Adjustment of Tension in Belts.—Belts which are made endless are uniformly stronger, and have better contact with the pulleys than when laced or hinged joints are used. The employment of endless belts requires that provision must be made for taking up the slack due to stretching, which develops rapidly with new belts and less rapidly with use.

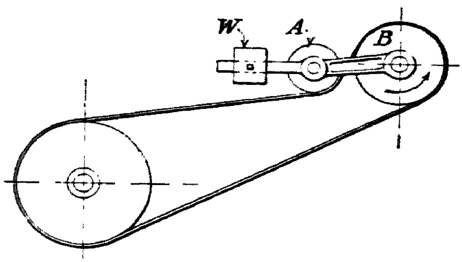


Fig. 18.—Gravity idler pulley.

When practicable, the belt tension is maintained by increasing the distance between pulleys by means of adjusting screws. Electric motors are adjustable in this manner with respect to their bed plates. Sometimes an idler pulley is employed to maintain constant belt tension, as shown in Fig. 18. Here a gravity idler pulley is held in place by the weight of the pulley *A* and the adjustable weight *W*. The idler pulley is located on the loose side of the belt, and increases the arc of contact of the pulley *B* in addition to taking slack out of the belt. Other types of idler pulleys depend upon springs to keep the idling pulley in contact with the belt. When the machines are not in use, idler pulleys are removed from contact with the belt to prevent unnecessary stretching of the belt. Laced or hinged belts are adjustable for tension by removing a short length of belt section from time to time.

318. Laced and Hinged Joints.—The length of a belt is found by passing a steel tape in correct alignment around the pulleys, pulling the tape as tight as possible, and noting the length. While factory-made endless belts are preferred to homemade

joints, operating conditions often compel the use of laced or hinged joints.

The laced joint is formed by punching holes in line across the belt, leaving a margin between the edge and the holes which is consistent with the strength and flexibility of the joint. A

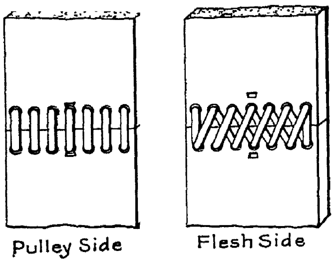


FIG. 19.—Straight stitch rawhide laced belt joint.

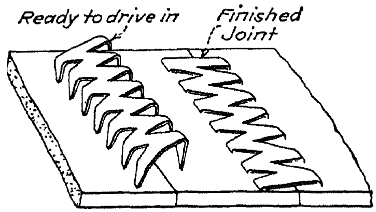


FIG. 20.—Steel belt fastener, Bristol type.

rawhide strip is used for lacing the two ends together to form a joint. There are several patterns which are used for lacing, the one shown in Fig. 19 being called the "straight stitch," in which the parallel stitches are made on the hair or grain side of the belt.

Metal "lacing" is manufactured in various forms which are applied by means of copper rivets, or the connection may be used as a staple, the points being driven through the flesh side of the belt and clinched on the inside. Figure 20 shows the Bristol type of metal belt lacing. Metal hinges may also be fastened to the belt ends and connected by a steel or fiber pin, as shown by the Clipper hinge in Fig. 21. The wire hooks are furnished by the manufacturer[1] properly spaced on cards, and are readily applied to the square ends of the belt to form a joint.

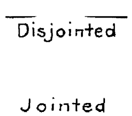


FIG. 21.—Hinged joint, Clipper type.

The relative strength of a factory-cemented joint is from 80 to 100 per cent, of metallic-hinged joints from 50 to 60 per cent, and of laced joints 30 to 50 per cent of the solid leather.

PULLEYS

319. General Considerations.—The transmission of power by belts is dependent upon the driving and driven pulleys as

[1] Clipper Belt Lacer Co., Grand Rapids, Mich.

much as upon the belt itself. The two pulleys must be in perfect alignment to allow the belt to travel in a line normal to the pulley faces, otherwise the belt will climb to the high side of the pulley.

Pulleys are classified according to the materials from which they are made, as follows:

(*a*) Cast-iron pulleys.
(*b*) Pressed-steel pulleys.
(*c*) Wood pulleys.

They are also classified according to the manner in which they are fastened to the shaft, as follows:

(*a*) Solid pulleys.
(*b*) Split pulleys.

Pulleys of the above three materials are made both solid and split.

320. Cast-iron Pulleys.—Cast-iron pulleys are designated as lightweight and heavy duty, according to the ply of the belt which is used with them, the belt being the criterion of the service which the pulley may be expected to withstand. Cast-iron pulleys are manufactured in stock sizes from 3 to 84 in. in diameter, and in a variety of face widths for each size, ranging from a 3-in. face for a 3-in. diameter to a 30-in. face for an 84-in. diameter.

Small cast-iron pulleys are made with a web connection between the flange and the hub. As the size increases, arms are provided; pulleys up to 8-in. diameter having four arms, those from 8 to 60 in. in diameter having six arms, and those larger than 60 in. having eight arms. A second set of arms is provided for pulleys having a face width greater than 20 in., but the pulleys with a single set of arms are made in all face widths. Two pulleys with single sets of arms and narrow faces, will, if placed side by side on the shaft, give the same result as a pulley with a wide face and a double set of arms.

Figure 22(*a*)[1] shows a cast-iron solid pulley with a single set of six arms, while Fig. 22(*b*) shows a split pulley of the same type. Figure 22(*c*) shows a cast-iron solid pulley with a wide face and a double set of six arms, while Fig. 22(*d*) shows a split pulley of the same type.

Cast-iron pulleys up to a diameter of 48 in. are balanced for speeds up to 300 r.p.m. Larger pulleys are balanced for a

[1] Figures 22, 25(*b*), 27, 28, 29, and 30 are used by permission of the Dodge Mfg. Corp., Mishawaka, Indiana.

maximum rim speed of 3,750 ft. per minute. C. H. Benjamin,[1] in experimenting with cast-iron pulleys, found that the maximum safe peripheral speed was about 1 mile per minute.

Rational formulas are not of great value in arriving at the proportions for cast-iron pulleys. Experience has determined the approximate proportions for the rim thickness, cross-section and taper of the arms, and size of the hub which will result in a sound casting and the least warpage. Design engineers follow the practice of using the empirical formulas which have been deduced by manufacturers who specialize in the production of

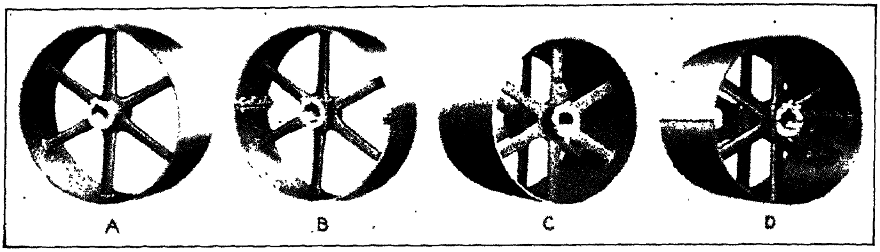


FIG. 22.—Cast iron pulleys, solid and split.

power-transmission machinery, of which pulleys form an important part and of considerable amount.

321. Width of Pulley Face and Height of Crown.—The width of the belt which is to run over a pulley determines the width of the pulley face. C. G. Barth[2] gives the following formula for the width of the pulley face:

$$F = 1\tfrac{3}{16}W + \tfrac{3}{8} \qquad (38)$$

in which F denotes the width of the pulley face, in inches.
W denotes the width of the belt, in inches.

If the width of the face as found by formula (38) is too wide to conform to design limits, the same authority suggests that the following formula be used:

$$F = 1\tfrac{3}{32}W + \tfrac{3}{16}. \qquad (39)$$

F. W. Taylor recommends that the face width of a pulley be made 25 per cent greater than the belt width. When tight and loose pulleys are used side by side, the face of the pulleys is made somewhat less than that obtained by formula (39).

[1] *Trans. A.S.M.E.*, p. 209, 1899; p. 168, 1902.
[2] *American Machinist*, Feb. 11, 1915.

Advantage is taken of the tendency of a belt to climb to the high side of a pulley, by increasing the thickness of the rim at the center to give it a *crown*. The crown is effective only when the slip of the belt is a minimum, because a slipping belt will run off a crowned pulley more easily than from a flat-faced pulley. Leather belts require higher crowns than rubber or cotton belts. Authorities do not agree on the amount of crown which should be used, one manufacturer using a crown based on ⅛-in. taper per foot of face. C. G. Barth recommends:

$$H = 0.03125\sqrt[3]{F^2}, \tag{40}$$

in which H denotes the height of the crown, in inches.
F denotes the width of pulley face, in inches.

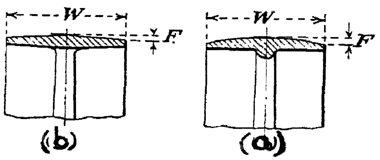


FIG. 23.—Pulley crowns.

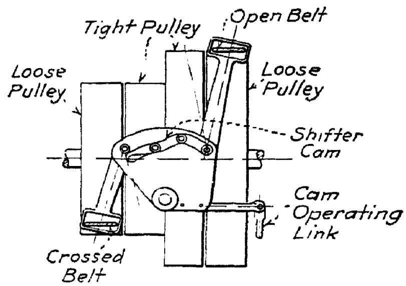


FIG. 24.—Belt shifter cam.

The crown should be rounded as shown in Fig. 23(a), but for simplicity in machining, the face is often tapered as shown in Fig. 23(b).

322. Tight and Loose Pulley.—By shifting a belt from a loose pulley on a shaft to one fastened to the shaft, the same result may be obtained as with a friction clutch. This form of friction clutch is especially adapted for driving machine tools of the planer type. The machine usually has two sets of driving and idler pulleys, one for the cutting stroke and the other for the return stroke, as shown in Fig. 24.

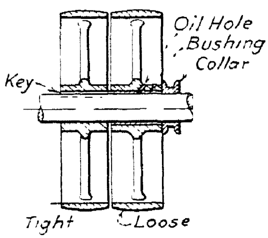


FIG. 25(a).—Tight and loose-pulley arrangement.

Loose pulleys are sometimes made 1 in. smaller in diameter than the tight pulley, to relieve the tension on the belt when the machinery is idle. In such cases an angular flange is provided on one side of the loose pulley, tapering up to the plane of the

face of the tight pulley, to enable the belt to be shifted. Figure 25(a) shows a cross-section of the standard tight- and loose-pulley arrangement, and Fig. 25(b) shows a Timken roller bearing on the loose pulley, with weights located on the inside of the rim to counterbalance the pulley. Figure 26 shows the ordinary method of shifting the belt from one pulley to the other. By locating the pins so that the distance between them is twice the distance A, the shifter lever will always hang in a vertical position.

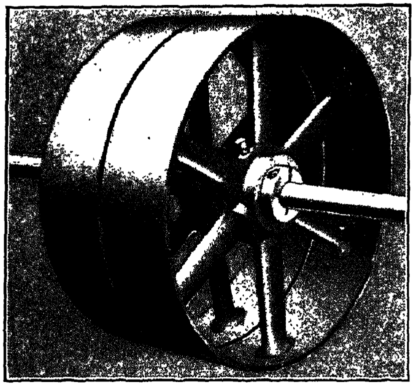


Fig. 25(b).—Tight and loose pulley. Loose pulley is mounted on Timkin roller bearings. (*Dodger Manufacturing Corp. Mishawaka, Ind.*)

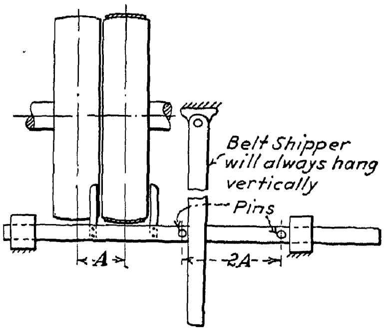


Fig. 26.—Belt shifter.

323. Steel Pulleys.—Steel pulleys are built up of pressed-steel plate, the rim and spokes being riveted together and securely fastened to a cast-iron hub, or riveted to a malleable-iron hub. Steel pulleys are lighter in weight and stronger than cast-iron pulleys, and are designed to run at very high speeds. They are made in two parts and riveted together to form a "solid" pulley, or they may be held together by bolts as shown in Fig. 27, similar to cast-iron pulleys.

Steel split pulleys are manufactured in diameters and face widths so as to be interchangeable with cast-iron pulleys, the largest size being 144 in. in diameter with a 40-in. face. Large steel pulleys are reinforced at the rim section where the greatest stress is likely to occur, as indicated in Fig. 28.

324. Wood Pulleys.—Wood is used for transmission pulleys because of its lighter weight and the high coefficient of friction

which develops between the pulley surface and the belt. These pulleys are built up of well-seasoned maple-wood segments, pinned with dowels and glued. Wood, because of its flexibility, is well adapted for pulleys which are subjected to shock loads which would be considered dangerous to a more rigid material. Wood pulleys should not be locatd in damp places.

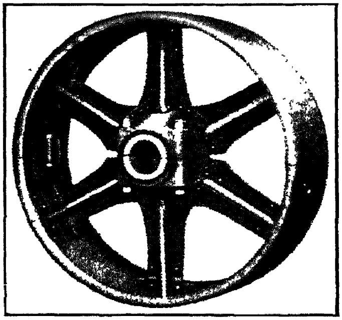


FIG. 27.—Steel pulley split type.

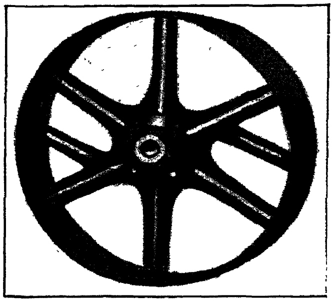


FIG. 28.—Steel pulley, reinforced at the rim.

Small wood pulleys are manufactured in a solid form as shown in Figs. 29(*a*) and 29(*b*). The metal core may be fitted with an interchangeable taper bushing, which when drawn into place by the bolts, clamps the pulley to the shaft. This form of pulley is made in stock sizes ranging from a 2-in. diameter with a 2-in.

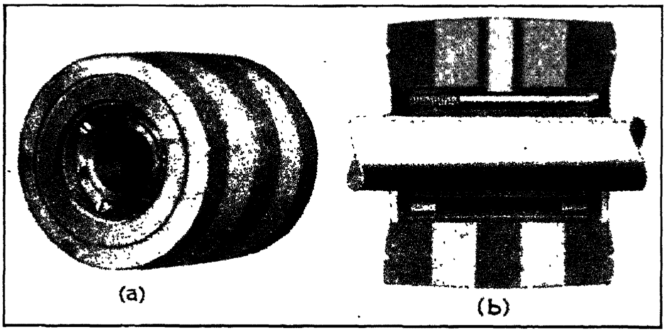


FIG. 29.—Wood pulley, solid form.

face, to a 12-in. diameter with a 12-in. face, and is particularly adapted for electrical machines.

A wood rim pulley with a cast-iron spider center, as shown in Fig. 30, is adapted to high speeds and severe service. This form of pulley is made with solid or split hubs.

Wood pulleys and steel pulleys are structurally very complex as compared with the simple cast-iron pulley, but they are manufactured in such large quantities that their cost compares favorably with cast-iron pulleys.

325. Variable-speed Belt Drives.—Stepped-cone pulleys placed opposite each other on the driving and driven shafts, offer a method of providing such machines as drill presses and engine lathes, with a range of driving speeds well adapted to the work which these machines must do. When a crossed belt is to be used, on a pair of stepped-cone pulleys, the sum of the radii of the mating pulleys must be constant. When an open belt is to be used, the distance between the shaft centers must be known. The solution of this problem is rendered simple by the use of one of the several graphical methods which are usually employed. The scheme shown in Fig. 31, and explained below, is credited to C. A. Smith.[1]

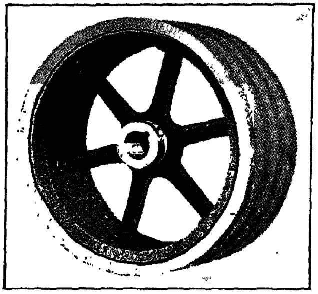


FIG. 30.—Wood rim pulley with cast-iron center.

A and B (Fig. 31) are the centers of the shafts, and L the distance between them in inches. At C, half way between A and B,

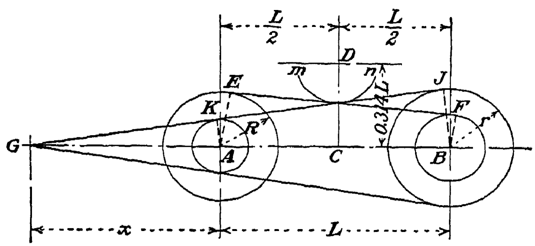


FIG. 31

a perpendicular CD of length equal to $0.314L$ is erected (this coefficient was determined experimentally).[2] A line EF is drawn tangent to the pulleys at E and F. With D as a center the arc mn is drawn of such a radius that it will be tangent to the line EF. BJ is the radius of one of the pulleys of the second pair of steps.

[1] *Trans. A.S.M.E.*, Vol. X, p. 269.

[2] When the angle between the belt and center line of pulley exceeds 18 deg., CD is taken as $0.298L$.

To find its mate the line JG is drawn tangent to the arc mn and tangent to an arc of radius BJ. From A, a perpendicular AK is dropped to this, line which will be the radius of the mate.

If instead of the diameter of one of the second pair of steps, the velocity ratio of the second pair together with the diameters of the first pair is given, the solution will be as follows:

The distance from where the line JK, extended, intersects the line of centers at G, to A, is called x. The velocity ratio of $A/B = BJ/AK = a$.

Then by similar triangles $\frac{x + L}{x} = \frac{BJ}{AK} \cdot x = \frac{L}{a - 1}$.

So that by taking $GA = x = \frac{L}{a - 1}$, and drawing a line through G tangent to FJ, pulleys for the required velocity ratio will be obtained.

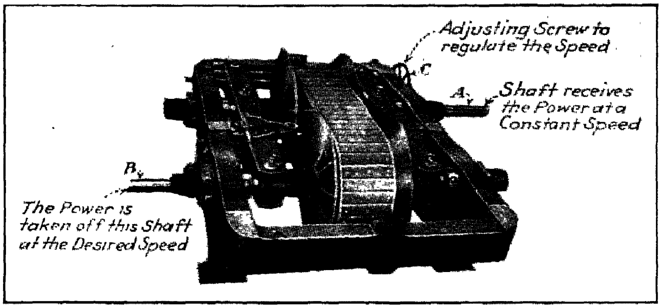


FIG. 32.—Variable speed belt drive. (*Reeves Pulley Company, Columbus, Ind.*)

When the smaller pulley is at B, the value of a becomes less than 1, and that of x negative, which indicates that it must be laid off from A toward the right in the figure.

A belt-driven variable-speed mechanism is shown in Fig. 32. Two pairs of cone-faced disks are mounted on parallel shafts, and controlled by levers so that they may be separated or moved closer together by changing the driving radius with respect to a V-shaped belt which connects them. The speed may be regulated while the machine is running by turning a hand wheel at C. These countershafts are manufactured in 14 sizes to transmit from 2 to 150 hp., and in seven classes to give speed ratios as low as 2 to 1 and as high as 10 to 1.

TRANSMISSION OF POWER BY ROPES

326. General Considerations.—Flat belts are limited to a distance between pulley centers of about 25 ft., and the transmission of large amounts of power would result in excessive cross-sections. Since the thickness of a flat leather belt is seldom greater than three ply, the width becomes excessive for large loads. For example, to transmit 1,500 hp. at 5,676 ft. per minute, would require a leather belt of three-ply thickness and 78 in. width. Ropes, on the other hand, will transmit large amounts of power between shafts which are 200 ft. apart, give service which is smooth and quiet, and will not be affected by out of door conditions. With rope drives the shafts may be out of strict alignment, and power may be readily taken off in any direction, and in fractional parts of the total amount. When comparison is made between belt and rope drives, relative economies are important, and the low initial cost and maintenance charge is favorable to rope transmission. There are two systems of rope transmission in use:

(*a*) The English, or separate rope system.

(*b*) The American, or continuous rope system.

327. The English System.—The English system employs separate loops of rope, each loop designed to carry its part of the load. If one rope breaks, or is taken off for any other reason, the remaining ropes carry the load. All ropes should be of the same diameter in order to ride at the same driving radius in the groove. Cotton cordage ropes are used, which are very strong and have little stretch. It is difficult to maintain a uniform tension in all the ropes, which results in overloading the tighter ropes. This system is the older one, but is now seldom used in American practice.

328. The American System.—The American system uses a single rope, which is wound around the driving and driven sheaves, forming as many belts as are required, and is then returned to the first groove by a sheave placed at the proper angle. A uniform tension is maintained by a traveling tension carriage, as shown in Fig. 33, moving on a track, which takes up slack in the rope as stretch occurs. A rope drive readily adjusts itself to variable loads, and the rope is able to absorb shock due to its flexible and elastic qualities. The further discussion of rope drives will refer to the single rope or American system.

329. Manila Rope.—Manila rope is made from long fiber Philippine Island hemp. The rope is made endless by splicing, and the splice should be long, with the rope diameter increased as little as possible. This prevents the large diameter from riding high in the wheel groove and becoming worn due to pinching. When the rope bends around the sheave wheel, there is some sliding of the fibers, causing the rope to wear and chafe internally. To minimize this defect the rope fibers are lubricated by a plumbago-tallow grease, which also renders the rope moisture proof. New rope will stretch considerably for a time, but after the fibers

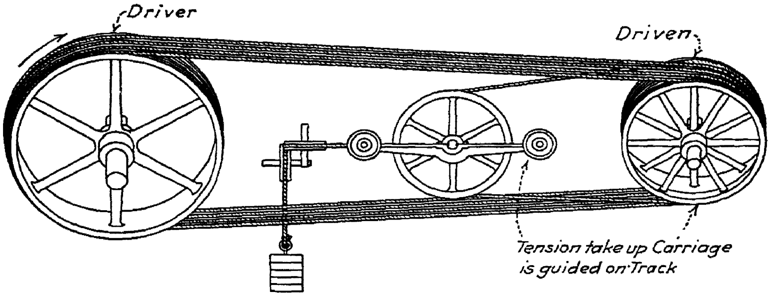


FIG. 33.—Rope drive, American system.

become firmily bedded, little further elongation takes place. The slack due to stretching of the rope may be taken out by removing a section of the rope.

330. Strength of Manila Rope.—The breaking strength of rope varies greatly, and may be from 7,000 to 12,000 lb. per square inch. A value of ultimate load which may be used as a basis of design is as follows:[1]

$$P = 7{,}000d^2,$$

in which P denotes the ultimate load, in pounds.
d denotes the diameter of the rope, in inches.

For continuous driving the maximum tension T_1 on the tight side should not exceed:

$$T_1 = \frac{7{,}000d^2}{35} = 200d^2. \qquad (41)$$

The notation to be used for ropes will be the same as that used in Sec. 308 for flat belts.

[1] HUNT, C. W., *Trans. A.S.M.E.*, Vol. VII, p. 230.

If a rope 1 in. in diameter and 1 in. long has a weight of w', the centrifugal force due to it, just as in the case of belts, is:

$$c = \frac{12w'v^2}{gr}.$$

The tension T_c produced in the rope due to the centrifugal force alone is found as follows, using a figure similar to Fig. 7(*b*):

$$cds = 2T_c \sin d\theta$$

$$\frac{12w'v^2}{gr}ds = T_c d\theta.$$

$$12w'v^2 d\theta = T_c d\theta.$$

$$T_c = \frac{12w'v^2}{\quad} = c''$$

The values of c'', based on a weight of rope 1 in. in diameter and 1 in. long, which is 0.028 lb., are given in Table XII.

TABLE XII.—VALUE OF COEFFICIENT (c'') FOR MANILA ROPE

Velocity in feet per minute...	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500	6,000	6,500	7,000
Coefficient (c'')	6.53	11.9	18.6	26.8	35.4	46.3	58.5	72.5	87.5	104	122	142

The difference between T_1 and the coefficient c'' is the net tension on the driving side, but a certain amount of tension is necessary on the loose side to give adhesion of the rope to the pulley. C. W. Hunt assumes that the tension necessary on the slack side, to give adhesion, is one-half of P, the effective tension which transmits power. On this basis:

$$T_1 = P + \frac{P}{2} + c''.$$

$$P = \frac{2}{3}(T_1 - c''). \tag{42}$$

The horsepower which is transmitted is:

$$\text{hp.} = \frac{2}{3}\frac{(T_1 - c'')V}{33{,}000} = 0.00002V(T_1 - c''). \tag{43}$$

This is the horsepower transmitted by a rope 1 in. in diameter.

TABL X DATA ON MAN 'E FOR POWER TRAN

Diameter of rope in inches, d	d^2	Weight pounds per foot, approximate	Breaking strength in pounds	Maximum allowable tension, $200 \times d^2$, T_1	Length required for splice in feet			Smallest diameter sheave for rope, $36 \times d$	Maximum r.p.m. of sheave	Area of rope in square inches	Equivalent in 1-in. diameter ropes
					Number of strands						
					3	4	6				
$\frac{3}{4}$	0.5625	0.21	3,950	112	6	8	..	28	760	0.4418	0.551
$\frac{7}{8}$	0.7656	0.27	5,400	153	6	8	..	32	650	0.6013	0.770
1	1.0000	0.36	7,000	200	7	10	14	36	570	0.7854	1.000
$1\frac{1}{8}$	1.2656	0.45	8,900	253	7	10	16	40	510	0.9940	1.270
$1\frac{1}{4}$	1.5625	0.56	10,900	312	7	10	16	46	460	1.2272	1.560
$1\frac{3}{8}$	1.8906	0.68	13,200	378	8	12	16	50	415	1.4849	1.890
$1\frac{1}{2}$	2.2500	0.80	15,700	450	8	12	18	54	380	1.7671	2.250
$1\frac{5}{8}$	2.6406	0.92	18,500	528	8	12	18	60	344	2.0739	2.640
$1\frac{3}{4}$	3.0625	1.08	21,400	612	8	12	18	64	330	2.4053	3.100
2	4.0000	1.40	28,000	800	9	14	20	72	290	3.1416	4.000
$2\frac{1}{4}$	5.0625	1.80	35,400	1,012	9	14	20	82	255	3.9761	5.000
$2\frac{1}{2}$	6.2500	2.20	43,700	1,250	10	16	22	90	230	4.9087	6.250

If the value of T_1 for a 1-in. rope be taken from formula (41), the above formula (43) becomes:

$$\text{hp.} = 0.00002V(200 - c''). \tag{44}$$

It will be noted that formula (44) is similar to formula (37) for flat belts, except that the constant K in formula (37) is taken as $\frac{2}{3}$ in formula (44), and is not computed on the basis of the coefficient of friction and the arc of contact. The reason for this is that reliable data on the coefficient of friction for ropes are meager. Since the ropes run in a grooved pulley the pressure against the rope will be greater than the radial pressure. If the frictional force for flat belts is μp, it is μp divided by the sine of $\theta/2$ for grooved pulleys, when the angle of the groove is θ. The coefficient of friction for ropes may therefore be taken as

$$\mu' = \frac{\mu}{\sin \frac{\theta}{2}}.$$

The angle θ for grooved pulleys is commonly taken as 45 deg. If μ is taken as 0.135 and θ as 45 deg., μ' is 0.353. If the constant K be determined from Table XI for a coefficient of friction of 0.35 and an arc of contact of 180 deg., the coefficient K is about $\frac{2}{3}$. This indicates that formula (44) may be looked upon as being based upon a coefficient of friction of 0.135, a 45-deg. groove, and a 180-deg. arc of contact.

The data given in Table XIII, taken from the Blue Book of the American Manufacturing Company of Brooklyn, New York, will be found useful.

Example.—How many manila ropes $1\frac{1}{2}$ in. in diameter are necessary to transmit 415 hp. at a velocity of 3,500 ft. per minute?

From formula (44) and Table XII, the horsepower for a 1-in. diameter rope is:

$$\text{hp.} = 0.00002V(200 - c'') = 0.00002 \times 3{,}500(200 - 35.4) = 11.5.$$

From Table XIII a $1\frac{1}{2}$-in. rope is equivalent to 2.25 1-in. ropes, hence:

$$\text{hp.} = 11.5 \times 2.25 = 25.9,$$

and

$$\frac{415}{25.9} = 16.02, \text{ say, sixteen } 1\tfrac{1}{2}\text{-in. ropes.}$$

The curves in Fig. 34 show the relation between horsepower transmitted and velocity in feet per minute, plotted from formula (44). They will be found useful for solving and checking problems.

331.—Rope Sheaves. The grooves in the pulleys are made narrow at the bottom, and as it bends around the sheave, the rope is pinched between the edges of the V-groove to increase the holding power of the rope on the pulley. The U-groove is used for

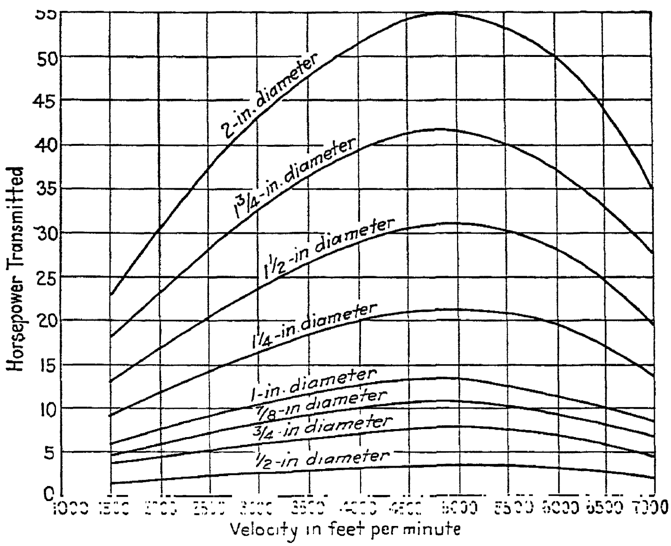


FIG. 34.—Horsepower transmitted by ropes.

idlers and tension pulleys, the rope riding at the bottom of the groove. The grooves should be finished smooth to avoid chafing of the rope. The diameter of the sheaves should be large to reduce the wear on the rope due to internal friction and bending

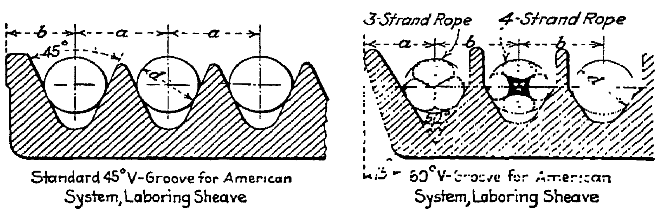


FIG. 35.—Sheave wheel grooves.

stresses. Manufacturers seem to be agreed upon 40 rope diameters as the proper size for sheave wheels, with 36 diameters as a minimum size. The standard 45-deg. V-groove is shown in Fig. 35. Referring to Fig. 35, the proportions of the grooves

are given in Table XIV, a being the distance from center to center of grooves, b being the distance from the first and last grooves to the edge of the sheave, and d being the diameter of the rope.

TABLE XIV.—DIMENSIONS OF GROOVES IN ROPE SHEAVES

Standard groove, English system	Diameter of rope in inches					
	1	1 1/8	1 1/4	1 1/2	1 3/4	2
a	1 1/2	1 3/4	1 7/8	2 1/4	2 1/2	2 3/4
b	1 1/8	1 1/4	1 7/16	1 11/16	1 7/8	2 1/16
45-deg. V-groove, American system						
a	1 1/2	1 11/16	1 7/8	2 1/4	2 1/2	2 3/4
b	1 1/8	1 1/4	1 3/8	1 5/8	1 13/16	2

The Dodge Manufacturing Corporation uses a V-groove with sides inclined at 60 deg., and the groove proportions are slightly less than those shown in the table for the 45-deg. groove.

It is important that all grooves have the same proportions to allow the ropes to assume the same position in the grooves. This means also that the stretch of the rope must be uniform so that the diameter will be the same throughout the length.

The design of rope-drive sheaves is similar to ordinary pulley design, and, usually, the design engineer who is dealing with problems of a general nature should specify the standard equipment which is available. Rope sheaves are manufactured in stock sizes from 24 to 54 in. in diameter, increasing by 2-in. increments. All sheaves are available in solid or split form with the number of grooves varying up to 24 as a maximum.

332. Wire-rope Transmission.—The general introduction of electrical transmission has retarded the extension of wire-rope drives, which had been used for transmitting power over distances of 100 ft. to 1 mile. Wire rope, because of its great tensile strength, will carry a heavy load with a relatively small loss. Wire-rope transmissions are low in first cost and will function in all weathers. Wire rope is flexible enough to allow it to run over sheaves which are made seventy-two to eighty times the diameter of the rope. The rope rides on the bottom of the grooves, which are usually padded with wood or hard-rubber blocks to avoid excessive wear on the rope.

333. Tex-rope Drives.—Tex rope[1] is a trade name for an endless belt which has many applications for the transmission of power when the distance between pulleys is small. The belt is trapezoidal in cross-section, and is made of rubberized cotton fabric and cord. The V-shaped belt runs in a grooved sheave, and by using a number of belts, relatively large amounts of power may be transmitted. These belts tend to absorb the shocks due to suddenly applied loads, have great frictional resistance due to their form, are noiseless, have a very small amount of slip, and may be used with pulley ratios as high as 7 to 1. Figure 36 shows a nine-rope drive, which illustrates the compactness of this form of transmission.

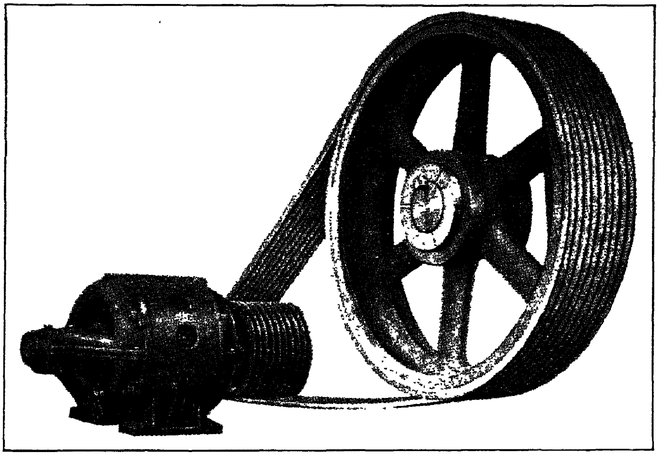


FIG. 36.—150 horsepower Tex-rope drive.

Tex ropes are manufactured in five sizes as shown in Fig. 37. The curves shown in this figure are plotted from the manufacturer's tables, but the formula for rope drives may be used to determine the horsepower which a Tex rope can transmit. Using formula (43):

$$\text{hp.} = 0.00002V(T_1 - c''),$$

in which hp. denotes the horsepower transmitted by 1 sq. in. of rope.

V denotes the velocity of the rope, in feet per minute.

T_1 denotes the tension on the tight side of the rope, in pounds.

[1] Manufactured by Allis-Chalmers Mfg. Co., Milwaukee, Wis.

c'' denotes the tension in the rope due to centrifugal force.

In the above formula, T_1 may be taken as 330 lb. for Tex rope of unit area. Since the factor c'' in Table XII is based upon a weight w' which was 0.028, and since w' is 0.04 lb. for a Tex rope 1 in. long and 1 in. in cross-section, c'' in the above formula may be obtained by multiplying the values in Table XII by 1.43.

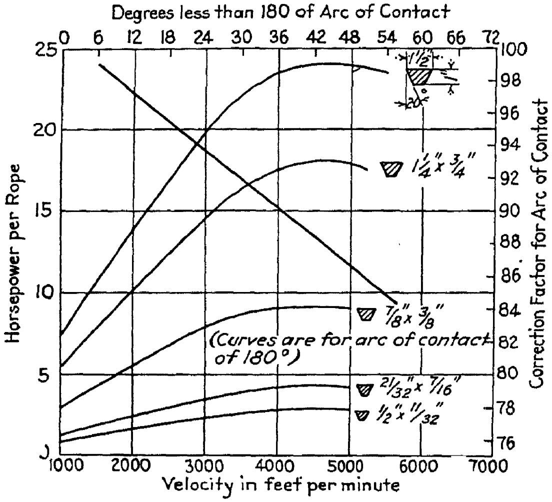


FIG. 37.—Horsepower transmitted by Tex ropes.

Problems

Two cast-iron friction wheels like those shown in Fig. 1 are used to transmit 2 hp. from one shaft to the other. The small wheel is the driver, is making 250 r.p.m., and is faced with leather fiber. The distance between shafts is 13 in. and the diameter of the large wheel is 18 in.

(a) What is the diameter of the small wheel?

(b) What is the speed of the large wheel?

(c) What is the bearing reaction at each bearing if the bearings are close to the wheels and the bending action of the shaft is neglected?

2 A brush plate and wheel like that shown in Fig. 2 are employed on a drill press for a variable speed drive. The larger wheel is keyed to the drill spindle and its minimum speed is 110 r.p.m. The diameter of the driver is 4 in., is faced with leather, and is turning at 200 r.p.m. Determine the value of the radius R.

3. The brake shown in Fig. 5 has the following proportions: $a = 12$ in., $b = 4$ in., and the diameter of the drum is 12 in. The center K is in a line tangent to the drum. What horsepower is absorbed by the brake

if the force W is 40 lb., and the drum is turning at 150 r.p.m.? Assume that the drum is made of cast iron, and that the wood blocks are poplar.

4. The smaller of a pair of grooved cast-iron friction wheels has a mean diameter of 4 in., has 4 grooves, and is turning at the rate of 165 r.p.m. The angle of the grooves is 25 deg. What horsepower will be transmitted if the reaction at the bearings is 200 lb. per square inch, and bending of the shaft is neglected? Assume the journals to be $1\frac{1}{2}$ by $2\frac{1}{2}$ in.
5. A band brake similar to the one shown by Fig. 8 has the following proportions: $a = 14$ in., $b = 3$ in., diameter of drum $= 10$ in., $\alpha = 200$ deg., and $\mu = 0.30$. What force will be required at the lever handle to absorb 5 hp. at 300 r.p.m.? The motion of the drum is counterclockwise.
6. A differential band brake similar to that shown by Fig. 10 has the following proportions: $a = 20$ in., $b_1 = 4$ in., $b_2 = 4$ in., $\alpha = 250$ deg., and $\mu = 0.35$. The force at the handle is 20 lb.
   (*a*) What horsepower is being absorbed by the drum which is 16 in. in diameter, when it is turning clockwise at the rate of 165 r.p.m?
   (*b*) What load will this brake hold suspended by a crane hook if the hoisting mechanism is 85 per cent efficient, and the mechanical advantage of the gears and hoisting drum is 24?
7. Show by means of a sketch and give all dimensions for a differential brake which will exert a torque of 4,000 in.-lb. The following data may be taken: $\alpha = 240$ deg., $\mu = 0.20$, $P = 25$ lb., and the diameter of the cast-iron drum is 10 in. The brake band is lined with leather and is slightly oily.
8. A Prony brake similar to the one shown in Fig. 11(*a*) is to be designed, which will be capable of measuring the output of a 125 hp. engine at 250 r.p.m. The belted flywheel is 12 ft. in diameter, and the weight on the scales is limited to 500 lb. The face of the brake pulley is 7 in., the poplar friction blocks are 3 in. wide, the friction surfaces are slightly oily, and the brake is water-cooled. Make a sketch and show all important dimensions.
9. An automobile dry-plate clutch is capable of transmitting 40 hp. when turning at 1,200 r.p.m. The maximum diameter of the disks is 9 in., and the inside diameter is $7\frac{1}{2}$ in. The surfaces are steel plates in contact with asbestos-lined steel plates. Assuming the mean pressure between the friction surfaces as 30 lb. per square inch, determine:
   (*a*) The number of disks required.
   (*b*) The pressure needed to disengage the clutch if the ratio of the pedal lever arm to the spring lever arm is $13\frac{1}{2}$ to 1.
10. A disk clutch is made up of eight pairs of contact surfaces formed by alternate brass and steel disks running in a bath of oil. The outside diameter of the surfaces is 10 in., and the inside diameter is 8 in. Assuming $\mu = 0.13$, and the axial force pressing the disks together is 160 lb., determine the horsepower transmitted at 650 r.p.m.
11. A leather-faced cone clutch transmits 15 hp. at a speed of 750 r.p.m. The cone angle is the standard $12\frac{1}{2}$-deg. inclination, the face width is 3 in., and μ is 0.25. What is the mean diameter of the clutch if the bearing pressure is 7 lb. per square inch?

12. A cast-iron cone clutch has a total angle of 19 deg. 20 min., and the friction surface is 2 in. wide. The mean diameter of the engaging surfaces is 18 in., and μ is 0.15.
(*a*) What horsepower will be transmitted at 175 r.p.m. if the normal pressure is 20 lb. per square inch?
(*b*) What spring pressure is necessary to transmit the above power?
13. A radial expanding clutch has four shoes of poplar wood, each 3 in. wide and 4 in. long, pressing against the inside of a cast-iron rim of 20 in. inside diameter, with a force of 30 lb. per square inch. The clutch has a maximum speed of 550 r.p.m. The clutch was installed 50 per cent oversize because of its frequent operation. Determine the capacity of the clutch.
14. A milling machine requires a 3 hp. motor to operate under maximum conditions. When the belt is driving the smallest pulley on the cone, the arc of contact is 165 deg. The fastest speed of the cone pulley on the machine is 215 r.p.m., and the smallest pulley is 8 in. in diameter. Assuming t_1 is 350, determine the width of a single-ply leather belt to drive the machine.
15. A belt which is ⅜ in. thick and 8 in. wide is running over a 60-in. and a 54-in. pulley. The 60-in. pulley is making 180 r.p.m. Assuming $\mu = 0.4$, and $t_1 = 300$, determine the horsepower which this belt will transmit.
16. A line shaft is to be connected to the driving motor by a two-ply leather belt. The pulley on the motor is running at 900 r.p.m., is 16 in. in diameter, and the pulley on the line shaft should reduce the speed in the ratio of 4 to 1. The diversity factor for this shop is 0.60, and by count the total maximum horsepower on the line shaft if all the machines were operating simultaneously is 68. Taking $t_1 = 300$ lb., $\mu = 0.45$, and α for the smaller pulley as 160 deg., determine:
(*a*) The width of the best grade belt which is required.
(*b*) The width and diameter of the line shaft pulley.
17. A five-ply 8 in. rubber belt is to be replaced by a high-grade leather belt. The operating conditions are as follows: the diameter of the driving pulley is 24 in., the diameter of the driven pulley is 36 in. and revolves at 180 r.p.m., and the distance between pulley centers is 18 ft. 6 in.
(*a*) Assume all other data and determine the width and thickness of the leather belt.
(*b*) If the life of the leather belt is double that of the rubber belt, and if the cost is . . . . and . . . . per foot, respectively, determine the saving, if any, effected by the replacement. Use catalogue prices for the cost of belting.
18. A three-ply leather belt, 30 in. wide, and traveling at the rate of 1 mile per minute, is to be replaced by a rope drive made up of 1 in. ropes. How many ropes are necessary?
19. (*a*) How many 1¼-in. ropes are required to transmit 175 hp. at the most efficient speed? (*b*) How wide should the sheaves be if the diameter of the driving pulley is 14 ft. and the diameter of the driven pulley is 60 in.? (*c*) What is the speed of each pulley in revolutions per minute? (*d*) With rope costing $0.05 per pound and the weight of a 1¼-in. diameter

rope equal to 50 lb. per 100 ft., determine the cost of the rope, allowing 40 ft. for the splice and tension take-up, and assuming a distance of 60 ft. between pulleys.

20. Design a Tex rope which will drive a 90-hp. air compressor. The data for the installation are as follows: motor pulley makes 600 r.p.m., and compressor pulley 200 r.p.m. Distance between centers is a minimum.

CHAPTER XV

DESIGN OF VARIOUS MACHINE ELEMENTS

SPRINGS

334. There are various applications of springs to structures and machines, because it is possible to control the distorsion of the spring under a known load. Springs may be used to absorb energy, as in railway cars and in automobiles; to store energy, as in clocks and other spring motors; to measure forces, as in spring balances; to hold machine parts in place, as in cams and their followers; and to produce normal pressure in friction devices, as in automobile clutches. The two types of springs most commonly used are *helical* springs and *laminated* springs.

335. Helical Springs.—The helical spring is commonly made by wrapping a rod of circular cross-section around a cylindrical surface, thus forming the axis of the rod into a helix, as shown in Fig. 1.

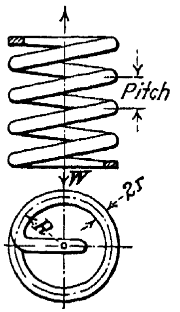


FIG. 1.—Helical spring.

If the spring is supporting an axial load W, and the mean radius of the coil is R, then it is evident that every cross-section of the coil is subjected to a twisting moment of WR. The stress produced on the cross-sections is a torsion stress and its value may be found from the torsion formula:

$$WR = \frac{S_s J}{r} \tag{1}$$

For a rod or wire of circular cross-section:

$$S_s = \frac{2WR}{\pi r^3}, \qquad \frac{TR}{I_p} \qquad \frac{Tr}{\pi r^4} = \frac{2T}{\pi r^3} \tag{2}$$

in which W denotes the axial load on the spring, in pounds.

R denotes the mean radius of the coil, in inches.

r denotes the radius of the wire or rod, in inches.

J denotes the polar moment of inertia of the wire or rod cross-section, in in.^{4}

S_s denotes the unit shear stress in the wire or rod, in pounds per square inch.

The above analysis may be used for closely coiled springs in which the obliquity of the helix may be neglected. When the coils are open so that the obliquity is appreciable, there will be produced a bending moment as well as a twisting moment.[1]

To determine the deflection of a close-coiled spring, consider the action as the twist of a wire of a length equal to the length of the spring, which permits the load W to descend. The angle of twist under torsion, from formula (19), Chap. VII is:

$$\theta = \frac{T_m L}{E_s J} \text{ in radians.}$$

The amount d, which the load W will descend, is $R\theta$, and the length L is the length of the spring, or the number of coils n multiplied by $2\pi R$. Hence:

$$d = \frac{R \times WR \times n \times 2\pi R \times 2}{E_s \times \pi r^4} = \frac{4WR^3 n}{E_s r^4} \qquad (3)$$

The formula shows that for a given load and number of coils the deflection varies as the cube of R. A large value of R therefore means a weak, flexible spring, and a small value of R means a strong, stiff spring.

336. Design of Helical Springs.—It is well to use a rod or wire of the smallest diameter, consistent with the other requirements, since the wires of smaller diameter have a higher elastic limit. The torsional modulus of elasticity, however, is constant, and may be taken as 12,000,000 lb. per square inch for steel and 5,000,000 lb. per square inch for brass. From the standpoint of strength it can be shown that a square wire with a side equal to the diameter of a round wire is 6 per cent stronger, but has in it 22 per cent more material. From the standpoint of economy, therefore, it is usually desirable to use a round cross-section.

From tests made by E. T. Adams at Sibley College on governor springs made from steel wire, varying in diameter from $\frac{3}{8}$ to $\frac{3}{4}$ in., it was found that the maximum safe stress for such springs was as follows:

$$S_s = 40,000 + \frac{7,500}{} \qquad (4)$$

[1] MAURER and WITHEY, "Strength of Materials," p. 274.
MORLEY, "Strength of Materials," p. 291.

Example.—A helical spring is 4 in. outside diameter, and has 8 effective coils of ½ in. round wire. The stress should not exceed 63,000 lb. per square inch. What deflection may be expected with a load of 1,000 lb.?

From formula (3):

$$d = \frac{4 \times W \times R^3 \times n}{E_s \times r^4} = \frac{4 \times 1{,}000 \times 1.75^3 \times 8}{12{,}000{,}000 \times 0.25^4} = 3.66 \text{ in.}$$

From formula (2), the unit stress will be:

$$S_s = \frac{2 \times 1{,}000 \times 1.75}{\pi \times 0.25^3} = 71{,}300 \text{ lb. per square inch.}$$

This unit stress seems to be excessive. Using formula (4):

$$S_s = 40{,}000 + \frac{7{,}500}{0.25} = 70{,}000 \text{ lb. per square inch.}$$

This value indicates that the spring should probably not be subjected to a load as great as 1,000 lb.

Example.—Using wire of circular cross-section, design a helical spring which will deflect 4 in. under a load of 1,000 lb. The wire is made of steel and the unit stress shall not exceed 60,000 lb. per square inch.

Substituting in formulas (2) and (3):

$$60{,}000 = \frac{2 \times 1{,}000 \times R}{r^3}.$$

$$4 = \frac{4 \times 1{,}000 \times R^3 n}{12{,}000{,}000 \times r^4}.$$

It is desirable to have a whole number for n, and convenient standard dimensions for r and R. Assuming $r = \frac{1}{4}$ in.:

$$R = \frac{60{,}000 \times \pi \times 0.25^3}{2 \times 1{,}000} = 1.47 \text{ in.}$$

Use 1.5 in. and solve for n:

$$n = \frac{4 \times 12{,}000{,}000 \times 0.25^4}{4 \times 1{,}000 \times 1.5^3} = 13.9 \text{ coils.}$$

Use 14 coils. The outside diameter of the coils will be 3.5 in., and the spring may be formed on an arbor 2.5 in. in diameter.

337. Length, Free Height, and Solid Height of Helical Springs.—The length of either a tension or compression spring is approximately:

$$L = 2\pi Rn. \tag{5}$$

The height of a compression spring when free or unloaded is approximately:

$$H = h + 0.02\text{h}\left(\frac{R}{r}\right)^2. \tag{6}$$

This formula is based upon the assumption that steel is the material used, and that the opening between coils will be closed up at a unit stress of 75,000 lb. per square inch.

The height when loaded with W, the load which will compress the spring solidly, is:

$$h = \frac{H}{1 + 0.02\left(\frac{R}{r}\right)^2}. \tag{7}$$

Combining formulas (2) and (3), it can be shown that the volume of a spring varies as the product of the load and deflection for given values of E_s and S_s:

$$\text{Volume} = 2Wd\frac{E_s}{S_s^2}. \tag{8}$$

The weight of a spring is, of course, directly proportional to the volume.

338. Data on Helical Springs.—It is evident from the above discussion that when only the load W and the deflection d are specified, there are a large number of springs which might meet the requirements of a design.

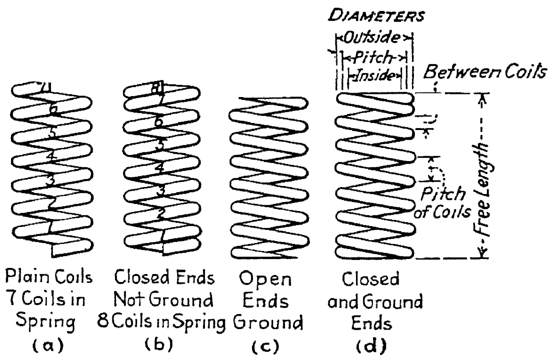


FIG. 2.—Compression springs.

High-class compression springs have the ends ground off square with the axis of the spring, as shown by Figs. 2(c) and 2(d), and the height should not be more than three times the outside diameter of the coil. Tension springs with end hooks and coils are shown in Figs. 3(a), 3(b), and 3(c). Steel springs are often japanned and baked to protect the material from corrosion. Springs are heat-treated after forming.

Some of the special alloy steels used for springs, such as vanadium steels, have elastic limits ranging from 180,000 to 225,000 lb. per square inch.

Brass is used for springs which must resist corrosion from moisture. These springs are more expensive than those made of steel, not only because brass weighs and costs more, but also because the permissible unit stress for brass is smaller, and the springs must therefore be larger for the same capacity.

Phosphor-bronze wire used in helical springs may have a maximum allowable unit stress ranging from 30,000 to 40,000 lb. per square inch.

Springs which act occasionally may carry a stress close to the elastic limit of the material, but when the push or pull is fre-

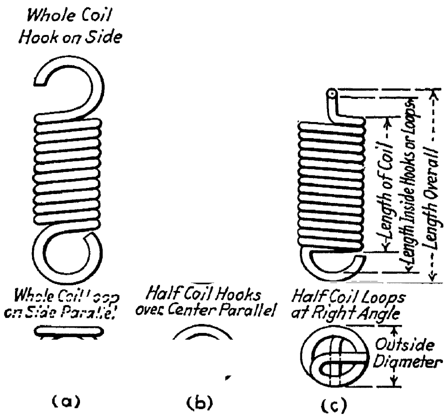


FIG. 3.—Tension springs.

quently repeated, a factor of safety must be used. The maximum unit torsional stress in a steel spring, closing the valve in a gas engine, should not exceed 40,000 lb. per square inch, which means that it should have a factor of safety of at least three.

339. Single-leaf Springs.—Flat springs consisting of a single leaf may be used as cantilever beams, or as beams with more than one support. The unit stresses which result will be tensile on one outer fiber and compressive on the other. Such springs may be designed by means of the beam theory given in Chap. VII, and they may be of uniform cross-section throughout, or may be designed as beams of approximately uniform strength.

The deflection formulas for beams of uniform strength and rectangular cross-section, are given in Table I for some of the common cases.

TABLE I

Kind of beam and loading	Maximum deflection, inches
1. Simple beam, supported at the ends. Concentrated load at center. Constant breadth, varying depth.	$d = \frac{P}{2bE}\left(\frac{L}{h}\right)^3$
2. Simple beam, supported at the ends. Concentrated load at center. Constant depth, varying breadth.	$d = \frac{3P}{8bE}\left(\frac{L}{h}\right)^3$
3. Cantilever beam. Concentrated load at free end. Constant breadth, varying depth.	$d = \frac{8P}{bE}\left(\frac{L}{h}\right)^3$
4. Cantilever beam. Concentrated load at free end. Constant depth, varying breadth.	$d = \frac{6P}{bE}\left(\frac{L}{h}\right)^3$

In the above table d denotes the maximum deflection, in inches.
P denotes the load on the beam, in pounds.
E denotes the modulus of elasticity in tension or compression, in lb. per square inch.
b denotes the maximum breadth, in inches.
h denotes the maximum depth, in inches.
L denotes the length of the beam, in inches.

Example.—A flat steel spring is to carry a load of 500 lb. and is to deflect 1 in. The length of the spring is 24 in., the maximum unit stress shall not exceed 50,000 lb. per square inch, the spring is supported at the ends and loaded with a concentrated load at the middle, and is to be of constant depth and varying breadth.

The material will be steel having a modulus of elasticity of 30,000,000 lb. per square inch.

From the stress formula for beams:

$$S = \frac{Mc}{I} \text{ and } M = \frac{PL}{4}.$$

$$50{,}000 = \frac{500 \times 24 \times 6}{4bh^2}.$$

From Table I:

$$d = \frac{3P}{8bE}\left(\frac{L}{h}\right)^3.$$

$$1 = \frac{3 \times 500 \times 24^3}{8 \times b \times 30{,}000{,}000 \times h^3}.$$

Solving for the breadth b in each case and putting these values equal to each other:

$$\frac{500 \times 24 \times 6}{4 \times 50{,}000 \times h^2} = \frac{3 \times 500 \times 24^3}{8 \times 30{,}000{,}000 \times h^3}.$$

$$h = 0.24 \text{ in.}, \quad b = 6.25 \text{ in.}$$

The spring would have a maximum breadth of 6.25 in. at the center, tapering to a point at the ends, and the constant depth would be approximately $\frac{1}{4}$ in.

340. Laminated Springs.—It is possible to design springs using several plates, and thus obtain a given deflection for a certain load without having excessive maximum breadth of spring.

Figure 4 represents a laminated spring having five plates. The plates are assumed to have been cut from a single diamond-shaped plate, shown in Fig. 5, which is a beam of uniform strength with constant depth and varying breadth. The spring in Fig. 4 will therefore be a beam of approximately uniform strength whose maximum deflection and unit stress will be the same as for the plate in Fig. 5.

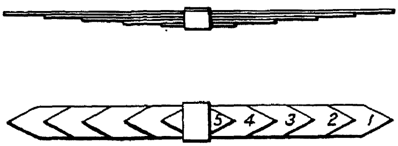


FIG. 4.

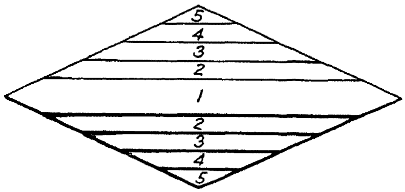


FIG. 5.

Laminated spring.

The formulas for unit stress and deflection in terms of the new breadth and number of plates are:

$$S = \frac{3PL}{2nb'h^2}, \tag{9}$$

$$d = \frac{3PL^3}{8Enb'h^3}, \tag{10}$$

in which n denotes the number of plates or leaves in the spring.
b' denotes the breadth of the laminated spring, in inches.

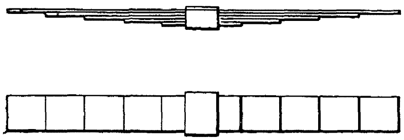


FIG. 6.—Spring with square ends.

Taking the spring designed in the preceding section and replacing it with five plates, the new width would be 6.25 in. divided by 5, or 1.25 in., which would make a much more practical design.

In laminated springs as actually used, the full-length leaf must usually have a square end by means of which it can be fastened to its supports. Sometimes the shorter leaves are also cut square at the ends, as shown in Fig. 6. This change makes

no error as far as formula (9) is concerned, but the deflection as given by formula (10) might not be quite correct, because the ends of the leaves are not pointed. The ends of the shorter leaves are sometimes rounded and made thinner so as to approximate the pointed condition. Figure 7 shows an automobile spring with the top leaf bent at the ends to form shackles.

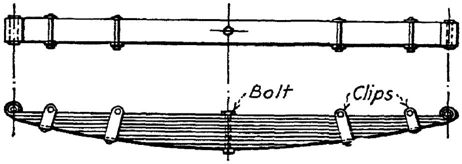


FIG. 7.—Automobile spring.

J. B. Peddle[1] has shown that the formulas for the deflection of full elliptic leaf springs is as follows:

$$d = \frac{4L^2S}{tE}K, \tag{11}$$

and for semi-elliptic springs:

$$d = \frac{2L^2S}{tE}K. \tag{12}$$

The notation is the same as before, and K is a factor which depends on the ratio r, which is the ratio of the number of full length leaves to the total number of leaves, that is:

$$r = \frac{\text{number of full-length leaves}}{\text{total number of leaves}} \tag{13}$$

$$K = \frac{1}{(1-r)^3}\left[\frac{1-r^2}{2} - 2r(1-r) - r^2 \log_e r\right]. \tag{14}$$

In Fig. 8(a) the width w, where the leaves are held together by a band, is neglected, and the length of the spring L is taken as indicated. Peddle points out that it is necessary to have the points of the shortened leaves tapered in width or in thickness, or both, as shown in Fig. 8(b), so as to make the transition from one leaf to the next, gradual.

The unit stress S to be used in formulas (11) and (12) may be calculated from the following formula, the load P being the one indicated in Fig. 8(a).

$$S = \frac{3LP}{nbh^2}. \tag{15}$$

[1] *American Machinist*, p. 645, Apr. 17, 1913.

It is evident from formulas (11) and (12) that the deflection of the full elliptic spring for the same load is double that of the semi-elliptic spring.

The leaves of a laminated spring are often given an initial curvature, so that they will tend to straighten under the load, and be straight when fully loaded. When this is done the deflection due to full load would determine the curvature of the plates.

It will be noted that the full-length plate is a beam of constant cross-section, while the shorter ones are not, with the result that the unit stress in the long plate will be greater than in the short plates. To overcome this defect E. R. Morrison[1] has suggested

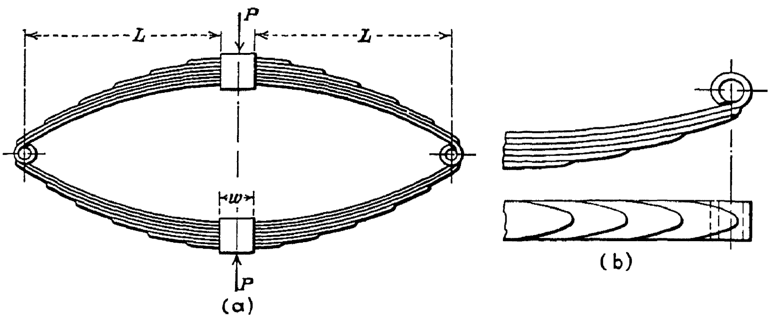


FIG. 8.—Elliptical spring.

that the shorter plates be given an additional initial curvature, equal to the difference in deflection which might be expected between the stiffer long plate and the shorter plates. After assembling the spring, the short plates would then have an initial stress, which, in addition to the stress induced in them by the load, would cause a uniformity of stress in the spring unit.

341. Data on Leaf Springs.—Figure 8(*a*) shows a full elliptic spring, so called on account of its form. A semi-elliptic spring would mean the upper or lower half of the spring shown in the figure. Springs with rounded ends for the shorter leaves are used in automobiles, and springs with square ends are used for locomotives.

Springs subjected to suddenly applied loads are often provided with rubber blocks to prevent metal-to-metal shock, while the rebound is absorbed by snubbers of various designs.

[1] *Machinery*, January, 1910.

A high-grade alloy steel is used in the manufacture of elliptic springs, and the heat treatment of the plates is an important part of the process of manufacture.

The maximum unit stress allowed in springs depends upon the thickness of the plates. It is usually taken as 100,000 lb. per square inch for $\frac{3}{16}$-in. plates, and 80,000 lb. per square inch for $\frac{3}{8}$-in. plates.

The rational design of all classes of springs should be checked by performance tests. This is especially true if the design contemplates the use of springs which are to function accurately with respect to load and deflection.

FLYWHEELS

342. General Considerations.—Flywheels are used on machines to prevent sudden changes in speed due to the intermittent reception of energy, or due to the variable rate at which work is being done by the machine. The flywheel of a gas engine tends to stabilize the energy which is received by the periodic explosion of the gas in the cylinders. The flywheel of a punch press prevents a sudden change in the speed of the machine due to the intermittent rate at which the punch operates.

Flywheels which carry belts run at relatively high speeds, to take advantage of the increased efficiency of belts when operating at high velocities. Such wheels have wide, shallow rims to accomodate the belt. When the flywheel is not to be used as a belt wheel, which is the usual case, the rim is rectangular in form, and is ordinarily proportioned with the thickness of the rim greater than the width.

Flywheels up to 10 ft. in diameter are usually cast in one piece, and although the initial stresses in the arms and rim due to cooling strains are uncertain, experiments[1] have demonstrated that these wheels are satisfactory for all ordinary purposes. Flywheels from 10 to 16 ft. in diameter are cast in two parts, and still larger wheels are cast in segments with one arm to each segment.

Flywheels have rarely failed due to any other cause than excessive rim stresses, therefore the correct design of the rim is far more important than any other consideration.

343. Energy Absorbed and Given Up by Flywheels.—When a flywheel absorbs energy its speed increases, and when it gives

[1] BENJAMIN, C. H., *Trans. A. S. M. E.*, Vol. XX, p. 209.

up energy its speed decreases. The total energy which a flywheel is capable of giving up is the amount which will bring the flywheel to rest. The total energy is expressed by the formula:

$$E = \tfrac{1}{2} I \omega^2, \tag{16}$$

in which E denotes the total energy of the flywheel, in foot-pounds.

I denotes the moment of inertia of the flywheel with respect to the axis of rotation, in slug ft.^{2}.

ω denotes the angular velocity of the flywheel, in radians per second.

Formula (16) is usually simplified by neglecting the moment of inertia of the hub and arms, and considering only that of the rim. Calculations on large flywheels have shown that about 65 per cent of the weight is in the rim, and about 35 per cent in the hub and arms. Because the hub and arms are nearer to the axis of the wheel than is the rim, the difference between the energies supplied by the rim and by the hub and arms, will be much greater than the weight percentages given above. The formula then becomes:

$$E = \frac{Wv^2}{2g}, \tag{17}$$

in which W denotes the weight of the rim, in pounds.

v denotes the linear velocity of a point on the rim at a mean radius, in feet per second.

g denotes the acceleration of gravity, in feet per second per second.

As the speed of the flywheel changes, the energy which is stored or given up is proportional to the difference between the squares of the initial and final speeds. The formula is:

$$E = \frac{W(v_1^2 - v_2^2)}{2g}, \tag{18}$$

in which E denotes the energy which is given up or absorbed due to the change in speed, in foot-pounds.

W denotes the weight of the flywheel, in pounds.

v_1 denotes the maximum linear velocity of a point on the rim at a mean radius, in feet per second.

v_2 denotes the minimum linear velocity of a point on the rim at a mean radius, in feet per second.

344. Flywheel Design.—The amount of energy which a flywheel must supply may be determined as follows:

(*a*) For machines which receive energy at a variable rate, it is necessary to determine the average energy supplied during each cycle, and then design the wheel to supply the difference between the average energy supplied and the maximum energy required. The indicator card of a steam or gas engine will furnish the data for laying out an energy diagram, from which the average energy supplied may be found. The maximum energy required depends upon the work which the machine does during a cycle.

(*b*) For machines whose source of energy is constant, but which do work at a variable rate, formula (18) will show the energy to be supplied by the flywheel, so that the speeds may be kept within certain limits.

345. Allowable Variation in Speed.—Experience has shown that variation of flywheel speeds should be kept within certain limits, and these limits are quite different for the several classes of service in which flywheels are employed. The ratio of the total variation in speed to the normal speed is called the *coefficient of fluctuation*, and may be expressed by the following formula:

$$k = v_1 - v_2$$

The usual range for this coefficient is given in Table II.

TABLE II.—COEFFICIENT OF FLUCTUATION OF FLYWHEEL SPEEDS

Type of machine or class of service	Coefficient of fluctuation, k
Air compressors	0.10 to 0.15
Electric machines (alternating current)	0.0003 to 0.003
Electric machines (direct current)	0.0065 to 0.0075
Punching, shearing, and power presses	0.10 to 0.15
Pumping machinery	0.03 to 0.05
Paper making, textile, weaving, and machine tools	0.025 to 0.030
Spinning machinery	0.015 to 0.025
Rolling mills and mining machinery	0.02 to 0.03

Example.—Determine the cross-section of the rim for the flywheel of a punching and shearing machine. The largest hole to be punched is 1 in. in diameter in a steel plate ½ in. thick. The maximum speed of the flywheel is 200 r.p.m., and because of the location of the shaft, the wheel may not be larger than 42 in. in diameter.

The force required at the punch depends to some extent upon the clearance between the punch and die. American practice assumes that the force required at the punch is 25 per cent larger than that found by the ordinary formula. The force required to punch a 1-in. hole in a ½-in. plate is:

$$F = \pi \times 1 \times 0.50 \times 50{,}000 \times 1.25 = 98{,}100 \text{ lb.}$$

The maximum punching effort will be at the moment when the punch begins to penetrate the metal. Experience has shown that when the punch has penetrated one-third of the plate thickness, the slug is practically free. Therefore, the work done by the punch is:

$$\text{Work} = 98{,}100 \times \tfrac{1}{3} \times \tfrac{1}{2} \times \tfrac{1}{12} = 1{,}360 \text{ ft.-lb.}$$

Assuming the machine to be 85 per cent efficient, the energy at the flywheel shaft must be:

$$E = \frac{1{,}360}{0.85} = 1{,}600 \text{ ft.-lb.}$$

The greatest permissible slowing up of the flywheel shaft is 15 per cent (see Table II).

Taking a mean diameter of 36 in.:

$$v_1 = \frac{\pi \times 36 \times 200}{12 \times 60} = 31.4 \text{ ft. per second.}$$

$$v_2 = 0.85 \times 31.4 = 26.7 \text{ ft. per second.}$$

From formula (18):

$$1{,}600 = \frac{W(31.4^2 - 26.7^2)}{2 \times 32.2},$$

$$W = 377 \text{ lb.}$$

Neglecting the effect of the hub and arms, the rim section will be:

$$b \times t \times \pi \times 36 \times 0.26 = 377.$$

$$bt = \frac{377}{29.4} = 12.8 \text{ in.}^2.$$

If b is taken as $3\frac{1}{2}$ in., $t = \dfrac{12.8}{3.5} = 3.66$ in., say, $3\frac{11}{16}$ in.

If the mean radius is taken as the sum of the inside and outside radii divided by two:

$$r_2 = 18 + \frac{3.69}{2} = 19.84 \text{ in.}$$

$$r_1 = 18 - \frac{3.69}{2} = 16.15 \text{ in.}$$

The diameters might be taken as $d_2 = 39\frac{5}{8}$ in., and $d_1 = 32\frac{5}{16}$ in.

The mean radius involved in formula (18) is the radius of gyration, which is $\dfrac{r_1^2 + r_2^2}{2}$ for a hollow cylinder. Using this formula, the computed value of r_1 was practically the same as that calculated in the above problem, so that the approximation used above is sufficiently correct.

346. Stresses in Flywheel Rim.—The safe speed for flywheels is dependent upon:

(*a*) The tensile stress which is induced by the centrifugal force.

(*b*) The tensile bending stress which is caused by the restraint of the arms.

(*c*) The localized stresses which are present in the casting due to the unequal rate of cooling. These stresses may be very high, but there is no easy method of arriving at their values. The presence of these stresses is acknowledged by the designer by the allowance of a reasonably large factor of safety in design.

347. Centrifugal Stress.—Considering the centrifugal tension alone, the rim is assumed to be a ring which is unrestrained by the arms. The ring is a thin cylinder which is subjected to an internal pressure as indicated by Fig. 9. The force which is pushing against each square inch of rim surface at the mean radius, tending to cause rupture at A and B is:

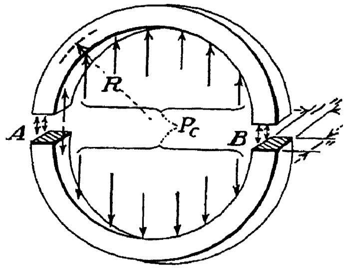


FIG. 9.

$$p_c = \frac{wtv^2}{Rg}, \tag{19}$$

in which p_c denotes the centrifugal force per unit area at the mean radius, in pounds per square inch.

w denotes the weight of the material per cubic inch, in pounds.

t denotes the thickness of the rim, in inches.

R denotes the mean radius of the rim, in feet.

g denotes the acceleration due to gravity, 32.2 ft. per second per second.

v denotes the linear velocity of the rim at the mean radius, in feet per second.

The tensile stress which is induced in the rim due to the centrifugal force p_c is, from formula (1), Chap. VIII:

$$S_c = \frac{p_c d}{2t}. \tag{20}$$

Substituting $24R$ for d, and for p_c from formula (19):

$$S_c = \frac{wtv^2 24R}{Rg2t} = \frac{12wv^2}{g} \tag{21}$$

The weight per cubic inch for cast iron and steel is 0.26 and 0.28 lb., respectively. Substituting these values in formula (21):

$$S_c = \frac{v^2}{10.3} \text{ for cast iron.} \tag{22}$$

$$S_c = \frac{v^2}{9.6} \text{ for steel.} \tag{23}$$

Both of these formulas may be taken approximately as:

$$S_c = \frac{v^2}{10}. \tag{24}$$

348. Bending Stress.—The determination of the unit tensile stress in the rim, due to the partial restraint of the arms, is based upon the assumption that each section of the rim which lies between a pair of arms behaves like a beam, fixed at the ends and uniformly loaded. Figure 10 indicates the condition of loading assumed. The maximum bending moment will be at the arms, so that:

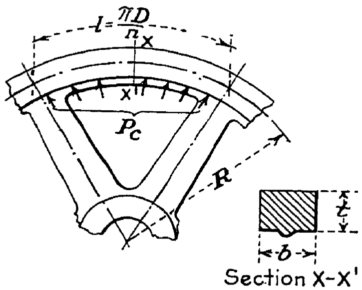


FIG. 10.

$$\frac{p_c l^2}{12} = \frac{S_b I}{c}. \tag{25}$$

The length of the rim between any pair of arms is:

$$l = 12\pi D \quad \begin{array}{l}(n = \text{number of arms}).\\ (D = \text{mean diameter, in feet}).\end{array}$$

From formula (19):

$$p_c = \frac{wtv^2}{Rg}.$$

Substituting these values in formula (25):

$$S_b = \frac{wtv^2 12^2 \pi^2 D^2 c}{12 Rgn^2 I} = \frac{7.34 wtv^2 Dc}{n^2 I}$$

Taking a rim of rectangular cross-section 1 in. wide, and using an average value of 0.27 for w:

$$S_b = \frac{11.9 v^2 D}{n^2 t}. \tag{26}$$

The resultant unit stress in the rim will be the sum of the stresses due to centrifugal force and bending. The third stress, due to shrinkage, is taken care of by a factor of safety, as already noted. Hence:

$$S = S_c + S_o. \tag{27}$$

If the arms of a flywheel did not stretch at all and were placed very close together, the centrifugal force would not set up stress in the rim and S_c in formula (27) would be zero. On the other hand, if the arms stretched enough to allow free expansion of the rim due to centrifugal action, there would be no restraint due to the arms, and S_b in formula (27) would be zero. G. Lanza[1] has shown that the arms of a flywheel stretch about three-quarters of the amount necessary for free expansion. Therefore, the assumption is made that in formula (27) the total stress is composed of three-quarters of S_c and one-quarter of S_b. Hence:

$$S = \tfrac{3}{4}S_c + \tfrac{1}{4}S_b.$$

Substituting from formulas (24) and (26):

$$S = \frac{3}{4} \times \frac{v^2}{10} + \frac{1}{4} \times \frac{11.9v^2D}{n^2t}.$$

Or approximately:

$$S = 3v^2\left(0.025 + \frac{D}{n^2t}\right). \tag{28}$$

Example.—Determine the stress which is induced in the rim of the problem in Sec. 345, if the speed is 500 r.p.m., and the number of arms is 6.

$$\text{Here } D = \frac{39.625}{12} = 3.30 \text{ ft.}$$

$$n = 6.$$

$$t = 3.69 \text{ in.}$$

$$v = \frac{\pi \times 3.30 \times 500}{60} \quad 86.4 \text{ ft. per second.}$$

From formula (28):

$$S = 3 \times 86.4^2\left(0.025 + \frac{3.30}{36 \times 3.69}\right) = 1{,}120 \text{ lb. per square inch.}$$

The speed used above was 500 instead of 200 r.p.m. as was given in the original example, and yet the unit stress is only 1,120 lb. per square inch. This stress indicates a factor of safety of about 16 if the ultimate strength of cast iron is taken as 18,000 lb. per square inch.

Formulas (24), (26), and (28) should be used for checking purposes only. Formula (24) shows that for any material the safe speed of a flywheel is independent of the amount of material used. Formula (26) shows that the stress due to bending is less with thick sections than with thin ones.

[1] *Trans. A.S.M.E.*, Vol. XVI, p. 208.

349. Flywheel Arms.—Rational analysis of stresses in the arms of flywheels should serve as a guide only. The shrinkage stresses in the arms are severe, especially with wheels of large diameter. The arms should be approximately three-quarters as strong as the shaft to resist successfully the maximum turning moment of the shaft.

Considering the arms to be cantilever beams, as shown in Fig. 11, and assuming that each arm takes its portion of the load, the maximum bending moment for all the arms will be:

$$M = Frn.$$

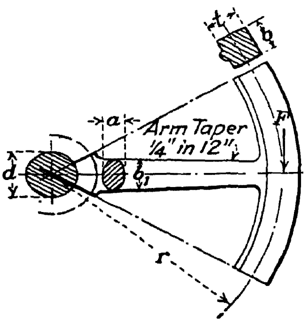


FIG. 11.

The maximum bending moment in the arms is also:

$$M = \frac{3}{4}\frac{S_2 J}{} = \frac{3}{4}\frac{S_2 \pi d^3}{16},$$

in which S_2 denotes the unit shearing stress in the shaft, in pounds per square inch.
d denotes the diameter of the shaft, in inches.

If the flywheel arm has an elliptical cross-section, with major and minor axes b and a, and bending taking place about the a axis, then the resisting moment of the shaft may be equated with the resisting moment of the arms:

$$\frac{\pi a b^2 S_1 n}{32} = \frac{3}{4}\frac{S_2 \pi d^3}{16}.$$

$$a = \frac{3}{2}\frac{S_2 d^3}{S_1 b^2 n}.$$

The usual proportions of the elliptical arms for pulleys and flywheels is $b = 2a$, hence:

$$a = \qquad (29)$$

in which S_1 denotes the tensile unit stress in the arms, in pounds per square inch.

If the ultimate tensile strength of cast iron is taken as 18,000 lb. per square inch, and the ultimate shear strength of steel as 48,000 lb. per square inch, formula (29) becomes:

For wheels with 6 arms, $a = 0.545d$, $b = 1.090d$.

For wheels with 8 arms, $a = 0.500d$, $b = 1.000d$.

350. Joints in Flywheel Rim.—Flywheels which are cast in halves are joined by fastenings at the hub and rim. The split hub is fastened by bolts of high tensile strength steel as shown in Fig. 12(*a*). Wheels which have a rim section which is relatively wide in proportion to its thickness, have the rim joint formed as shown in Fig. 12(*b*), the number of bolts being proportional to the width of the rim. The added weight caused by this form of joint becomes a serious defect in wheels which

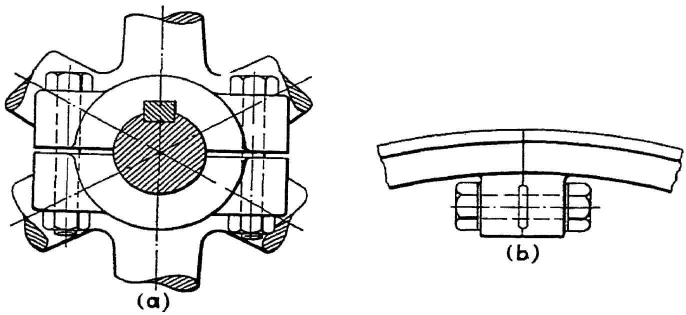


FIG. 12.—Flywheel hub and rim joints.

rotate at high speeds, and for this reason this form of joint is often located near a pair of arms.

Thiok rims are held together across the joint by a fastening of the form shown in Fig. 13. The keyway in the rim is cast and prepared for a shrink fit of the key. A pair of keys, carefully fitted and shrunk into place, form the fastening for each rim joint.

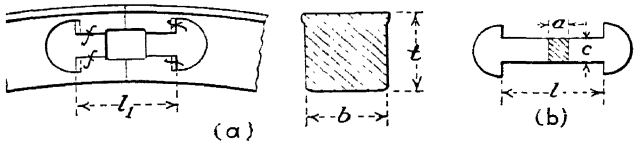


FIG. 13.—Fly-wheel rim fastner.

Assuming the relative strength of the joint to be 0.60, the rim to be of cast iron, the links of steel, and the unit tensile stresses of the cast iron and steel to be S_1 and S_2, respectively:

$$0.60tbS_1 = 2acS_2 \quad \text{(see Fig. 13).}$$

If S_1 is 18,000 lb. per square inch for cast iron, and S_2 is 70,000 lb. per square inch for steel:

$$ac = 0.0772tb. \tag{30}$$

The cross-section of the two keys should evidently each be approximately 0.08 of the cross-section of the cast-iron rim.

351. Construction of Flywheels.—Large flywheels are constructed in various ways, and some have assumed highly complicated forms. Several of the larger manufacturers of power transmission equipment have developed special designs, which have given satisfactory service while running at relatively high speeds. The "Handbook for Machine Designers and Draftsmen"[1] shows several of these large wheels in detail. The student should bear in mind that the design and construction of flywheels offers a problem which must be worked out with great care. Rational formulas, while important for guidance and for checking purposes, are subordinated to successful practical experience.

THICK CYLINDERS

352. Thick Cylinders.—When the ratio of the diameter to the thickness of a cylinder is 15 or greater, the maximum unit stress due to internal pressure is not very much greater than the average unit stress, and the assumption of uniform stress across the thickness of the cylinder wall may be safely made. This assumption cannot be made for thick cylinders under internal pressure, because there is a considerable difference between the maximum tensile unit stress on the inside fiber and the minimum stress on the outside fiber. Besides the circumferential tensile stress there exists also a radial compressive stress. When a thick cylinder is subjected to external pressure, the circumferential stress is of course compressive.

There are several formulas which are used to determine the stresses in thick cylinders, those of Barlow and Lamé having been used considerably by design engineers. Lamé's formula is used by many engineers for thick-walled cast-iron and steel cylinders, and Barlow's formula is used for medium thick-walled cylinders.

353. Barlow's Formula.—Barlow's formula assumes that the volume of metal in a cylinder does not change during the expansion of the cylinder, so that the unit stress S varies inversely as the square of the radius.

Therefore:

$$Sr^2 = S_1 r_1^2,$$

[1] Halsey, F. A., McGraw-Hill Book Company, Inc., New York.

in which S_1 denotes the unit stress at the inside fiber, in pounds per square inch.
r_1 denotes the inside radius, in inches.
S denotes the unit stress at any radius r, in pounds per square inch.
r denotes any radius, in inches.

For equilibrium in Fig. 14:

$$2r_1p_1 = 2\int_{r_1}^{r_1+t} Sdr,$$

and

$$p_1 = \frac{S_1t}{r_1 + t}, \tag{31}$$

in which p_1 denotes the internal pressure, in pounds per square inch.
t denotes the thickness of the cylinder wall, in inches.

Formula (31) may be expressed:

$$p_1 \quad 2S_1t \tag{32}$$

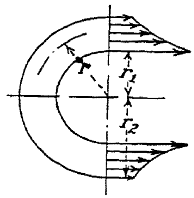


FIG. 14.

in which d_2 is the outside diameter, in inches. It will be noted that formula (32) is the same as that for thin cylinders, except that the outside diameter d_2 is used instead of the inside diameter.

354. Lamé's Formula.—For very thick cylinders, many design engineers make use of Lamé's formula. In developing the formula, Lamé assumed that each particle of metal is subjected to a radial compressive stress, and to a longitudinal and circumferential tensile stress. The formula is as follows:[1]

$$S = p\,\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}, \tag{33}$$

in which S denotes the maximum unit circumferential stress on the inside of the cylinder, in pounds per square inch.
p denotes the internal pressure, in pounds per square inch.
r_2 denotes the outside radius, in inches.
r_1 denotes the inside radius, in inches.

[1] MAURER and WITHEY, "Strength of Materials," p. 331.
MORLEY, "Strength of Materials," p. 303.

Solving for r_2:

$$r_2 = r_1 \tag{34}$$

Substituting $r_1 + t$ for r_2 and solving for t:

$$t = r_1\left[\sqrt{\frac{S + p}{S - p}} - 1\right]. \tag{35}$$

When cast iron is used for hydraulic cylinders, the iron should be close grained, and for such cast iron an allowable tensile fiber stress as high as 5,000 lb. per square inch may be used, and for steel cylinders an allowable tensile fiber stress of 15,000 lb. per square inch is recommended. Hydraulic cylinders are usually lined with brass or bronze bushings to prevent the corrosive action of water on iron.

Example.—A cast-iron hydraulic cylinder has a bore of 10 in., and is subjected to an internal pressure of 1,200 lb. per square inch. Determine the wall thickness if the tensile stress is not to exceed 5,000 lb. per square inch.

According to Barlow's formula, formula (32):

$$t = \frac{1{,}200d_2}{2 \times 5{,}000} = 0.12d_2.$$

Since

$$d_2 - d_1$$

$$d_2 - 10 = 0.24\ d_2.$$
$$d_2 = 13.2 \text{ in. and } t = 1.58 \text{ in.}$$

According to Lamé's formula, formula (35):

$$t \quad 5\left[\sqrt{\frac{5{,}000 + 1{,}200}{5{,}000 - 1{,}200}} - 1\right].$$
$$t = 1.40 \text{ in.}$$

355. Steam Cylinders.—Steam engine and pump cylinders are not classified as thick or thin cylinders, although it is probably safer to apply the thick-cylinder formulas in design. The following empirical formulas have been used.

J. H. Barr[1] gives the formula:

$$t = 0.05d + 0.3 \text{ in.} \tag{36}$$

Kent's "Mechanical Engineers' Handbook" gives the formula:

$$t = 0.0004dp + 0.3 \text{ in.} \tag{37}$$

[1] *Trans. A.S.M.E.*, Vol. XVIII, p. 741.

Marks' "Mechanical Engineers' Handbook" gives the formula:

$$t = 0.0005dp + 0.3 \text{ in.}^{1} \tag{38}$$

In the above formulas t denotes the wall thickness, in inches.
d denotes the bore of the cylinder, in inches.
p denotes the initial steam pressure, in pounds per square inch.

FLAT PLATES

356. Circular Flat Plates.—Cylinder heads, pistons, and other circular flat plates are designed according to rational formulas which have been developed from the investigations of Grashof, Bach, and others. These formulas give results which agree within reasonable limits.

For circular flat plates, *supported at the edges* and uniformly loaded:[2]

$$S \quad \frac{39}{32} p \frac{r^2}{t^2}, \tag{39}$$

in which S denotes the maximum radial and tangential unit stress, in pounds per square inch, which occurs at the outer fibers at the center of the plate.
p denotes the pressure on the plate, in pounds per square inch.
r denotes the radius of the plate, in inches.
t denotes the thickness of the plate, in inches.

Formula (39) is based upon a value of 0.25 for Poisson's ratio.

For circular flat plates *fixed at the edges* and uniformly loaded, the formula is:

$$S = \frac{3}{4} p \frac{r^2}{t^2}. \tag{40}$$

Formual (40) gives the maximum unit stress in the plate, which is the radial stress at the circumference.

For circular flat plates *supported at the edges* and with a load uniformly distributed over a small circle of radius r_0 at the center of the plate, the formula is:

$$S = \frac{P}{\pi t^2}\left(\frac{3}{2} + \frac{15}{8} \log_e \frac{r}{r_0} - \frac{9}{32} \frac{r_0^2}{r^2}\right), \tag{41}$$

in which P denotes the center load.

[1] This value of t allows for reboring the cylinder due to wear.

[2] MAURER and WITHEY, "Strength of Materials," p. 328.
MORLEY, "Strength of Materials," p. 382.

Formula (41) gives the maximum unit stress in the plate, which is the radial and the circumferential stress at the center.

Formula (41) may be written:

$$S \quad \frac{}{\pi t^2} K, \tag{42}$$

in which K equals the parenthesis in formula (41).

Table III shows the value of K as the ratio of r to r_0 changes.

TABLE III

Ratio $\frac{r}{r_0}$	10	20	30	40	50
Value of K	5.79	7.12	7.86	8.42	8.83

The above formulas on flat plates do not take into account the practice of adding reinforcing ribs to large cylinder heads. These added stiffening members reduce the maximum stress and are on the side of safety.

357. Rectangular Flat Plates.—Rectangular flat plates are usually ribbed for added strength and stiffness, and are so proportioned that the casting will be sound and true. Ribbed flat plates, such as steam chest covers, are designed according to empirical rules.

Rectangular flat plates without ribs may be designed according to the following rational formulas:

For rectangular plates *supported at the edges* and uniformly loaded:

$$S = \frac{pl^2b^2}{2t^2(l^2 + b^2)}, \tag{43}$$

in which S denotes the maximum unit stress, in pounds per square inch.

p denotes the pressure on the plate, in pounds per square inch.

l denotes the length of the plate, in inches.

b denotes the width of the plate, in inches.

t denotes the thickness of the plate, in inches.

For rectangular flat plates *fixed at the edges* and uniformly loaded:

$$S = \frac{pl^2b^2}{3t^2(l^2 + b^2)}. \tag{44}$$

COLUMNS

358. Machine Parts Designed as Columns.—Piston rods, connecting rods, and shafts sometimes act as columns. Section 139 of Chap. VII includes a brief discussion of axially loaded columns. Euler's formula for long columns is:

$$P = \frac{mEI}{L^2}. \tag{45}$$

$$\frac{P}{A} = \frac{mE}{\left(\frac{L}{r}\right)^2}. \tag{46}$$

In this formula m is a constant which depends upon end conditions, and experiments have shown that this constant has the following average values:

Round ends $m = 10$.
Pin or hinged ends $m = 16$.
Square or flat ends $m = 25$.
One end fixed and the other free $m = \frac{\pi^2}{4}$ (theoretical).

For columns which are not long enough to fall into the Euler class, empirical formulas have been derived, and in Amercian practice the so-called straight-line formulas are most commonly used.

The criterion which is applied to a column to determine whether the Euler formula or a straight-line formula should be used is as follows:

$$X_1 = \frac{\sqrt{3mE}}{S} \tag{47}$$

in which X_1 denotes the slenderness ratio, $\frac{L}{r}$

m denotes the constant for end conditions, mentioned above.

E denotes the modulus of elasticity of the material, in pounds per square inch.

S denotes a value of ultimate sterss determined by experiment, usually taken as the yield point for ductile materials, in pounds per square inch.

The value of X_1 in formula (47) is the slenderness ratio at which the straight line becomes tangent to the Euler curve. When the slenderness ratio of a column is greater than X_1

the Euler formula should be used, and when the slenderness ratio is less than X_1 a straight line formula should be used.

Formulas (45) and (46) are ultimate load formulas, and a factor of safety of 3 or 4 should be applied to obtain safe working loads.

The following are T. H. Johnson's ultimate-load straight-line formulas for steel:

Flat ends:

$$\frac{P}{A} = 52{,}500 \quad 179\frac{L}{r}, \quad \left(\text{limit of } \frac{L}{r} = 195\right). \tag{48}$$

Hinged ends:

$$\frac{P}{A} = 52{,}500 - 220\frac{L}{r}, \quad \left(\text{limit of } \frac{L}{r} = 159\right). \tag{49}$$

Round ends:

$$\frac{P}{A} = 52{,}500 - 284\frac{L}{r}, \quad \left(\text{limit of } \frac{L}{r} = 123\right). \tag{50}$$

The following is J. B. Johnson's ultimate-load formula for cast-iron columns, derived from experimental data:

$$\frac{P}{A} = 34{,}000 - 88\frac{L}{r}, \quad \left(\text{Limit of } \frac{L}{r} = 125\right). \tag{51}$$

The American Railway Engineering Association formula for steel columns with hinged ends, for working loads, is:

$$\frac{P}{A} = 15{,}000 - 50\frac{L}{r}. \tag{52}$$

In formula (52) P/A must not exceed 12,500 lb. per square inch, and L/r must not exceed 200.

The American Bridge Company uses the following formulas for steel columns with hinged ends, for working loads:

$$\frac{P}{A} = 13{,}000, \quad \left(\frac{L}{r} \text{ up to } 60\right). \tag{53}$$

$$\frac{P}{A} = 19{,}000 - 100\frac{L}{r}, \quad \left(\frac{L}{r} \text{ from 60 to 120}\right). \tag{54}$$

$$\frac{P}{A} = 13{,}000 - 50\frac{L}{r}, \quad \left(\frac{L}{r} \text{ from 120 to 200}\right). \tag{55}$$

The New York Building Code formula is a satisfactory one to use for working loads on cast-iron columns:

$$\frac{P}{A} = 9{,}000 - 40\frac{L}{r}, \quad \left(\text{Limit of } \frac{L}{r} = 70\right). \tag{56}$$

The American Railway Engineering Association formula, and the American Bridge Company formula, given above, while based upon hinged end constants, are used without reference to particular end conditions.

The methods discussed in Sec. 208 of Chap. VII for shafting under column action, may also be employed to advantage in certain column problems involving piston rods and connecting rods.

359. Eccentrically Loaded Columns.—For axially loaded columns, neither the unit stress in the column nor the deflection of the column can be computed by formulas. However, when the column is eccentrically loaded, both the unit stress and the deflection may be computed.

For a column with *one end fixed and the other end free,* the maximum compressive unit stress is:[1]

$$S = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\sqrt{\frac{PL^2}{EI}}\right) \qquad (57)$$

in which S denotes the maximum unit stress, in pounds per square inch.

P denotes the load on the column, in pounds.

A denotes the cross-sectional area of the column, in in.^{2}.

e denotes the eccentricity of the load, in inches.

c denotes the distance from the centroid of the section to the extreme compressive fiber, in inches.

L denotes the length of the column, in inches.

E denotes the modulus of elasticity of the material, in pounds per square inch.

I denotes the moment of inertia of the column cross-section, in in.^{4}.

For a column with *both ends round* the formula is:

$$S = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\sqrt{\frac{PL^2}{4EI}}\right). \qquad (58)$$

For a column with both ends round in which the tensile stress would govern, as would be the case for cast iron, the formula is:

$$S = \frac{P}{A}\left(\frac{ec}{r^2}\sec \qquad - 1\right) \qquad (59)$$

[1] MAURER and WITHEY, "Strength of Materials," p. 310.
BOYD, "Strength of Materials," p. 229.

Example.—The cylinder of a low-speed steam engine is 8 in. in diameter, and the steam pressure is 125 lb. per square inch. If the length of the piston rod is 30 in., determine the diameter of the rod.

In this case the piston rod acting as a column will be hinged at one end and restrained at the other. The theoretical constant m for Euler's formula for one end fixed and the other end free is $\pi^2/4$ and this will be adopted.

The load on the column is:

$$P = 125 \times \pi \times 8^2 \qquad 6{,}280 \text{ lb.}$$

Experience shows that a factor of safety of 6 for high-speed engines and 4 for low-speed engines is satisfactory. Therefore, using formula (45):

$$6{,}280 \times 4 = \frac{\pi^2 \times 29{,}000{,}000 \times I}{4 \times 30^2}$$

$$0.316.$$

For a circle

$$I = \frac{\pi d^4}{64} = 0.316.$$

$$d^4 = 6.43,\ d = 1.59 \text{ in., use } 1\tfrac{5}{8} \text{ in.}$$

In this problem the radius of gyration is $d/4$ or 0.40 in., and $\frac{L}{r} = \frac{30}{0.40} = 75.$

From formula (47), using $S = 40{,}000$ lb. per square inch for steel:

$$X_1 \qquad \sqrt{\frac{3\pi^2 \times 29{,}000{,}000}{4 \times 40{,}000}} = 75.$$

This shows that the column investigated is an Euler column, and may be designed by the Euler formula.

Example.—A connecting rod for the engine in the previous problem is 40 in. long, and of rectangular cross-section with the width twice the thickness. The dimensions of the cross-section are to be determined.

If the piston rod was considered free at the crosshead end, then the connecting rod should be considered free at that end also. At the crank end the connecting rod may be considered hinged in the plane of the motion, and fixed in the lateral plane. For the case of one end free and the other end hinged a value of $m = \pi^2/6$ will be used, and for the case of one end free and the other end fixed, a value of $m = \pi^2/4$ will be used.

The column will be designed first for bending in the plane of the motion:

$$6{,}280 \times 4 \qquad \frac{\pi^2 \times 29{,}000{,}000 \times I}{6 \times 40^2}$$

$$I = 0.844.$$

For bending about the t axis:

$$\frac{tb^3}{12} = 0.844.$$

Or, since b is to be twice t:

$$\frac{\ }{12} \quad 0.844.$$

$$t^4 = 1.266,\ t = 1.06 \text{ in., } b \quad 2.12 \text{ in.}$$

For bending in the lateral plane:

$$6{,}280 \times 4 = \frac{\pi^2 \times 29{,}000{,}000 \times I}{4 \times 40^2}.$$

$$I = 0.563.$$

For bending about the b axis:

$$I = \frac{bt^3}{12} = 0.563.$$

$$\frac{2t^4}{12} = 0.563.$$

$$t^4 = 3.38,\ t = 1.36 \text{ in.},\ b = 2.72 \text{ in.}$$

This result indicates that the dimensions to resist bending in the lateral direction govern the design.

If the above connecting rod were of circular cross-section:

$$I = \frac{\pi d^4}{64} = 0.844.$$

$$d^4 = 17.2,\ d = 2.04 \text{ in.}$$

MACHINE FITS

360. Machine Fits.—To facilitate the assembling and erection of machine parts, it is essential that the parts be fitted together with a definite relation existing between the sizes of the parts. This relation varies to some extent among manufacturers engaged in the same line of production. The growth of the interchangeable system of manufacture demands that some system of classification and standardization of fits be established, to enable design engineers to specify and dimension the several classes of fits, so that manufacturers may assemble the parts of a machine without fitting one part to another. The following discussion of fits, tolerances, and allowances is extracted from the report of the Committee on the Standardization of Plain Limit Gages for General Engineering Work.[1]

Definitions.—To understand the following discussion of machined fits the definitions of the terms used are first given:

Allowance.—Allowance is the intentional difference in the dimensions of mating parts, or the minimum clearance space which is intended between the mating parts. Its purpose is to provide for different classes of fit.

[1] The committee studied the best available practices, and where differences were noted a compromise was made. The report of the committee was approved by the American Engineering Standards Committee in December, 1925. This study was sponsored by The American Society of Mechanical Engineers. The complete report may be obtained in pamphlet form, from the A. E. S. C. and A. S. M. E., 29 W. 39th St., New York, N. Y.

Basic Size.—Basic size is the exact theoretical size from which all limiting variations are made.

Standard.—A standard is a physical representative of a form, dimension, or size, established by law or by general usage and consent.

Tolerance.—Tolerance is the permissible variation in size of a part.

Gaging.—Gaging is the process of measuring manufactured materials to assure the uniformity of size and contour required by the industries.

Limits.—Limits are the extreme permissible dimensions of a part.

Gage.—A gage is a device for determining whether or not one or more of the dimensions of a manufactured part are within specified limits.

Ring Gage.—A ring gage is one which has the inside measuring surfaces circular in form. The measuring surfaces may be cylindrical or conical. Figure 15 shows two forms of ring gages.

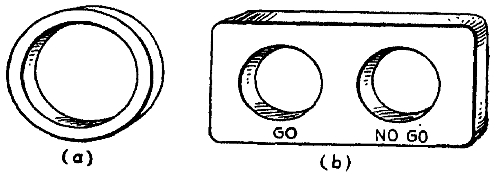


FIG. 15.—Ring gages.

Plug Gage.—A plug gage is one which has outside measuring surfaces, arranged to verify the specified uniformity of holes. A plug gage may be straight or tapered and of any cross-sectional shape. Figure 16 shows three forms of plug gages.

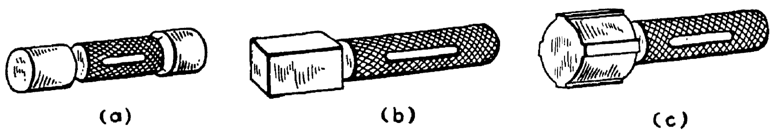


FIG. 16.—Plug gages.

Receiving Gage.—A receiving gage is one which has inside measuring surfaces arranged to verify specified uniformity of size and contour of manufactured material. Figure 17 shows two forms of receiving gages.

Indicating Gage.—An indicating gage is one which shows the variations in the uniformity of dimensions or contour, the amount of variation being indicated on a graduated scale or dial. Two forms of indicating gages are shown in Fig. 18.

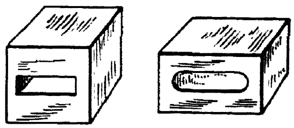
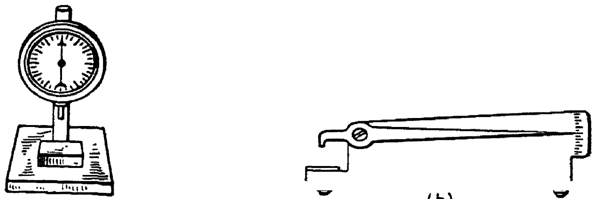


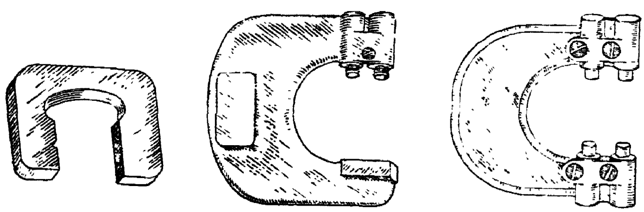
FIG. 17.—Receiving gages.



(a) (b)

FIG. 18.—Indicating gages.

Snap Gage.—A snap gage is a fixed gage with inside measuring surfaces for calipering diameters, lengths, and thicknesses. Figure 19 shows three forms of snap gages.



(a) (b) (c)

FIG. 19.—Snap gages.

Caliper Gage.—A caliper gage is one which is similar to a snap gage, for measuring internal dimensions, and similar to a plug gage, for measuring external dimensions. Figure 20 shows three forms of caliper gages.

361. Classification of Fits.—Machine fits are classified as follows:

Class 1, *Loose Fit.*—A loose fit is made with a large allowance, provides for considerable freedom of movement between the surfaces of the members, and is recommended where accuracy is

not essential. Examples are: Machine fits of agricultural and mining machinery; machinery used in the textile and rubber industries, and other machinery of a similar grade; and some ordnance material.

Class 2, *Free Fit.*—A free fit is made with a liberal allowance, and should be employed for running fits when the speed is 600 r.p.m. or over, and the pressure between the journal and bearing is 600 lb. per square inch or over. Examples are: dynamos, engines, many machine tools, and some automotive parts.

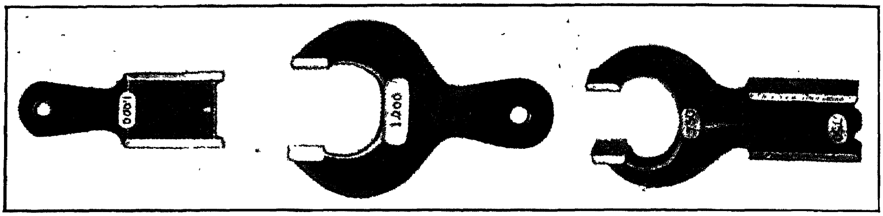


FIG. 20.—Caliper gages.

Class 3, *Medium Fit.*—A medium fit is made with a medium allowance and is recommended for running fits, when the speed is under 600 r.p.m., and the pressure between the journal and bearing is less than 600 lb. per square inch. This class of fit is also recommended for sliding fits, and the more accurate machine tool and automotive parts.

Class 4, *Snug Fits.*—A snug fit is made with zero allowance. It is the closest fit which can be assembled by hand and requires considerable precision. It should be used where no perceptible shake is permissible, and where moving parts are not intended to move freely while loaded.

Class 5, *Wringing Fit.*—A wringing fit is also known as a "tunking" fit, and gives metal to metal contact. Assembly is selective and not interchangeable.

Class 6, *Tight Fit.*—A tight fit is one in which light pressure is required to assemble the parts, resulting in a more or less permanent assembly. This fit is used for drive fits in thin sections, in extremely long fits in other sections, and for shrink fits on very light sections. This fit is used in automotive, ordnance, and general machine manufacturing.

Class 7, *Medium Force Fit.*—A medium force fit is made with a negative allowance, that is, the shaft or internal member is

turned larger than the hole or external member. Considerable pressure is required to assemble these fits, and the parts are permanently assembled. This fit is used to fasten locomotive wheels, passenger and freight-car wheels, armatures of dynamos and motors, and crank disks to their axles or shafts. This fit is used for shrink fits on medium sections and on long fits, and is the tightest fit which is recommended for cast-iron holes or external members, since the cast iron is stressed to its elastic limit.

Class 8, *Heavy Force and Shrink Fit.*—Heavy force and shrink fits are made with considerable negative allowance for steel holes, where the metal can be highly stressed within the elastic limit. This fit causes excessive stress for cast-iron holes. Shrink fits are used where it is impracticable to use heavy-force fits, as on locomotive-wheel tires, and on heavy crank disks of large engines.

Formulas for arriving at the size of the hole and of the shaft for the above eight classes of fits are given in Table IV. The student will note that the allowance is diminished as the tightness of the fit increases, and that for a snug fit, class 4, the allowance is zero. When the shaft is larger than the hole into which it is to fit, there is interference of metal, which increases from zero for a wringing fit, class 5, to $0.001d$ for a heavy force or shrink fit. The desired condition for a fit of classes 5, 6, 7, and 8 is the average interference which is given in column 4, Table IV, and must be obtained by mating large shafts in large holes, and small shafts in small holes.

The length of engagement between the surfaces is important, a short engagement being fitted tighter than a long one. When the external member is long compared with the diameter of the fit, it is usually recessed at the middle, so that the actual length of the surface in contact is approximately equal to the diameter of the internal member.

Example.—Determine the diameters for the tightest, loosest, and average condition for a 6-in. shaft and a cast-iron hub, using a medium force fit, class 7.

The allowance is negative, that is, there is interference of shaft and hole metal. The hole size is basic, so that it should be 6 in.

The shaft should be larger than the hole by $0.0005d$ (column 4, Table IV), or 0.0030 in.

TABLE IV.—FORMULAS FOR RECOMMENDED ALLOWANCES AND TOLERANCES
(American Standard)

(The formulas for allowance values give the ideal condition of fit for classes 1 to 4. The formulas for selected average interference of metal gives the ideal condition of fit for classes 5 to 8. d denotes the diameter of fit in inches. A denotes allowance.)

1	2	3	4	5	6	Hole stress in pounds per square inch		Force for pressing steel shaft in	
Class of fit	Method of assembly	Allowance[1]	Selected average interference of metal per inch of mean size	Hole[2] tolerance	Shaft[2] tolerance	7 Steel hole	8 Cast-iron hole	9 Steel hole, tons	10 Cast-iron hole, tons
1. Loose	Strictly Interchangeable	$0.0025\sqrt[3]{d^2}$	....	$0.0025\sqrt[3]{d}$	$0.0025\sqrt[3]{d}$				
2. Free		$0.0014\sqrt[3]{d^2}$	....	$0.0013\sqrt[3]{d}$	$0.0013\sqrt[3]{d}$				
3. Medium		$0.0009\sqrt[3]{d^2}$	....	$0.0008\sqrt[3]{d}$	$0.0008\sqrt[3]{d}$				
4. Snug		0.0000	....	$0.0006\sqrt[3]{d}$	$0.0004\sqrt[3]{d}$				
5. Wringing	Selective Assembly	....	0.0000	$0.0006\sqrt[3]{d}$	$0.0004\sqrt[3]{d}$				
6. Tight		....	$0.00025d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$	$\frac{29,000,000A}{d}$	$\frac{10,432,000A}{d}$	$1,298A$	$747.2A$
7. Medium force		....	$0.0005d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$				
8. Heavy force or shrink		....	$0.001d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$	$\frac{29,000,000A}{d}$	....	$1,298A$	

[1] Allowance represents the condition of the tightest permissible fit or the largest internal member (shaft) mated with the smallest external member (hole).

[2] Tolerance must tend towards greater looseness, for example, if a tolerance is 0.001 in. on each member, they would be dimensioned as follows:
Shaft 0.874 +0.000 in. This defines a condition where the greatest looseness is 0.003 in. and the greatest tightness gives a clearance of 0.001 in.
−0.001 in.
Hole 0.875 +0.001 in.
−0.000 in.

Therefore the shaft should be 6.003 in. plus the tolerance. The tolerance is (column 6):

$$0.0006\sqrt[3]{d} = 0.00109, \text{ say } 0.0011 \text{ in.}$$

With this tolerance, the shaft size will be designated $6.003\begin{smallmatrix}+0.0011\\-0.000\end{smallmatrix}$, or the upper limit will be a diameter of 6.0041 in., and the lower limit will be a diameter of 6.0030 in.

The hole size is basic, so that it will be $6.00\begin{smallmatrix}+0.0011\\-0.0000\end{smallmatrix}$ (column 5), or the upper limit will be a diameter of 6.0011 in., and the lower limit will be a diameter of 6.0000 in.

SUMMARY OF DIMENSIONS

	Hole, inches	Shaft, inches	Difference, inches
Tightest fit....	6.0000	6.0041	0.0041
Loosest fit.....	6.0011	6.0030	0.0019
The selected fit	6.0011	6.0041	0.0030
or..........	6.0000	6.0030	0.0030

The hole stress is (column 8):

$$\text{Tightest fit} \quad \frac{10{,}432{,}000 \times 0.0041}{6} = 7{,}130 \text{ lb. per square inch.}$$

$$\text{Loosest fit} \quad \frac{10{,}432{,}000 \times 0.0019}{6} = 3{,}310 \text{ lb. per square inch.}$$

$$\text{Selected fit} \quad \frac{10{,}432{,}000 \times 0.0030}{6} = 5{,}220 \text{ lb. per square inch.}$$

The force required for pressing in the shaft is (column 10):

Tighest fit.................. $747.2 \times 0.0041 = 3.06$ tons.
Loosest fit.................. $747.2 \times 0.0019 = 1.42$ tons.
Selected fit.................. $747.2 \times 0.0030 = 2.24$ tons.

Example.—A locomotive tire is to be fitted to a cast-steel wheel center with a shrink fit, class 8. The diameter of the fit is 64 in., and the tire is to be fitted by selective assembly (column 2).

The tire should be bored $64.000\begin{smallmatrix}+0.0024\\-0.0000\end{smallmatrix}$ (column 5).

The wheel should be turned $64.064\begin{smallmatrix}+0.0024\\-0.0000\end{smallmatrix}$ (columns 4 and 6).

TABLE V.—STANDARD FITS

Summary of Tightest, Loosest, and Selected Conditions for All Classes of Fit

Size			Loose fit (Class 1)		Free fit (Class 2)		Medium fit (Class 3)		Snug fit (Class 4)		Wringing fit (Class 5)		Tight fit (Class 6)	Medium force fit (Class 7)	Heavy force and shrink fit (Class 8)
			Tightest fit	Loosest fit	Tightest fit	Loosest fit	Tightest fit	Loosest fit	Tightest fit	Loosest fit	Tightest fit	Loosest fit	Selected fit	Selected fit	Selected fit
From	Up to and inclusive	Mean	Allowance	Allowance + tolerances	Allowance	Allowance + tolerances	Allowance	Allowance + tolerances	Allowance	Allowance + tolerances	Allowance	Allowance + tolerances	Average interference of metal	Average interference of metal	Average interference of metal
			+[1]	+[1]	+[1]	+[1]	+[1]	+[1]		+[1]	−[1]	+[1]	−[2]	−[2]	−[2]
0	3/16	1/8	0.001	0.003	0.0004	0.0018	0.0002	0.0010	0.0000	0.0005	0.0002	0.0003	0.0000	0.0001	0.0001
3/16	5/16	1/8	0.001	0.005	0.0006	0.0022	0.0004	0.0014	0.0000	0.0007	0.0003	0.0004	0.0001	0.0001	0.0003
5/16	7/16	3/8	0.001	0.005	0.0007	0.0025	0.0005	0.0017	0.0000	0.0007	0.0003	0.0004	0.0001	0.0002	0.0004
7/16	9/16	1/2	0.002	0.006	0.0009	0.0029	0.0006	0.0018	0.0000	0.0008	0.0003	0.0005	0.0001	0.0003	0.0005
9/16	11/16	5/8	0.002	0.006	0.0010	0.0032	0.0007	0.0021	0.0000	0.0008	0.0003	0.0005	0.0002	0.0003	0.0006
11/16	13/16	3/4	0.002	0.006	0.0012	0.0036	0.0007	0.0021	0.0000	0.0009	0.0004	0.0005	0.0002	0.0004	0.0008
13/16	15/16	7/8	0.002	0.006	0.0013	0.0037	0.0008	0.0024	0.0000	0.0010	0.0004	0.0006	0.0002	0.0004	0.0009
15/16	1 1/16	1	0.003	0.009	0.0014	0.0040	0.0009	0.0025	0.0000	0.0010	0.0004	0.0006	0.0003	0.0005	0.0010
1 1/16	1 3/16	1 1/8	0.003	0.009	0.0015	0.0043	0.0010	0.0026	0.0000	0.0010	0.0004	0.0006	0.0003	0.0006	0.0011
1 3/16	1 3/8	1 1/4	0.003	0.009	0.0016	0.0044	0.0010	0.0028	0.0000	0.0010	0.0004	0.0006	0.0003	0.0006	0.0013
1 3/8	1 5/8	1 1/2	0.003	0.009	0.0018	0.0048	0.0012	0.0030	0.0000	0.0012	0.0005	0.0007	0.0004	0.0008	0.0015
1 5/8	1 7/8	1 3/4	0.004	0.010	0.0020	0.0052	0.0013	0.0033	0.0000	0.0012	0.0005	0.0007	0.0004	0.0009	0.0018
1 7/8	2 1/8	2	0.004	0.010	0.0022	0.0054	0.0014	0.0034	0.0000	0.0013	0.0005	0.0008	0.0005	0.0010	0.0020

2 ⅛	2 ⅜	2¼	0.004	0.010	0.0024	0.0058	0.0015	0.0035	0.0000	0.0013	0.0005	0.0008	0.0006	0.0011	0.0023
2 ⅜	2 ¾	2½	0.005	0.011	0.0026	0.0062	0.0017	0.0039	0.0000	0.0013	0.0005	0.0008	0.0006	0.0013	0.0025
2 ¾	3 ¼	3	0.005	0.013	0.0029	0.0067	0.0019	0.0043	0.0000	0.0015	0.0006	0.0009	0.0008	0.0015	0.0030
3 ¼	3 ¾	3½	0.006	0.014	0.0032	0.0072	0.0021	0.0045	0.0000	0.0015	0.0006	0.0009	0.0009	0.0018	0.0035
3 ¾	4 ¼	4	0.006	0.014	0.0035	0.0077	0.0023	0.0049	0.0000	0.0016	0.0006	0.0010	0.0010	0.0020	0.0040
4 ¼	4 ¾	4½	0.007	0.015	0.0038	0.0080	0.0025	0.0051	0.0000	0.0017	0.0007	0.0010	0.0011	0.0023	0.0045
4 ¾	5 ½	5	0.007	0.015	0.0041	0.0085	0.0026	0.0054	0.0000	0.0017	0.0007	0.0010	0.0013	0.0025	0.0050
5 ½	6 ½	6	0.008	0.018	0.0046	0.0094	0.0030	0.0060	0.0000	0.0018	0.0007	0.0011	0.0015	0.0030	0.0060
6 ½	7 ½	7	0.009	0.019	0.0051	0.0101	0.0033	0.0063	0.0000	0.0019	0.0008	0.0011	0.0018	0.0035	0.0070
7 ½	8 ½	8	0.010	0.020	0.0056	0.0108	0.0036	0.0068	0.0008	0.0020	0.0008	0.0012	0.0020	0.0040	0.0080

All dimensions in inches.

[1] NOTE: (+) denotes clearance or amount of looseness.

[2] NOTE: Obtained by fitting or selection.

NOTE: It is not necessary that both shaft and hole member be made to the same class of fit. EXAMPLE: Shaft members of class 2 may be used with hole members of class 1 and 3 or *vice versa*.

Summary of Dimensions

	Hole, inches	Shaft, inches	Difference, inches
Tightest fit	64.0000	64.0664	0.0664
Loosest fit.	64.0024	64.0640	0.0616
Selected fit.	64.0024	64.0664	0.0640
or.......	64.0000	64.0640	0.0640

The stress in the tire will be (column 7):

$$\text{Tighest fit} \quad \frac{29{,}000{,}000 \times 0.0664}{64} = 30{,}100 \text{ lb. per square inch.}$$

$$\text{Loosest fit} \quad \frac{29{,}000{,}000 \times 0.0616}{64} = 27{,}900 \text{ lb. per square inch.}$$

$$\text{Selected fit} \quad \frac{29{,}000{,}000 \times 0.0640}{64} = 29{,}000 \text{ lb. per square inch.}$$

If the tire is forced on, the required force is (column 9):

Tightest fit.................. $1{,}298 \times 0.0664 = 86.1$ tons.
Loosest fit.................. $1{,}298 \times 0.0616 = 79.9$ tons.
Selected fit.................. $1{,}298 \times 0.0640 = 83.0$ tons.

The allowance and average interference of metal for all classes of fits are given in Table V for all sizes up to 8 in. in diameter, having been determined by use of the formulas given in Table IV.

362. Taper Force Fits.—Taper fits may be readily measured by means of a plug gage so graduated that it will indicate the fit. The usual taper is 1/16 or 1/8 in. per foot, and has no effect upon the security of the fitted surfaces. The usual practice is to ream the external member to size, making the necessary allowance on the diameter of the external member. The allowances and tolerances used for tapered surface fits are approximately the same as those used for parallel surface fits. With taper surfaces it is possible to obtain more effective lubrication when making a force fit, since there is a much larger surface contact between the parts when interference of metal starts. The usual lubricant is a mixture of white lead and linseed oil.

363. Temperature of Metal for Shrink Fits.—The coefficient of linear expansion determines the temperature to which the external member of a fit should be heated. The coefficient of expansion is 0.0000065 for steel and 0.0000062 for cast iron, which is the change in length in 1 in. of metal for 1° F. change of temperature.

To expand the tire of the problem given in Section 361, so that it will slip into place by expanding it 0.006 in. oversize, and allowing it to cool in position, the following change of temperature is necessary.

$$\text{Total expansion} = 0.064 + 0.006 = 0.070 \text{ in.}$$

$$\text{Change of temperature} = \frac{0.070}{0.0000065} = 1077° \text{ F.}$$

If the room temperature is 70° F., the required temperature for the tire would be 1147° F. According to temperature color charts, this temperature corresponds to a red color distinguishable in sunlight. There is such a wide variation in color-chart temperatures given by different authorities, that the determination of temperature by color is not dependable. Pyrometers should be used to measure temperatures for shrink fits, and parts to be shrunk on should not be heated above the temperature required to make the fit.

Problems

1. A helical spring is to be elongated 4 in. by a load of 500 lb. The mean diameter of the coils is not to be greater than 4 in., and the unit stress is not to exceed 50,000 lb. per square inch. What diameter of steel wire should be used, and how many coils should the spring have?
2. A helical spring for an automobile clutch has 8 coils of circular wire, and is subjected to a pressure of 340 lb. when the clutch surfaces are engaged. The mean diameter of the coils is 3¾ in., and the spring is compressed 2 in. in assembling the clutch. When the clutch is thrown out the spring is compressed an additional ½ in. Determine the diameter of the wire, the maximum unit stress induced in the wire, the free height of the spring, and the length of the pedal lever for a foot pressure of 28 lb. if the fulcrum is 1¼ in. from the pressure point on the spring.
3. A typical spring system for a freight car consists of concentric helical spring nests of two springs each, and coiled opposite to avoid binding. The data for each spring are:

	Outside Spring	Inner Spring
Diameter of wire	1 in.	⅝ in.
Height when solid	4 in.	4 in.
Free height	5¼ in.	5⅓₂ in.
Outside diameter	5 in.	2⅞ in.

(*a*) Assuming a maximum shear stress according to formula (4), determine the maximum load for each spring.
(*b*) What load is each spring subjected to when the springs go solid?
(*c*) Determine the number of coils, the length of the wire, and the weight of each spring.

4. The steel in a laminated spring is stressed to 80,000 lb. per square inch when the spring is deflected 4 in., the unsupported length of the spring being 34 in. For a load of 400 lb. determine the thickness of the plates if they are 3 in. wide and have rounded ends, and determine the number of leaves required for the spring. Assume $E = 30,000,000$ lb. per square inch.
5. For an automobile weighing 3,000 lb. the S. A. E. front spring which is recommended is 40 in. long, 2 in. wide, and has six leaves. Assuming that the car will carry six people, and that 60 per cent of the weight is taken by the rear axle, and 40 per cent by the front axle, determine: (*a*) the thickness of the plate in the spring, and the deflection of the spring. (*b*) the load which the car could carry on a smooth and level road for a maximum value of 70,000 lb. per square inch fiber stress in the spring steel.
6. A locomotive spring is built up of eight leaves, each 5 in. wide and 3/8 in. thick. The two top leaves are full length, that is, 36 in. long. The maximum unit stress is not to exceed 68,000 lb. per square inch. What load would this spring take in deflecting 4 in.?
7. (*a*) What would be the safe speed in revolutions per minute of a cast iron flywheel whose outside diameter is 48 in., and whose rim dimensions are 6 in. wide and 6½ in. deep?
   (*b*) What would be the theoretical bursting speed?
   (*c*) What would be the safe speed if the flywheel were made of steel?
8. What probable unit stress would be induced in the rim of a flywheel with an outside diameter of 12 ft., and a rim which is 14 in. wide and 2½ in. thick, if the speed is 300 r.p.m.? The wheel has six elliptical arms and is cast in one piece. Is this stress too high?
9. The diameter of the wheel of Problem 8 is increased to 13 ft., which made it necessary that the wheel be cast in two parts, which were fastened together by bolts at the hub and rim. The rim joint was located at the middle point between the two arms. (*a*) What probable stress would be induced in the rim? (*b*) What would be the safe speed in r.p.m. for the flywheel?
10. A flywheel is to be used on a punching machine, and it is desired to design the flywheel so that one punching operation will bring the flywheel to rest after the power has been shut off. The maximum hole to be punched is a 1½-in. hole in a ¾-in. plate. The wheel is not to be larger than 44 in. in order to clear the floor. The punch is capable of punching 25 holes per minute, and the velocity ratio of the driving shaft to the eccentric shaft operating the punch is 6 to 1. The flywheel rim is to be 10 per cent deeper than it is wide. (*a*) Determine the weight of the flywheel rim. (*b*) Determine the cross-section of the rim. (*c*) Determine the maximum unit stress in the rim.
11. Design the elliptical arms for a 14-ft. flywheel which is to turn 90 r.p.m. on a shaft which is 7 in. in diameter. The shaft is made of a good grade of steel with an ultimate shear stress value of 54,000 lb. per square inch. The maximum tensile stress in the cast iron rim of the wheel is not to exceed 4,000 lb. per square inch.

12. A flywheel rim is held together by "dumb-bell" fasteners shrunk into place. The cast-iron rim is 9 in. wide and 10½ in. deep, and the ultimate tensile strength of the material is 22,000 lb. per square inch. The steel rim fasteners are made of forged steel having a maximum tensile strength of 73,000 lb. per square inch. What cross-section should the steel fasteners have, and what is the probable relative strength of this joint?

13. A cast-iron cylinder is 8 in. in diameter and has walls 4½ in. thick. What may the internal pressure be if the maximum allowable tensile stress is 2,400 lb. per square inch?

14. Determine the thickness of the walls of a cylinder 12 in. in diameter for an internal working pressure of 1,200 lb. per square inch, using a factor of safety of 10.

(*a*) The cylinder is made of close-grained cast iron having an ultimate tensile strength of 28,000 lb. per square inch.

(*b*) The cylinder is made of cast steel having an ultimate tensile strength of 62,000 lb. per square inch.

15. The plunger of a wheel press is 12 in. in diameter, and the cylinder wall is 3 in. thick.

(*a*) What pressure in tons may be applied if the steel cylinder is subjected to a working pressure of 3,500 lb. per square inch?

(*b*) What is the maximum tangential stress induced in the cylinder walls?

16. A locomotive cylinder is 22 by 26 in., and the safety valves on the boiler are set to pop at 215 lb. per square inch. Using a good grade of cast iron, having a tensile strength of 26,000 lb. per square inch, what should be the thickness of the cylinder walls using a factor of safety of 6, and making an allowance of ¼ in. for reboring the cylinder?

17. The steam cylinder of a simplex pump is 8 in. in diameter and the initial steam pressure is 175 lb. per square inch. The stroke of the plunger is 12 in., and the pump is used to furnish boiler feed water against a boiler pressure of 200 lb. per square inch. What should be the thickness of the steam and of the water cylinders, if the water cylinder is bored to a diameter of 8 in.?

18. What probable stress would be induced in the cylinder head of the steam end of the pump of problem 17, if the thickness of the head is ¾ in.?

19. What would be the thickness of the cast-iron cylinder head for the locomotive cylinders of problem 16 if the allowable radial and tangential unit stress is 4,000 lb. per square inch? If 50 per cent is added to the strength of the cylinder head by well-designed radial ribs, what thickness should be recommended for the edges of the cylinder head?

20. A piston rod has a diameter of 2⅞ in. and its length is 32 in. It must be reduced in diameter to 2⅝ in. during its life because of truing-up due to uneven wear. The maximum steam pressure is 200 lb. per square inch on a piston which is 18 in. in diameter. What factor of safety does this design imply?

21. The plunger rod is to be designed for a hydraulic press which has a maximum capacity of 80 tons. The maximum unsupported length of the plunger rod is 26 in. What should be its diameter if it is turned out

of 0.20 per cent carbon steel, and the allowable unit stress is 8,000 lb. per square inch?

22. The connecting rod of a Corliss engine is circular in cross-section, and is forged from steel having an ultimate tensile strength of 68,000 lb. per square inch. The ratio of the length of the connecting rod to the crank is 5 to 1, and the cylinder proportions are 16 by 30 in. The initial steam pressure is 175 lb. per square inch, and the crank turns at the rate of 100 r.p.m. Design the connecting rod for a stress which implies a factory of safety of 7.

23. What allowance should be made for a fit in a 6-in. steel shaft of an ore crusher. The shaft is to turn in a cast-iron bearing 8 in. long and makes 125 r.p.m. (class 1, loose fit).

24. A free fit is to be made between the main journals and their bearings for a high-speed steam engine. The bearings are bored and finished to a diameter of 4½ in. Determine the clearance for the fit, and also the tolerances.

25. The ram of a shaper is to be fitted to the guides. The distance between the side guides is 12 in., and the distance between the top and bottom guides is 2¼ in. Determine the allowances and tolerances.

26. A snug fit is required between the arbor support and its bearing on a milling machine. The bearing is finished to 4½ in. in diameter. What is the finished size of the support bar?

27. A medium-force fit is to be made between a motor armature and its shaft. The shaft is 4 in. in diameter, the bearings are 5 in. long, and the distance between bearings is 14 in. What should be the diameter of the shaft?

28. A cast-iron crank disk is to be fitted to its 7½-in. shaft. (*a*) If a medium force fit is employed, what will be the finished size of the shaft? Determine also the force required to make the fit, and the unit stress induced in the disk after it has been forced into place. (*b*) If a shrink fit is made, what should be the finished size of the hole in the disk? Determine also the shaft size and the unit stress induced in the cast-iron disk.

29. What should be the temperature of the cast-iron disk in Problem 28 to make a good shrink fit between the shaft and crank disk?

CHAPTER XVI

MACHINE FRAMES

364. The stresses which occur in machine frames are often of such a complex nature that they are difficult to analyze with any great degree of mathematical exactness. The frame of a machine is designed to resist the acceleration effects of the moving parts and the forces which the machine transmits in doing its work. These forces may vary from zero at no load and a low speed, to a maximum value due to a heavy load combined with an increase of speed of the moving parts. A frame should be designed to resist the stresses induced under maximum operating conditions, even though the machine may not be required normally to attain its maximum performance.

Machine frames must be strong enough to resist the stresses induced, and they must be stiff enough to maintain alignment of the bearings. The frames of machine tools, especially, must present adequate surfaces to resist the wear of rubbing parts, react against the working loads, and maintain alignment of parts to insure the machining of surfaces which are within close limits of accuracy.

Minor bearings, lever fulcrums, and supports for various attachments are cast as separate parts, to simplify the making of patterns and castings, and to eliminate the possibility of breakage during then process of machining and trasportation.

Modern machine frames partake of the cabinet form, which encloses many of the moving parts, making them accessible by means of hinged doors or removable plates, which are flush with the frame surface when in place. Machine frames should be adapted to the requirements of the moving parts of a machine, and should be as simple as possible.

The student should refer freely to manufacturers' catalogues for information, and he should be guided by the general lines and proportions of existing machines of the same general type as the one being contemplated in his design.

365. Material for Machine Frames.—Cast iron is used extensively for machine frames because it is inexpensive, easily

machined and finished, and offers good resistance to wear. Its brittleness and low tensile strength are disadvantages. Wall sections of cast-iron frames are limited in thickness to about 4 in., because of the lack of homogeneity of the metal in thick cast-iron sections.

Large machines which operate under severe shock-loading conditions are provided with cast-steel frames, because cast steel has greater strength in proportion to its weight. The rapid development in the art of welding has led to the introduction of fabricated-steel frames, bed plates, and supports for large machines, such as steam turbines, motors, generators, water wheels, and for small machines which are used in temporary locations.

366. Design of Machine Frames.—Modern machine design is the result of evolution in that field, and machine frames,

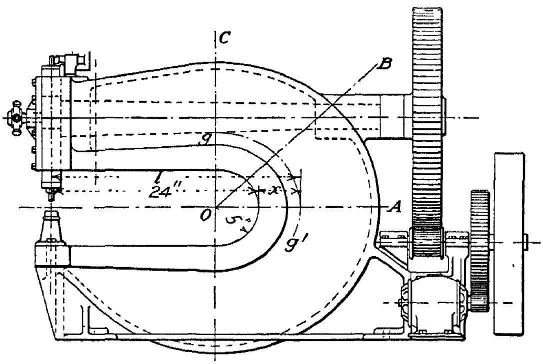


FIG. 1.—Punching and shearing machine.

especially, have developed from crude wooden structures built by cabinet makers. The wooden frames were later replaced by cast-iron frames, but these followed the same general lines. The more recent trend in design is characterized by the absence of ornamentation, and the placing of metal where it will be most effective.

Those parts of machine frames which are subjected to the greatest stresses should be checked for strength and distribution of the load. Very often this can be done most effectively by the trial and error method. For instance, a beam of approximately uniform strength may be designed by determining the dimensions at several different cross-sections, and then connecting these cross-sections in such a way as to make the beam adequate

from the standpoint of strength, also of pleasing outline, and of such form as to be fabricated most conveniently and economically.

There are several types of machine frames which may be designed by using rational formulas. The punching and shearing machine frame shown in Fig. 1 is one of these, and the maximum stresses in the frame are readily found for the maximum working conditions. Cast iron is the metal used for machine frames of this type, except for machines which are designed for very heavy work.

The box section shown in Fig. 2 is a satisfactory form to resist the induced stresses, and the cored center is convenient for the installation of the shaft. Cast iron is about four or five times as strong in compression as it is in tension, and this accounts for the heavy flange used in the box section on the side at which the tensile stresses occur. The distance from the neutral axis to the tensile fiber will be smaller than the distance to the compressive fiber, thus helping to reduce the flexural tensile stress. A machine frame of cast iron usually cannot be designed so that the maximum tensile and compressive stresses will be in proportion to their ultimate strengths, because such a section would be difficult to produce on account of the great differences in wall thickness. It is considered good practice to design the section so that the tensile unit stress will be within safe limits, without attempting to get the full benefit of the high compressive strength.

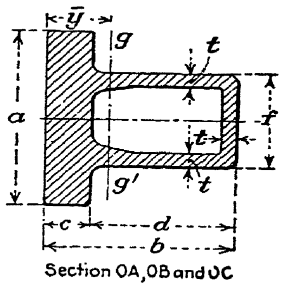


FIG. 2.

Steel has a much higher resistance to bending than has cast iron, and the allowable tensile and compressive unit stresses are equal, hence the frame section when made of steel would be in the form of a modified I section, reinforced by ribs to resist the eccentric loading which occurs in a shearing machine.

In the frame shown by Fig. 1 the maximum unit stresses will occur at the cross-section on the line OA. The load applied at the punch is an eccentric lôad for this section, and therefore subjects the section to a direct tensile stress over the entire cross-section, and a bending stress due to the bending moment. In Fig. 1 the bending moment would be the force at the punch times a moment arm equal to $(24 + x)$ in. The basis of this analysis is the theorem in mechanics which states that a force

may be resolved into a force at a given point and a couple. If the force at the punch in Fig. 1 is called P, and the distance x is the distance from the edge of the frame to the centroid of the cross-section, then at the centroidal point there may be placed two opposite forces equal to P. It is then evident that the cross-section is under a direct tension due to P and a moment equal to the moment of the couple. The formulas for this case were derived in Sec. 135, Chap. VII.

If the sides of the punch frame in Fig. 1 at the section OA are vertical, then the following formulas give the unit stresses:

$$S_t = \frac{P}{A}\left(1 + \frac{ec_1}{r^2}\right) = \frac{P}{A} + \frac{Pec_1}{I}, \tag{1}$$

$$S_c = \frac{P}{A}\left(1 - \frac{ec_2}{r^2}\right) = \frac{P}{A} - \frac{Pec_2}{I}, \tag{2}$$

in which S_t denotes the maximum tensile unit stress, in pounds per square inch.

P denotes the load at the punch, in pounds.

e denotes the eccentricity of P with respect to the centroid of the cross-section, in inches.

c_1 denotes the distance from the centroid to the tensile fiber, in inches.

r denotes the radius of gyration of the cross-section, in inches.

S_c denotes the maximum compressive unit stress, in pounds per square inch.

c_2 denotes the distance from the centroid to the compressive fiber, in inches.

If the sides of the frame are curved, as they are in Fig. 1, then the maximum stress is greater than that given by formula (1) because of this curvature, the stress increasing as the radius of curvature decreases. The formulas for curved beams were discussed in Sec. 136, Chap. VII, and are as follows:

$$S_t = \frac{Pe}{R(A' - A)}\left(\frac{R}{R - c_1} - \frac{A'}{A}\right) + \frac{P}{A'}, \tag{3}$$

$$S_c = \frac{Pe}{R(A' - A)}\left(\frac{A'}{A} - \frac{R}{R + c_2}\right) - \frac{P}{A'}, \tag{4}$$

in which S_t denotes the maximum tensile unit stress, in pounds per square inch.

P denotes the load at the punch, in pounds.

e denotes the eccentricity of the load with respect to the centroid of the cross-section, in inches.

R denotes the radius of curvature of the centroidal axis of the cross-section, in inches.

A' denotes a factor which depends upon the shape and size of the cross-section, in in.^{2}.

A denotes the area of the cross-section, in in.^{2}.

c_1 denotes the distance from the centroid to the tensile fiber, in inches.

S_c denotes the maximum compressive unit stress, in pounds per square inch.

c_2 denotes the distance from the centroid to the compressive fiber, in inches.

The factor A' in formulas (3) and (4) is given by the following formula:

$$A' = R\int\frac{dA}{R + y} \tag{5}$$

Example.—The frame of a punching and shearing machine as shown in Fig. 1 is to be designed. The throat is 24 in., and the punching capacity is a hole 1 in. in diameter, in metal ¾ in. thick. The shearing capacity is 6- by ⅞-in. flats, and 1¾-in. rounds. The frame is to be cast of close grained iron having ultimate tensile and compressive strengths of 25,000 and 100,000 lb. per square inch, respectively. The shearing strength of the steel is taken as 48,000 lb. per square inch. The maximum unit tensile stress in the frame will be limited to 5,000, and the maximum unit compressive stress to 20,000 lb. per square inch.

The force required to punch a 1-in. hole in a ¾-in. plate is:

$$F = \pi dtS = 3.14 \times 1 \times \tfrac{3}{4} \times 48{,}000 = 113{,}000 \text{ lb.}$$

The force required at the punch depends to some extent upon the clearance between the punch and die. American practice assumes that the force required at the punch is 25 per cent greater than that found above, hence:

$$F = 113{,}000 \times 1.25 = 141{,}000 \text{ lb.}$$

Shear blades are beveled to reduce the force required for shearing flat plates. Instead of shearing off the full width of the plate, the top blade comes in contact with the plate at one edge and begins to cut into only a small portion of the full width. Experiment has shown that this force is:

$$F = \frac{0.225St^2}{\tan\alpha} \tag{6}$$

in which α denotes the angle of the bevel, which is usually made from 5 to 10 deg.

Assuming an angle of 8 deg. for the upper plate, the force required for the flat plate is:

$$F = \frac{0.225 \times 48{,}000 \times 0.875^2}{0.141} = 58{,}600 \text{ lb.}$$

For the circular bar:

$$F = \frac{\pi \times 1.75^2}{4} \times 48{,}000 = 116{,}000 \text{ lb.}$$

The maximum load is therefore determined by the force required for punching the hole.

It is evident that the box section in Fig. 2 and also formulas (3) and (4) are too complicated to make a direct solution possible. The method of trial and error will therefore be used. A cross-section with given dimensions will be assumed, and a calculation for the maximum unit stress will then be made to determine whether the stresses are within safe limits.

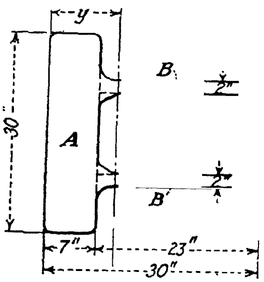


FIG. 3.

Some ideas as to the relative dimensions for box sections may be obtained from "Machinery's Handbook."[1]

The dimensions shown in Fig. 3 will be used for a trial calculation.

The position of the centroid must first be determined, and for this purpose the section in divided into the four rectangles A, B, B, C. The fillets are neglected.

$$A = 210 \text{ in.}^2 \quad B = 42 \text{ in.}^2 \quad C = 32 \text{ in.}^2 \quad \text{Total area} = 326 \text{ in.}^2$$

$$\bar{y} = \frac{210 \times 3.5 + 2 \times 42 \times 17.5 + 32 \times 29}{210 + 2 \times 42 + 32} = \frac{3{,}132}{326} = 9.61 \text{ in.}$$

The moment of inertia with respect to the centroidal axis will be determined next, to be used later for an approximate calculation.

$$\text{For } A, \quad I_y = \frac{30 \times 7^3}{12} + 210 \times 6.11^2 \quad 8{,}677 \text{ in.}^4$$

$$\text{For } 2B, \quad I_y \quad 2\left(\frac{2 \times 21^3}{12} + 42 \times 7.89^2\right) = 8{,}326 \text{ in.}^4$$

$$\text{For } C, \quad I_y \quad \frac{16 \times 2^3}{12} + 32 \times 19.39^2 = 12{,}031 \text{ in.}^4$$

Total $I_y = 8{,}677 + 8{,}326 + 12{,}031 = 29{,}034$ in.^{4}

An approximate calculation will be made first, neglecting the curvature and using the simple expressions in formulas (1) and (2).

$$S_t = \frac{141{,}000}{326} + \frac{141{,}000 \times 33.61 \times 9.61}{29{,}034} =$$
$$432 + 1{,}570 \quad 2{,}002 \text{ lb. per square inch.}$$

$$S_c = \frac{141{,}000}{326} - \frac{141{,}000 \times 33.61 \times 20.39}{29{,}034} =$$
$$432 - 3{,}320 = 3{,}752 \text{ lb. per square inch.}$$

[1] "Machinery's Handbook," 6th Ed., p. 356, Industrial Press, New York, N. Y.

The value of 2,000 lb. per square inch in tension indicates that the section may perhaps be large enough even when the curvature is taken into account, and a trial calculation will therefore be made on this basis.

The factor A' must next be determined. In Fig. 1 the center of curvature lies on the flange side of the section, and the values of y in the formula are negative to the left of the centroidal axis, and positive to the right of that axis.

$$\text{For rectangle } A,\ A' = R\int\frac{dA}{R+y} = 14.61\int_{-9.61}^{-2.61}\frac{30dy}{14.61+y}.$$

$$= 438\log_e\left(\frac{14.61-2.61}{14.61-9.61}\right) = \frac{438\times 0.380}{0.434} = 383\text{ in.}^2$$

$$\text{For } 2B,\ A' = 14.61\int_{-2.61}^{+18.39}\frac{4dy}{14.61+y} =$$

$$58.44\log_e\left(\frac{14.61+18.39}{14.61-2.61}\right) = \frac{58.44\times 0.439}{0.434}.$$

$$= 59.1\text{ in.}^2$$

$$\text{For } C,\ A' = 14.61\int_{18.39}^{20.39}\frac{16dy}{14.61+y} =$$

$$23.4\log_e\left(\frac{14.61+20.39}{14.61+18.39}\right) = \frac{23.4\times 0.0253}{0.434}.$$

$$= 1.36\text{ in.}^2$$

$$\text{Total } A' = 383 + 59.1 + 1.36 = 443\text{ in.}^2$$

Using formula (3):

$$S_t = \frac{141{,}000\times 33.61}{14.61(443-326)}\left(\frac{14.61}{14.61-9.61} - \frac{443}{326}\right) + \frac{141{,}000}{326}.$$

$$= 4{,}330 + 432 = 4{,}762\text{ lb. per square inch.}$$

Using formula (4):

$$S_c = \frac{141{,}000\times 33.61}{14.61(443-326)}\left(\frac{443}{326} - \frac{14.61}{14.61+20.39}\right)$$

$$= 2{,}610 - 432 = 2{,}178\text{ lb. per square inch.}$$

Since the maximum tensile unit stress is within the allowable value of 5,000 lb. per square inch the design will be considered satisfactory.

Comparing the result obtained when the curvature is taken into account, with the approximate method in which curvature is neglected, the value of maximum unit tensile stress is 4,762 lb. per square inch in one case, and 2,000 lb. per square inch in the other. The flexural unit stress in the one case is almost three times as great as in the other. It is evident, therefore, that the approximate method is not a safe method for design.

Investigation shows that the section at OA in Fig. 1 is subjected to the greatest bending moment and should be the strongest section. Any other section, such as OB, at an angle of 45 deg., will be subjected to a smaller moment but an increasing shear stress. In this case the force producing tension over the entire cross-section is:

$$F = 141{,}000 \times 0.707 = 99{,}600\text{ lb.}$$

The force producing shear on this section is equal to the same amount.

The moment arm of the force producing bending is decreased by an amount y (see Fig. 4), so that the moment arm is:

$$\begin{aligned} \text{Arm} &= 19 + 14.61 \times 0.707 \\ &= 19 + 10.3 = 29.3 \text{ in.} \end{aligned}$$

If the cross-section is the same at OB in Fig. 1 as it was at OA, the direct tensile stress is:

$$\frac{P}{A} = \frac{99{,}600}{326} = 306 \text{ lb. per square inch.}$$

The shear stress may be computed from formula (10), Sec. 133, Chap. VII.

$$S_s = \frac{V}{Ib} A'y'.$$

$V = 99{,}600$ lb., $I = 29{,}034$ in.^{4}, $b = 4$ in.

$A'y' = 16 \times 2 \times 19.39 + 2 \times 18.39 \times 2 \times 9.19 = 620 + 675 = 1{,}295$ in.^{3}.

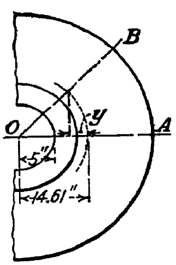


FIG. 4.

Substituting in the formula:

$$S_s = \frac{99{,}600 \times 1{,}295}{29{,}034 \times 4} = 1{,}110 \text{ lb. per square inch.}$$

The maximum bending stress will be in the ratio of the bending moments or moment arms at the sections OB OA and, therefore:

$$S_t = \frac{4{,}330 \times 29.3}{33.61} = 3{,}770 \text{ lb. per square inch.}$$

The maximum tensile unit stress is therefore:

$$S = 306 + 3{,}770 = 4{,}076 \text{ lb. per square inch.}$$

The shear stress of 1,110 lb. per square inch is amply safe.

The fact that the maximum unit stress at the section OB is less than at the section OA, indicates that the outside radius of the frame may be gradually decreased for sections between OA and OC.

The portion of the frame to the left of OC acts as a beam, and might be gradually tapered as the free end of the beam is approached, if it were not for the shaft, bearings, and the mechanism of the ram.

367. Load Eccentric in Two Directions.—For the cross-section shown in Fig. 5, the usual loading is such that the load P has its action line coinciding with the center line AB. The load is then eccentric only with respect to the centroidal axis gg'. In a shearing machine or in a drill press the load may be eccentric with respect to the axis gg' and also with respect to the axis AB, as shown in the figure. To determine the maximum unit stress induced in the frame for such a case, it is necessary to know the location of the principal axes through the center of gravity. The principal axes for an area, are the axes through a given point with respect to which the moments of inertia are respectively

a maximum and a minimum. For the section in Fig. 5 the axes AB and gg' are the principal axes.

For the eccentricities of the load shown in Fig. 5 the maximum unit stress would occur at corner D.[1] The bending moment with respect to axis gg' would be P times y_1, and to obtain the unit stress at corner D, formula (3) should be used, because for this axis the beam is a curved beam. The bending moment with respect to axis AB is P times x_1, and the unit stress may be obtained by the formula $S = Mc/I$, because the beam is not a curved beam with respect to this axis. For corner D these two flexural stresses will both be tension, and to them must be added the direct stress, which is also tension and is equal to P/A.

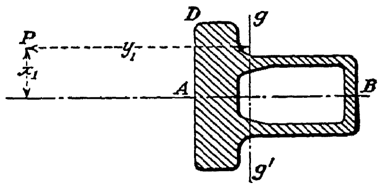


FIG. 5.

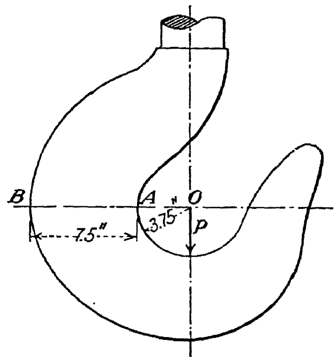


FIG. 6.

368. Hooks.—While the subject of hooks does not logically belong in this chapter, it will be discussed here because the action of a load on a hook is similar to that just discussed for punch frames. In Fig. 6 the load P induces a direct tension and a flexural stress on the section at AB. The proportions which are commonly used in the design of hooks may be found in the various engineering handbooks.[2]

Example.—For the hook shown in Fig. 6 the load is 30,000 lb., and the hook has been given dimensions which are commonly used. The maximum unit stresses in tension and compression are to be determined.

The dimensions of the cross-section at BA in Fig. 6 are shown in Fig. 7. For convenience of calculation the cross-section of the hook is transformed

[1] MAURER and WITHEY, "Strength of Materials," p. 309.
BOYD, "Strength of Materials," p. 247.

[2] HALSEY, "Handbook for Machine Designers and Draftsmen," p. 486, 2nd Ed., McGraw-Hill Book Company, Inc.

into the trapezoidal section as indicated, the area which is subtracted being made approximately equal to the area which is added.

$$\text{Area} = \frac{(5 + 3.5) \times 7.5}{} = 31.8 \text{ in.}^2$$

$$\bar{y} = \frac{(5 + 2 \times 3.5) \times 7.5}{3(5 + 3.5)} = 3.53 \text{ in.}$$

$$' = R\int\frac{dA}{R + y} = 7.28\int\frac{b'dy}{7.28 + y}.$$

The variable width $b' = 4.3 - 0.2y$.

$$A' = 7.28\int_{-3.53}^{+3.97}\frac{(4.3 - 0.2y)dy}{7.28 + y} = 7.28\int_{-3.53}^{+3.97}\left(-0.2 + \frac{5.76}{y + 7.28}\right)dy.$$

$$= 7.28\left(-1.5 + 5.76 \log_e \frac{11.25}{3.75}\right) = 35 \text{ in.}^2$$

Substituting in formula (3):

$$S_t = \frac{30{,}000 \times 7.28}{7.28(35 - 31.8)}\left(\frac{7.28}{7.28 - 3.53} - \frac{35}{31.8}\right) + \frac{30{,}000}{31.8}.$$

$$= 7{,}860 + 940 = 8{,}800 \text{ lb. per square inch.}$$

$$S_c = \frac{30{,}000 \times 7.28}{7.28(35 - 31.8)}\left(\frac{35}{31.8} - \frac{7.28}{7.28 + 3.97}\right) - \frac{30{,}000}{31.}$$

$$= 4{,}230 + 940 = 5{,}170 \text{ lb. per square inch.}$$

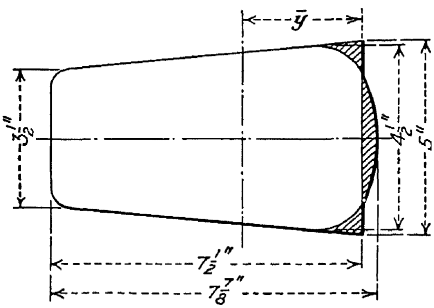


FIG. 7.

Halsey states that for a high grade of wrought iron, properly heat treated, a maximum unit tensile stress of 17,000 lb. per square inch is permissible. For wrought iron with an ultimate tensile strength of 48,000 lb. per square inch this cannot be considered safe, especially when it is remembered that a hook may be subjected to a shock load, and that there is considerable danger to life and limb in case of failure. Using a factor of safety of seven the allowable unit stress is about 7,000 lb. per square inch for wrought iron, and about 8,500 lb. per square inch for steel. The above computed maximum unit stress may therefore be considered safe for a hook which is made of a good grade of steel.

If the curvature is neglected in the above problem, the maximum unit tensile stress is as follows:

$$I_y = \frac{5^2 + 4 \times 5 \times 3.5 + 3.5^2}{36(5 + 3.5)} \times 7.5^3 \quad 148 \text{ in.}^4$$

$$S_t = \frac{Mc}{I} + \frac{P}{A} \quad \frac{30{,}000 \times 7.28 \times 3.53}{148} + \frac{30{,}000}{31.8}$$

$$= 5{,}220 + 940 = 6{,}160 \text{ lb. per square inch.}$$

This calculation shows again that the flexural unit stress is much too low when determined by this approximate method.

Sometimes the cross-section of a hook, punch frame, or other curved beam is of such shape that it cannot be conveniently broken up into areas for which the factor A' may be computed. In such cases the value of A' may be determined by employing graphical or semi-graphical methods.[1]

Problem

1. *Punch and Shearing Machine.*—Design and make a complete set of drawings for a $\left\{\begin{array}{l}\text{single-end}\\\text{double-end}\end{array}\right.$ punch and shearing machine in accordance with the following specifications:

Capacity.—The capacity must be equal to that given in Table I for machine No. .... The general specifications must be consistent with those capacities and dimensions given in Table II and for a *throat depth* of ... in.

Frame.—The frame of the machine is to be made of a good grade of cast iron, free from defects, and to be of box section amply designed for its rated capacity. The permissible maximum unit stresses to be used for this design are .... lb. per square inch for tensile and shear and .... lb. per square inch for compression.

Shafts.—The main shaft is to be forged of open-hearth steel containing from 0.20 to 0.30 per cent carbon. The journals to be large enough to insure ample bearing surface and good lubrication.

Bearings.—The main shaft bearings are to be bushed with bronze.

Pendulum.—The pendulum is to be a steel casting, finished to fit accurately at the eccentric and ram, and provision made to insure good lubrication. The pendulum block is to be of phosphor-bronze.

Ram.—The ram is to be of cast steel and proportioned to present ample sliding surfaces. A bronze gib should be provided to allow for adjustment for wear. The punching and shearing tools are to be fastened to the ram by through bolts made easily accessible.

Clutch.—The clutch is to be of .... fitted and attached to the main shaft by two feather keys.

[1] MAURER and WITHEY, "Strength of Materials," p. 218.
MORLEY, "Strength of Materials," p. 339.

Gears.—The gear and pinion are to be made of a good grade of cast iron. The gear is to be made . . . . and the jaws are to be . . . . The gear teeth are to be machine molded. The gear ratio is to be . . . . to . . . .

Flywheel.—The flywheel is to be made of a good grade of cast iron. The outside diameter to be . . . . .in. and the face . . . . in.

Pulleys.—Tight and loose pulleys are to be provided. The diameter of the pulleys are . . . . in. and the face . . . . in. The driving belt is to be two-ply leather and . . . . in. wide.

Pinion Shaft.—The pinion shaft is to be cold rolled steel shafting provided with ample journals. The boxes are to be split and lined with babbitt metal.

Back Frame.—The back frame is to be made of a good grade of cast iron and to conform with the general lines of the main frame. The back frame is to be fastened to the main frame by through bolts.

Other Details.—The machine is to be provided with a treadle connected to the clutch operating mechanism by adjustable links. The ram is to be provided with an adjustable counterbalance. All pins, studs, and levers are to be designed to withstand reasonably hard usage. The machine should conform with all local and state safety rules and regulations.

TABLE I.—PUNCH AND SHEAR CAPACITY

Size number	Punching pressure, tons	Area sheared, square inches	Maximum hole punched	Bars sheared, inches	Rounds sheared, inches	Angles sheared, inches	Thickness of plate split, inches
1	7.5	0.30	$\frac{3}{8} \times \frac{1}{4}$	$2\frac{3}{4} \times \frac{1}{4}$	$\frac{3}{4}$	$1\frac{1}{4} \times 1\frac{1}{4} \times \frac{3}{16}$	$\frac{1}{4}$
2	20.0	0.80	$\frac{1}{2} \times \frac{1}{2}$	$3 \times \frac{1}{2}$	1	$1\frac{3}{4} \times 1\frac{3}{4} \times \frac{1}{4}$	$\frac{3}{8}$
3	30.0	1.20	$\frac{5}{8} \times \frac{5}{8}$	$3 \times \frac{5}{8}$	$1\frac{1}{8}$	$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$	$\frac{1}{2}$
4	50.0	2.00	$\frac{7}{8} \times \frac{3}{4}$	$5 \times \frac{3}{4}$	$1\frac{1}{2}$	$4 \times 4 \times \frac{1}{4}$	$\frac{3}{4}$
5	78.0	3.12	1×1	$6 \times \frac{7}{8}$	$1\frac{3}{4}$	$6 \times 6 \times \frac{1}{4}$	$\frac{7}{8}$
6	100.0	4.00	$1\frac{1}{4} \times 1$	7×1	2	$6 \times 6 \times \frac{3}{8}$	1
7	137.0	5.50	$1\frac{3}{4} \times 1$	$8 \times 1\frac{1}{8}$	$2\frac{1}{4}$	$6 \times 6 \times \frac{1}{2}$	$1\frac{1}{8}$
8	200.0	8.00	$2\frac{1}{4} \times 1\frac{1}{8}$	$9 \times 1\frac{1}{4}$	$2\frac{1}{2}$	$6 \times 6 \times \frac{3}{4}$	$1\frac{1}{4}$
9	242.0	9.68	$2\frac{1}{4} \times 1\frac{3}{8}$	$10 \times 1\frac{1}{4}$	$2\frac{3}{4}$	$6 \times 6 \times \frac{7}{8}$	$1\frac{3}{8}$
10	300.0	12.00	$2\frac{1}{2} \times 1\frac{1}{2}$	$11 \times 1\frac{1}{2}$	$3\frac{1}{4}$	$8 \times 8 \times \frac{3}{4}$	$1\frac{1}{2}$
11	475.0	19.00	3×2	$10 \times 2\frac{1}{2}$	$4\frac{1}{4}$	$8 \times 8 \times 1\frac{1}{4}$	$2\frac{1}{2}$

TABLE II.—SPECIFICATIONS OF PUNCH AND SHEAR PRESSES. SINGLE OR DOUBLE END

Machine size number	1	2	3	4	5	6	7	8	9	10	11
Stroke, inches	¾	⅞	1	1¼	1½	1¾	2	2	2¼	2½	3½
Die space, inches	6¾	6 15/16	8¼	10½	10	11	12	12	13	13	19
Ram face, inches	2⅜×8	3×9½	3¼×10	4¼×13	5¼×15	6×18	7×19½	8×21	9×22	10×23	15×24
Size of table, inches	5×10¾	7½×11½	8×13	8×16½	10½×20	14×23	13×25	12×28	13×29	13×30	16×33
Size hole in table, inches	2×3	2×2	2×3	3×5	3×6	3×10½	3×12	3½×14	3½×9	3½×9	6 diam.
Bar shear blade length, inches	5	5	5½	6	7	8	9	10	11	12	15
Splitting shear blade length, inches	8	8½	9	13	15	18	19	20	22	23	
Size of pulley, inches	10×3¼	12×4¼	14×4¼	16×5¼	18×5¼	20×6½	30×6½	30×8½	32×8½	32×9½	40×10½
Speed of pulley, revolutions per minute	276	276	241	207	243	215	200	187	176	400	405
Motor, horsepower	1	2	2	5	6	8	10	15	20	20	25
Strokes per minute	40	40	35	30	30	30	25	25	22	20	15
Throat depths, inches	6	6	8	8							
			12	12	12						
			16	16	15	15	15	15	15	15	
			20	20	20	20	20			18	20
			25	25	25	25	25	25	25	25	
			30	30	30	30	30	30			
			36	36	36	36	36	36	36	36	36
						42	42	42			
			48	48	48	48	48	48	48	48	
				60	60	60	60	60	60		
						72	72	72			

CHAPTER XVII

SAFETY ENGINEERING DESIGN

369. Any moving part of a machine which is within the zone of a worker is considered an accident hazard, and may be the cause of an injury. Machines employed on heavy work, or machines which are operated at high speeds, are especially hazardous, unless precautions have been taken to guard against accidents.

The records of one of the state[1] accident boards shows that during one year there were 90,168 accidents in the industries of that state, of which 474 were fatal. Machinery was responsible for 32,088 accidents, 215 of which were fatal. It is estimated by safety engineers that from 50 to 75 per cent of all accidents are preventable, one-half by the removal of dangerous hazards by the application of safety devices, and one-half by safety education.

Most of the states have enacted legislation which prescribes reasonable protection for machine operators and other workmen who may be exposed to injury. States having such laws hold the employer of the workers responsible for injuries to their employees, provided workers are injured while performing their regular duties.

Industrial concerns and other employers usually carry casualty insurance for the protection of the workers. Insurance premium rates are lower in well-regulated industries where reasonable precautions are taken to prevent accidents, by providing machinery and shop equipment with safety appliances, than in industries where these appliances are lacking.

The Industrial Commissions or Accident Boards of the several states are charged with the enforcement of the laws, which hold the employers liable for compensation for the temporary disability of the worker, or for an injury of a permanent nature. These commissions and boards are often given quasi-judicial powers to make rules and regulations for the protection of

[1] *Annual Report* of the Massachusetts Industrial Accident Board, 1913.

workers, without imposing unnecessary or unreasonable hardships upon the employers. These boards also act as arbiters on disputed points, and have legal power to penalize employers who fail to provide reasonable protection for their workers.

Purchasers of machinery and mechanical equipment demand that the builders provide safety devices in accordance with the accepted standard of local requirements, and it frequently happens that the safety features of one machine are the deciding factor in its choice over another machine offered in competition.

Safety appliances should in no way interfere with the operation of the machine, should not slow up production, nor inconvenience the worker unnecessarily.

The lack of uniform state and underwriter company requirements has been partially overcome by the adoption, in 1927, of a safety code by the American Engineering Standards Committee,[1] and the following discussion has been abstracted from the first revision of the Safety Code for Mechanical Power-Transmission Apparatus.

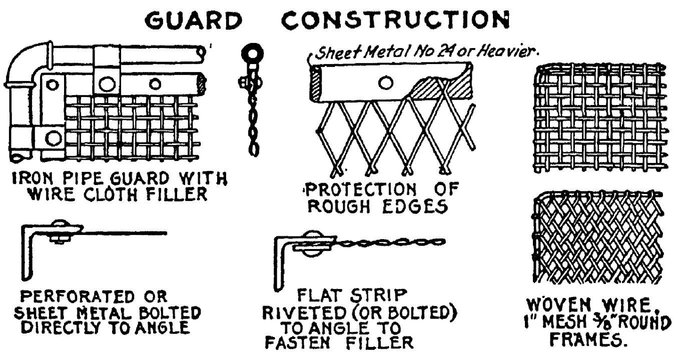
370. Purpose of the Code.—The purpose of the code is to provide reasonable safety for life, limb, and health. The suggestions made in the code are intended to serve as a guide for design engineers, and apply especially to new machines and installations. The rules given should be liberally construed and applied to secure the best results, and in case of practical difficulty or unnecessary hardship, exception to the literal requirements may be granted if equivalent protection is secured. Where specific devices or methods are mentioned, other devices or methods which will secure equally good results may be used, subject to the approval of the enforcing authority.

371. Guards for Prime Movers.—Prime movers include steam, gas, oil, and air engines, motors, steam turbines, and hydraulic turbines. When a flywheel is located so that any part is 6 ft. or less above the floor, it should be guarded in one of the following ways:

(*a*) With an enclosure of sheet, perforated, or expanded metal, or woven wire, as shown in Fig. 1. The specifications for this material are given in Table I.

[1] The sponsor organizations are: National Bureau of Casualty and Surety Underwriters, International Association of Industrial Accident Boards and Commissions, and the American Society of Mechanical Engineers.

TABLE I.—MATERIALS AND DIMENSIONS FOR ALL GUARDS EXCEPT THOSE FOR HORIZONTAL OVERHEAD BELTS



Material	Clearance from moving part at all points, inches	Largest mesh or opening allowable, inches	Minimum gage (U. S. Stand.) or thickness	Minimum height of guard from floor or platform level, feet and inches
Woven wire	Under 4	½	No. 16	6–0
	4–15	2	No. 12	5–0
Expanded metal	Under 4	½	No. 18	6–0
	4–15	2	No. 13	5–0
Perforated metal	Under 4	½	No. 20	6–0
	4–15	2	No. 14	5–0
Sheet metal	Under 4	. . .	No. 22	6–0
	4–15	. . .	No. 22	5–0
Wood or metal strip crossed	Under 4 4–15	½	Wood ¾ in. Metal No. 16	6–0
		2	Wood ¾ in. Metal No. 16	5–0
Wood or metal strip not crossed	Under 4	½ width	Wood ¾ in. Metal No. 16	6–0
	4–15	1 width	Wood ¾ in. Metal No. 16	5–0
Standard rail	Min. 15 Max. 20			

(*b*) With guard rails as shown in Fig. 2, placed not less than 15 in. nor more than 20 in. from the rim. Specifications for guard rails are included in the code. When a flywheel extends into a pit, or is within 12 in. of the floor, a toe board should be provided which is at least 4 in. high, made of wood, metal, or metal grill. Toe boards at flywheel pits should be placed close to the edge of the pit.

(*c*) The upper part of a flywheel which protrudes through a working floor should be entirely enclosed, or surrounded by a guard rail and toe board.

(*d*) For flywheels[1] with smooth rims which are 5 ft. or less in diameter, when the preceding methods cannot be applied, the following may be used: A sheet-metal disk is attached to the flywheel so as to cover the spokes on the exposed side, presenting a smooth surface and edge. A 4-in. open space may be left

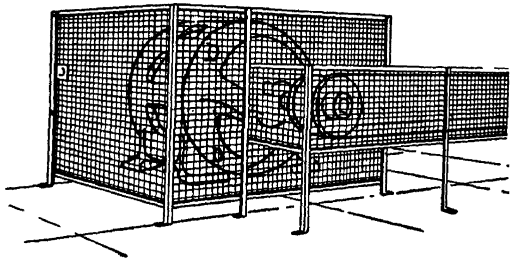


FIG. 1.—Guard of woven wire.

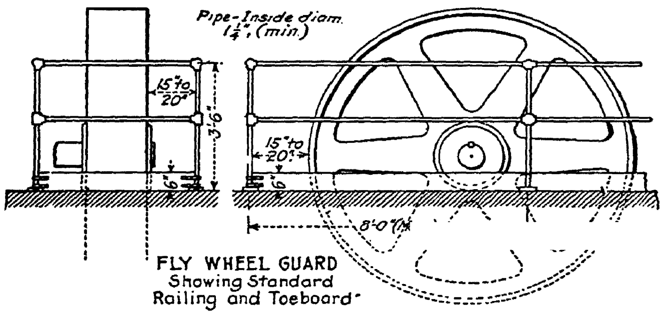


FIG. 2.—Guard rail enclosure.

between the outside edge of the disk and the rim of the wheel, to facilitate turning the wheel over. When a disk is used, the keys or other dangerous projections should either be cut off or covered.

Engine cranks, connecting rods, tail rods, or extension piston rods, which are exposed to contact, should be enclosed by a guard rail or a metal guard, fastened to the frame of the machine or to the floor.

[1] It may be of interest to the student to know that casualty insurance rates are higher for flywheels than for boilers, which indicates that accidents due to the bursting of flywheels are more frequent than those due to boiler explosions.

Governor balls which are 6 ft. or less from the floor or other working level, if exposed to contact, should be enclosed for the full working range of the governor.

372. Guards for Shafting.—Each continuous line of shafting should be secured in position against excessive endwise movement. Inclined and vertical shafts, especially inclined idler shafts, should be securely held in position against end thrust. All exposed parts of horizontal shafting which is 6 ft. or less from the floor or working platform (excepting runways used exclusively for oiling or for making running adjustments) should be protected by a stationary casing which encloses the shaft completely, as shown in Fig. 3(*a*), or by a trough enclosing the sides and top as shown in Fig. 3(*b*), or by a trough enclosing the sides and bottom, as shown in Fig. 3(*c*).

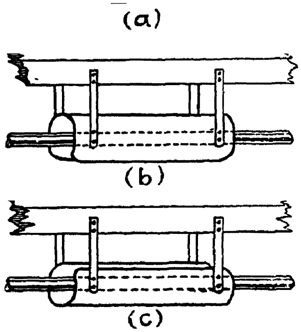


FIG. 3.—Shaft guards.

Shafting which extends over a driveway should be protected as indicated above, unless it is 15 ft. or more above the driveway.

Shafting under bench machines should be enclosed by a fixed casing, as shown in Fig. 4. The sides of the trough should come within 6 in. of the under side of the table, or, if near the floor,

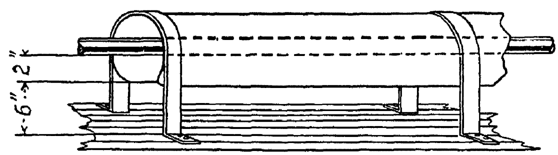


FIG. 4.—Guard for shaft when located near the floor.

within 6 in. of the floor. The trough should extend at least 2 in. below or above the shafts.

Projecting shaft ends should present a smooth edge, and the shaft should not project a distance greater than one-half its diameter, unless provided with a non-rotating cap or safety sleeve. Unused keyways should be filled up or covered.

373. Pulley Guards.—Any parts of a moving pulley which are 6 ft. or less from the floor or working platform should be guarded.

Pulleys which serve as flywheels, on which the point of contact between the belt and pulley is more than 6 ft. from the floor or platform, should be guarded with a disk as described for flywheels. Pulleys with cracks, or pieces broken out of the rims, should be replaced.

374. Location of Pulleys.—When the distance between a shaft bearing and a fixed pulley is less than the width of the belt, a guide should be provided to prevent the belt from leaving the pulley on the side where insufficient clearance exists.

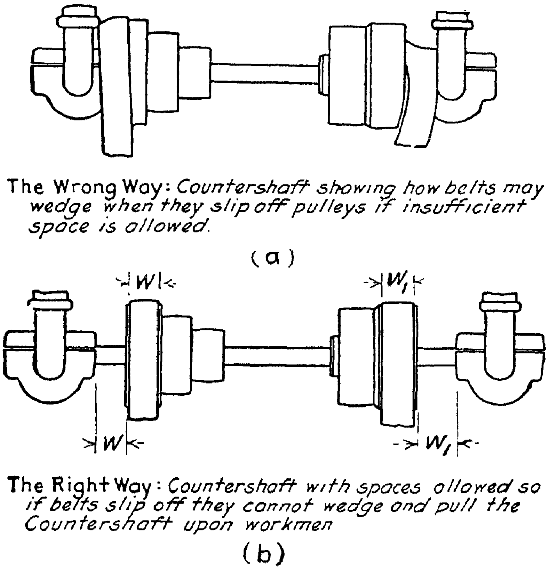


FIG. 5.—Wrong and right ways of locating pulleys.

When a pulley overhangs a bearing, a guide should be provided on the outer side of the pulley, to prevent the belt from running off the pulley.

Pulleys should be located on the shaft so that the distance between the pulley and the shaft bearing is equal to the width of the belt. Figure 5(a) shows how pulleys should *not* be located on the shaft with respect to the bearings, and Fig. 5(b) shows the correct location.

Ordinary pulleys should not be operated at rim speeds in excess of 4,000 ft. per minute. When pulleys operate at rim speeds in excess of 4,000 ft. per minute they should be balanced for the speed at which they operate.

Composition pulleys and laminated-wood pulleys should not be used in locations which are continually subjected to the action of moisture. Pulleys used under conditions producing active corrosion should be constructed of corrosion-resisting material.

Pulleys which are permanently out of service should be removed from shafting which is in use.

375. Belt, Rope, and Chain Guards.—Horizontal belts and ropes which are 6 ft. or less from the floor level should be guarded. The guard should be constructed of material according to

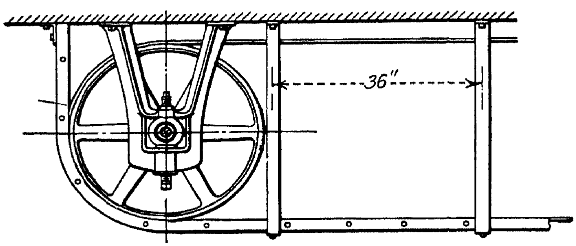


FIG. 6.—Belt guard.

Table I, and should extend at least 15 in. above the belt. Horizontal belts and ropes having both runs 42 in. or less from the floor, should have guards fully enclosing the belt or rope. In power plants or rooms in which power is developed guard rails may be used.

Overhead horizontal belts, with the lower strand 7 ft. or less from the floor or platform, should be guarded on the sides and bottom, as shown in Fig. 6. The guard material and dimensions are given in Table II.

Overhead horizontal belts which are more than 7 ft. above the floor, should be guarded for their entire length under the following conditions:

(*a*) If located over passageways or work places, and if the speed of the belt is 1,800 ft. or more per minute.

(*b*) If the center to center distance between pulleys is 10 ft. or more.

(*c*) If the belt is 8 in. or more in width.

When the upper and lower runs of horizontal belts are so located that passage of persons between them would be possible, the passage should be completely barred by a guard rail or other barrier, or a platform should be constructed over the lower run.

Table II.—Materials and Dimensions for Guards for Horizontal Overhead Belts, Ropes, and Chains 7 Ft. or More Above Floor or Platform

Members	Width			Material
	Over 10 to 14 in. inclusive	Over 14 to 24 in. inclusive	Over 24 in.	
Framework	1½″ × 1½″ × ¼″	2″ × 2″ × 5⁄16″	3″ × 3″ × ⅜″	Angle iron
Filler (belt guards)	1½″ × 3⁄16″	2″ × 3⁄16″	2″ × 5⁄16″	Flat iron
Filler and vertical side member	No. 20 A. W. G.	No. 18 A. W. G.	No. 18 A. W. G.	Solid sheet metal
Filler supports	2″ × 5⁄16″ flat iron	2″ × ⅜″ flat iron	2½″ × 2½″ × ¼″ angle	Flat and angle
Guard supports	2″ × 5⁄16″	2″ × ⅜″	2½″ × ⅜″	Flat iron
Fastenings				
Filler supports to framework	(2) 5⁄16″	(2) ⅜″	(3) ½″	Rivets
Filler flats to supports (belt guards)	(1) 5⁄16″	(1) 5⁄16″	(2) ⅜″	Flush rivets
Filler to frame and supports (rope and chain guards)	3⁄16″ rivets spaced	8″ centers on sides	4″ centers on bottom (2)	
Guard supports to framework	(2) ⅜″	(2) 7⁄16″	⅝″	Rivets or bolts
Guard and supports to overhead ceiling	¼″ × 3½″ lag screws or ½″ bolts	⅝″ × 4″ lag screws or ⅝″ bolts	¾″ × 6″ lag screws or ¾″ bolts	Lag screws or bolts
Details—Spacing, Etc.				
Width of guards	One-quarter wider than belt, rope, or chain drive			
Spacing between filler supports	20″ C. to C.	16″ C. to C.	16″ C. to C.	
Spacing between filler flats (belt guards)	2″ apart	2½″ apart	3″ apart	
Spacing between guard supports	36″ C. to C.	36″ C. to C.	36″ C. to C.	
Other Belt Guard Filling Permitted				
Sheet metal fastened as in rope and chain guards	No. 20 A. W. G.	No. 18 A. W. G.	No. 18 A. W. G	Solid or perforated
Woven wire, 2″ mesh	No. 12 A. W. G.	No. 10 A. W. G.	No. 8 A. W. G.	

Clearance from outside of belt, rope, or chain drive to guard				
Distance center to center of shafts	Up to 15′ inclusive	Over 15′ to 25′ incl.	Over 25′ to 40′ incl.	Over 40′
Clearance from belt, rope, or chain to guard	6″	10″	15″	20″

This platform should be guarded by a railing completely filled with wire mesh or other filler, or by a solid barrier. The upper run should be guarded to prevent contact with the belt by the worker or by objects carried by him. In power plants, only the lower run of a belt need be guarded.

Overhead chain and link belt drives which are in excess of 2 in. wide should be guarded as described for belts.

The American or continuous system of rope drives, which are so located that the condition of the rope (especially the splice) cannot be constantly and easily observed, should be equipped with a "telltale" device which will give warning when the rope begins to fray. A "telltale" device which rings an electric bell when the rope begins to fray is shown in Fig. 7.

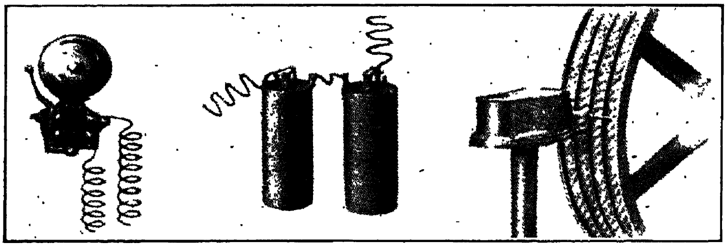


FIG. 7.—"Tell tale" device for indicating frayed rope.

Cone-pulley belts should be equipped with a belt shifter so constructed that it will guard the nip-point of the belt and pulley. The nip-point is the point where the belt running on to a pulley meets the face of the pulley. If the belt is of the endless type or laced with rawhide laces, the belt will be considered guarded if the nip-point guard is located in front of the cone, and extending to the top of the largest step of the cone, and so formed that it will show the contour of the cone in order to give the nip-point of the belt and pulley full protection. If the cone pulley is located less than 3 ft. from the floor, the pulley-and belt should be guarded to a height of 3 ft.

Belt tighteners of the counterbalanced form should be of substantial construction and securely fastened, and the bearings should be capped. Means should be provided to prevent the pulley from falling in case the belt should break. If suspended counterweights are used which are not safe by location, they should be fully guarded.

376. Gear Guards.—Power-driven gears should be guarded by (*a*) a complete enclosure constructed of sheet metal, as shown in Fig. 8, or wire mesh as shown in Fig. 9; by (*b*) a band guard constructed from the materials given in Table I; or by (*c*) a band guard covering the width of the gears and having flanges extending beyond the bottom of the teeth in the exposed sides.

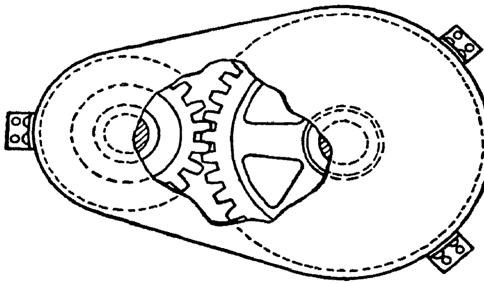


FIG. 8.—Sheet metal gear guard.

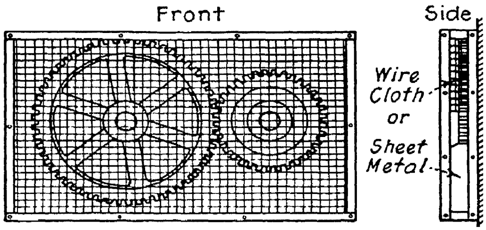


FIG. 9.—Wire mesh gear guard.

377. Sprocket Wheels and Chains.—Power-driven sprocket wheels and chains should be enclosed unless they are at least 7 ft. above the floor.

378. Openings in Guards for Oiling.—When parts which are guarded by enclosed guards require frequent attention and lubrication, the enclosure should be provided with hinged or sliding self-closing covers.

379. Guarding Friction Drives.—The driving portion of friction drives, if exposed to contact, should be guarded. All friction drive wheels, having arms, spokes, or holes in the wheel web, should be enclosed. Projecting bolts or other parts which might be dangerous should be guarded.

380. Keys, Set Screws, and Other Projections.—Keys and set screws, or other projecting parts of revolving members,

should be guarded by a metal cover, or made flush with the circular contour of the moving member. Safety set screws and the method of their application are shown in Fig. 10. When set screws, keys, or oil cups are on the hubs of wheels and pulleys

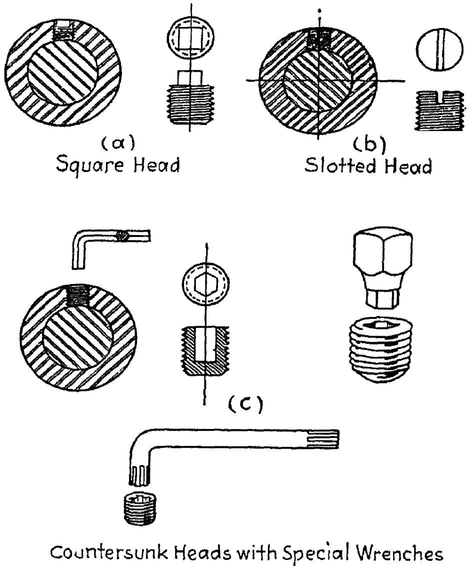


FIG. 10.—Various forms of safety set screws.

which are 20 in. in diameter or less, and which are inside the plane of the rim edges, they are not considered dangerous.

381. Collars and Couplings.—Revolving collars, including the split type, should be cylindrical in form. Set screws or

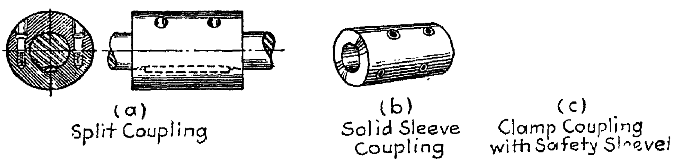


FIG. 11.—Safety couplings.

bolts should not project outside of the large periphery of the collar. Figure 11 shows three types of safety couplings.

Shaft couplings should present no hazards from bolts, nuts, set screws, or revolving surfaces. Three types of safety-flanged couplings are shown in Fig. 12. The shifting parts of jaw

clutches and the shifting or mechanism part of friction clutch couplings should be attached to the driven section of the shaft, that is, the section of the shaft which is idle when the clutch is disengaged.

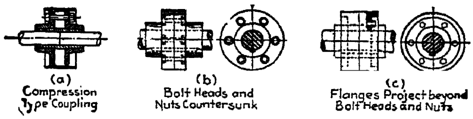


FIG. 12.—Safety-flanged couplings.

382. Bearings.—Self-lubricating bearings are recommended for safety. All oil-drip pans and drip-oil cups should be securely fastened.

383. Starting and Stopping Devices.—A permanent belt-shifting device should be provided for a tight and loose pulley combination, so arranged that it will prevent the belt from creep-

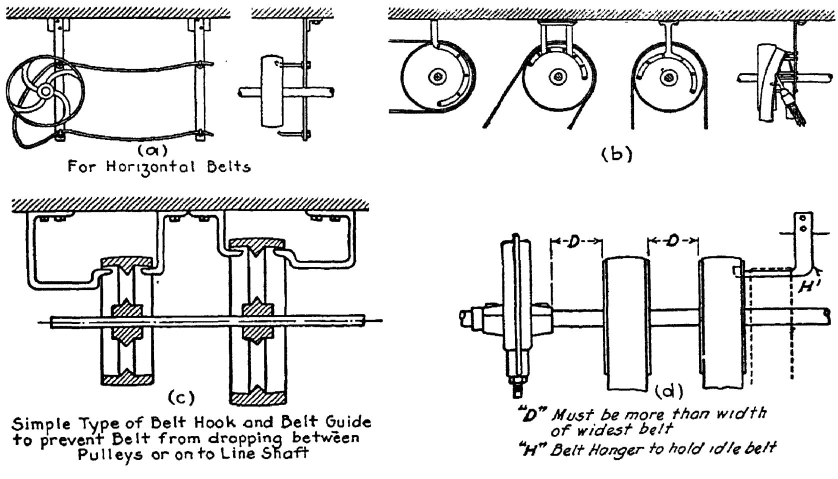


FIG. 13.—Types of belt perches.

ing from the loose to the tight pulley. Belt shifter and clutch handles should be rounded, and located within easy reach of the machine operator. When a belt shifter is not located directly over a machine or bench, the handles should be 6 ft. 6 in. above the floor. Belt and clutch shifters of the same type should move in the same direction to stop machines. This suggestion

does not apply to shifters which have three positions, the handle being in the neutral or central position when the machine is idle.

Belt poles should not be substituted for mechanical belt shifters. If pole belt shifters are used they should be large enough to enable the worker to grasp and hold them securely. A 2-in. diameter or a 1½- by 2-in. rectangular section is suggested. Poles should be made of straight-grained wood such as ash or hickory, and finished smooth, with the edges of rectangular sections rounded. Poles should be long enough to extend from the top of the pulley to within 40 in. of the floor.

Belt perches should be provided to keep idle belts, which are off their moving pulleys, from coming in contact with the pulley or shaft. Several types of belt perches are shown in Fig. 13. Perches should be substantially made, and designed to allow the belt to be shipped easily from the pulley to the perch, and back to position on the pulley.

When the shifting of a belt by hand must be done of necessity, the joint in the belt should not be formed by metal fasteners, or any other fastening, which, by its construction or due to wear, would be considered a hazard.

384. Abrasive Wheels.[1]—All abrasive wheels which operate at speeds in excess of 2,000 surface ft. per minute, should be provided with one of the following forms of protection, which are listed in the order of their importance.

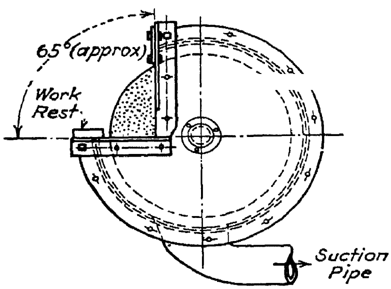


FIG. 14.—Protection hood for abrasive wheel.

(*a*) *Protection Hood.*[2]—A protection hood is an enclosure consisting of a peripheral and two side members, enclosing as much of the grinding wheel as the nature of the work will permit. If the wheel surface speed is below 7,000 ft. per minute, the hood may be made of gray iron, malleable iron, or steel castings. If the surface speed is above 7,000 ft. per minute, the hood should be constructed of fabricated steel, as shown in Fig. 14.

[1] American Standard. Safety Code for the Use, Care, and Protection of Abrasive Wheels. Adopted July 7, 1926, American Engineering Standards Committee.

[2] Specifications for materials for hoods may be procured from the American Society for Testing Materials, Philadelphia, Pa.

(*b*) *Protection Flange.*—A protection flange is designed for a wheel of special shape, so that it will retain the parts of the wheel if the wheel should break in operation.

(*c*) *Protection Bands.*—A protection band is a continuous band placed around a cup, cylinder, or sectional ring wheel, to retain effectually the pieces of such a wheel which might break in operation.

(*d*) *Protection Chuck.*—A protection chuck is used for mounting cup, cylinder, or sectional ring wheels, and is so designed that the jaws enclose the wheel, up to a point which depends upon the thickness of the wheel. The distance which any wheel extends beyond the jaws of the chuck should not exceed 2 in.

Hoods on wheels used for dry grinding should be provided with a connection to an exhaust system, in order to carry away the particles and dust of the grinding operation.

All new wheels should be run at full speed for 1 minute before applying the work, during which time the operator should stand at one side. Abrasive wheels should not be operated at speeds in excess of those which are recommended by the wheel manufacturer.

The above discussion does not apply to metal, cloth, or paper wheels or disks having a layer or layers of abrasive on the surface.

385. Power and Foot-press Guards.—Punch and forming presses are a particularly dangerous type of machine tool to operate. The work must be placed and removed for each machine operation by one of the following methods:

(*a*) *Manual feeding*, or placing the material under the ram by hand, or by the use of hand tools.

(*b*) *Semi-automatic feeding*, or placing the material under the ram by some mechanical device. The device requires the attention of the operator at each stroke of the ram.

(*c*) *Automatic feeding*, or placing the work under the ram by a mechanical device which does not require the attention of the operator at each stroke of the ram.

Press guards are classed as gate or sweep guards, depending upon whether they completely enclose the point of operation before the operating clutch becomes engaged, or move across the point of operation after the operating clutch has become engaged.

Every power press should be provided with some means for disconnecting all power from the press, and from the pulley on the press. Large presses should be provided with means of

stopping the machine instantly at any point in the stroke. Treadles or pedals should be guarded against accidental tripping.

Automatic or semi-automatic feeding devices are the best means of safeguarding the operation of power presses, and they usually result in an increase of production. Automatic feeding devices are actuated by one of the following movements: roll, push, pull, or plunger.

Gate guards are usually preferable to sweep guards, because when the treadle or hand lever is operated, the gate completely encloses the work. Gate guards are usually connected to the foot treadle, and may expand upward, downward, or sidewise.

Two-hand tripping devices require the use of both hands to start the working operation, taking the hands of the operator out of the danger zone.

386. Suggestions for Die Designers, Makers, and Setters.—Feeding devices, knockouts, and guards are usually attached to the die, or must be made to conform to the die, therefore, the designer of the die should have the element of safety constantly in mind while planning for the work. The following suggestions should be given consideration:

1. The method of feeding should be determined before starting the design, to insure the safe operation of the machine.
2. Holes should be tapped in heavy dies for the insertion of screw-eyes for lifting.
3. Guide pins should be used to insure alignment of the dies.
4. The stripper or guard enclosure should be set not more than $\frac{3}{8}$ of an in. above the lower die, and the enclosure should be at least as high as the highest travel of the ram.
5. The front, both sides, and the back of the die should be guarded, so that the operator will not be tempted to reach around the ram if the material sticks.
6. When setting dies or making adjustments (except on large presses which cannot be turned by hand) the power should first be disconnected. If the power is disconnected by the throwing of a belt shifter, clutch lever, or electric switch, these should be locked or blocked in the open position.
7. The dies and die holders should be designed to allow the operator the greatest visibility of the work.

387. Woodworking Machinery.—Machines should be used on work and material for which they are designed. Many woodworking machines accomplish a variety of operations satis-

factorily, and because of this variety it is sometimes difficult to provide effective guards for the point of operation. Whenever practicable, a machine should be selected and guarded for a specific operation, and other work of a different character, involving special hazards, should be assigned to other machines suitable for the work. This may require additional machinery, but usually the cost is offset by the economy in time which would otherwise be required to change the setting of machines, and by the prevention of delays which this causes in a busy shop. All woodworking machines should be stopped immediately upon the completion of an operation, and the accidental starting of machines should be guarded against.

388. Hot Forging and Stamping.—Where practicable, metal guards should be provided at the point of operation, and a guard should be provided to stop all flying particles. This class of machinery includes drop hammers, steam hammers, pneumatic hammers, hydraulic presses, trimming presses, bulldozers, upsetting machines, bolt- and rivet-heading machines, and hot saws.

389. Identification of Piping Systems by Colors.[1]—Various color schemes have been introduced by industrial plants and organizations for the identification of piping systems, and in general the key colors used were satisfactory to those using them. Such color schemes could not be uniform, and their significance was lost to outside agencies such as fire departments, and caused confusion in the minds of those who changed employment from one plant to another. Many mills and plants, desirous of securing the advantages of an identification scheme, did not install one because there was no generally accepted standard available.

390. Definitions.—Piping systems include, in addition to the pipes of any kind, fittings, brackets, valves, and pipe coverings. Pipes are conduits for transporting gases, liquids, and semi-liquids or plastics, but not solids carried in air or gas.

For general identification purposes piping systems are classified as follows:

1. *Fire Protection, Materials, and Equipment.*—Materials and equipment for fire protection include sprinkler systems and other fire-fighting equipment.

[1] Extracted from the report of the Sectional Committee on the Identification of Piping Systems, American Engineering Standards Committee. The sponsor organizations are The National Safety Council, 108 E. Ohio St., Chicago, Ill., and the American Society of Mechanical Engineers, 29 W. 39th St., New York City.

2. *Dangerous Materials.*—Dangerous materials are those which in themselves are hazardous to life or property by virtue of being easily ignited, productive of poisonous gases, or are in themselves poisonous. They include materials which are known ordinarily as fire producers or explosives.

3. *Safe Materials.*—Safe materials are those involving no hazard in their handling, and are of no extraordinarily high value. A workman making repairs on a piping system will run no undue risk in breaking into a pipe bearing a safe material, even though the pipe had not been emptied by previous arrangement.

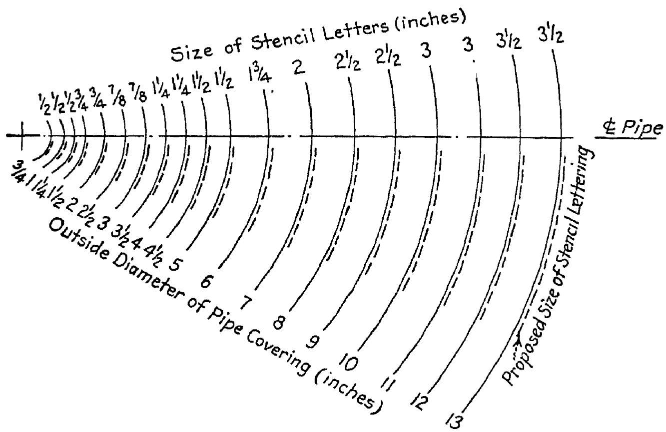


FIG. 15.—Location and size of lettering on pipe.

4. *Protective Materials.*—Protective materials are those which are piped through plants to be available either to prevent or minimize the hazards of dangerous materials. A plant may have special gases, which are antidotes to poisonous fumes, piped through the shop to be released in case of danger.

5. *Extra Valuable Material.*—Extra valuable material might be classed with safe materials, but because of its high value it is given a separate major classification.

391. Method of Identification.—At conspicuous places in a piping system *color bands* should be painted on the pipes to designate to which one of the five main classes the contents belong. If desired, the entire length of the piping system may be painted according to the main classification color.

The actual contents of pipes may be indicated, preferably by a stenciled legend, giving, the name of the contents in full or in

abbreviated form. These legends should be placed on the colored bands. The identification may be extended by the use of colored stripes placed at the ends of the colored bands. Figure 15 shows the size of stencil letters to be used, and their location with respect to the center line of the pipe. The bands, legends, and stripes should be placed at intervals throughout the piping system, preferably adjacent to valves and fittings, to insure ready recognition during operation, while making repairs, and at times of emergency. Figure 16 shows the position of legends with respect to visibility from the floor.

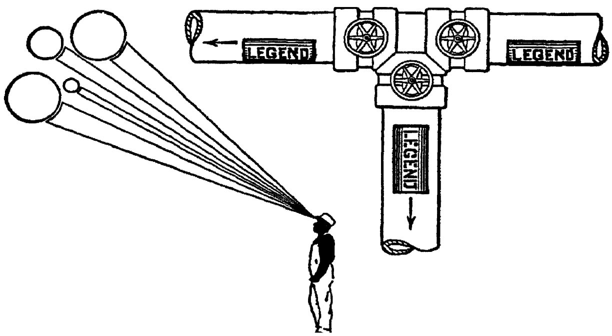


FIG. 16.—Position of pipe legends with respect to visibility from the floor.

392. Color Scheme.—The following colors have been chosen for marking purposes because they are readily distinguishable one from the other. *Red* has been assigned to fire-protection equipment because of long established custom. *Yellow* and *orange* have been assigned to dangerous materials because they have a high coefficient of reflection under white light, and can be readily recognized under the poorest condition of illumination. Yellow is the traditional color of the quarantine flag and has been adopted as the caution signal for railroads and road traffic. The assignment of the other three colors follows in natural order.

1. (*F*) Fire Protection—*red*
2. (*D*) Dangerous Materials—*yellow* or *orange*
3. (*S*) Safe Materials—*green* (or the achromatic colors *white, black, gray,* or *aluminum*).
4. (*P*) Protective Materials—*bright blue.*
5. (*V*) Extra Valuable Materials—*deep purple.*

The classification of some of the ordinary materials contained in or transported by piping is given in Table III.

TABLE III.—CLASSIFICATION OF MATERIALS CARRIED IN PIPES

KEY:	*F*-FIRE PROTECTION	*D*-DANGEROUS	*S*-SAFE	*P*-PROTECTIVE	*V*-EXTRA VALUABLE
	Red	Yellow	Green	Blue	Purple

Material piped	Physical state	Temperature of material degrees Fahrenheit	Pressure: Pounds per square inch	Pressure: Vacuum in water or mercury	Classification
Acetic acid	Liquid	Normal	30	28 Hg	*D*
Alcohol	Liquid	Cool	....	......	*D*
Alum solution	Liquid	Cool	....	......	*D*
Acetylene gas	Gas	0 to 200	0.5 to 250	......	*D*
Benzol	Liquid	Cold	up to 80	......	*D*
Brine	Liquid	Cold	about 60	......	*S*
Blau gas	Gas	00 to 200	0.5 to 250	......	*D*
Compressed air	Gas	80	300 to 3,000	......	*S*
CO_2	Gas and liquid	-30 to -100	0 to 250 (gage)	......	*D*
Caustic soda sol	Solution	....	....	......	*D*
Chlorine	Gas, liquid and solution	....	....	......	*D*
Carbon bisulphide	Gas and liquid	....	....	......	*D*
Chloroform	Gas and liquid	....	....	......	*D*
Carbon tetrachloride	Liquid	....	....	......	*S*
Coal gas	Gas	0 to 200	0.5 to 250	......	*D*
Dyes	Solution	Cool	....	......	*D*
Flue gas (waste heat)	Gas	0 to 200	0.5 to 250	......	*D*
Glycerine	Liquid	Hot and cold	up to 60	......	*S*
Hydrogen	Gas	....	....	......	*D*
H_2S gas	Gas	0 to 150	0.5 to 100	......	*D*
Lactic acid	Liquid	Normal	60	20 to 22	*V*
Miscl. solvent	Liquid	Cool	....	......	*D*
Mercury	Gas and liquid	....	29 in. to 100 lb.	......	*D*
Muriatic acid	Liquid	....	....	......	*D*
Natural gas	Gas	0 to 200	0.5 to 200	......	*D*
Nitric acid	Liquid	Normal	10	......	*D*
Oils (petroleum)	Liquid	below 580	below 1,200	......	*D*
Paper sizing solution	Liquid	....	....	......	*S*
Pintsch gas	Gas	0 to 200	0.5 to 250	......	*D*
Producer gas	Gas	0 to 200	0.5 to 250	......	*D*
Paint	Liquid	Cold	30	......	*D*
Soda ash solution	Liquid	Cold	60	......	*S*
Sugar juices and syrup	Liquid	195 to 200	0 to 50	......	*S*
Steam	Vapor	below 212	below atmos.	......	*S*
Steam	Vapor	212 to 800	above atmos.	......	*D*
SO_2	Gas and liquid	-30 to 100	0 to 250 (gage)	......	*D*
Turpentine	Liquid	Cold	up to 80	......	*D*
Tar	Liquid	....	....	......	*D*
Varnish	....	Cold and hot	30	......	*D*
Water	Liquid	Cold	any pressure	......	*S*
Water gas	Gas	0 to 200	0.5 to 250	......	*D*

Example.—Applying the color scheme to the stream and water piping system of a power plant, the colors and additional stripes are suggested as shown by Table IV.

TABLE IV.—IDENTIFICATION OF STEAM AND WATER PIPING

Material piped	Pressure	Color band	Additional color stripes
Steam....	Saturated (100 lb. and higher)	Orange	White
	Superheated (100 lb. and higher)	Orange	White-gray
	Saturated (20 to 75 lb.)	Orange	White-black
	Superheated (20 to 75 lb.)	Orange	White-black
	Saturated (atmosphere to 20 lb.)	Orange	White-green
	Superheated (atmosphere to 20 lb.)	Orange	White-green
	Vacuum (atmosphere to 1 in. absolute)	Yellow	White-buff
Water....	Fire protection	Red	
	Fresh	Green	
	Circulating	Green-blue	
	Boiler feed	Green	Black
	Blow-off	Green	Brown
	Make-up	Green	Gray
	Treated	Green	Yellow
	Filtered	Green	
	Waste	Green	White
	Hot	Green	
	Wash	Green	
	Return condensate	Green	Black
	Hydraulic piping—high pressure	Yellow	Green-red
	Hydraulic piping—low pressure	Green	
	High pressure 50 to 125 lb.	Yellow	Black-white
	Low pressure atmosphere to 3 lb.	Green	Black-white

Problems

1. Design a flywheel guard for the. . . . . .engine in the mechanical laboratory. The guard is to be constructed from pipe and standard guard rail fittings similar to the guard shown in Fig. 2. Make a pencil sketch showing all important details and dimensions.
2. The. . . . . .motor in. . . . . .laboratory is to be fully guarded as shown by Fig. 1. Make a pencil sketch giving all dimensions and specifications which are required in order to construct and erect the guard in place.
3. Design a gear guard with metal-cloth filler for the exposed gears on the . . . . .machine in the engineering shop laboratories.
4. Design a gear guard of sheet metal enclosing the gears on the. . . . . machine in the engineering shop laboratories.

References

1. BEYER, D. S., "Industrial Accident Prevention," Houghton Mifflin Company, Boston and New York, 1920.

2. "The Principles and Practice of Safety." A handbook for technical schools and universities. Issued by the National Safety Council.
3. "Safety Devices," Sheet Metal Ware Association, 280 Madison Avenue, New York.
4. CHANEY, L. W. and H. S. HANNA, "Accidents and Accident Prevention in Machine Building," Bull. 216, U. S. Bureau of Labor Statistics, Washington, D. C.
5. National Safety Council. Safe Practices, bulletins as follows:
   No. 4, Cranes.
   No. 5, Belt Shifters and Belt Shippers.
   No. 7, Belts and Belt Guards.
   No. 8, Shafting, Couplings, Pulleys, Gearing.
   No. 9, Engine Guarding and Engine Stops.
   No. 10, Oiling Devices and Oilers.
   No. 13, Grinding Wheels.
   No. 15, Freight Elevators.
   No. 18, Power Presses.
   No. 20, Wood Working Equipment.

APPENDIX

TABLE I.—TABLE OF DECIMAL EQUIVALENT PARTS OF AN INCH
Fractional inch equivalents

Fractional inch	Decimal inch	MM	Fractional inch	Decimal inch	MM
1/64	0.015625	0.3968	33/64	0.515625	13.0966
1/32	0.031250	0.7937	17/32	0.531250	13.4934
3/64	0.046875	1.1906	35/64	0.546875	13.8903
1/16	0.062500	1.5874	**9/16**	0.562500	14.2872
5/64	0.078125	1.9843	37/64	0.578125	14.6841
3/32	0.093750	2.3812	19/32	0.593750	15.0809
7/64	0.109375	2.7780	39/64	0.609375	15.4778
1/8	0.125000	3.1749	**5/8**	0.625000	15.8747
9/64	0.140625	3.5718	41/64	0.640625	16.2715
5/32	0.156250	3.9686	21/32	0.656250	16.6684
11/64	0.171875	4.3655	43/64	0.671875	17.0653
3/16	0.187500	4.7624	**11/16**	0.687500	17.4621
13/64	0.203125	5.1592	45/64	0.703125	17.8590
7/32	0.218750	5.5561	23/32	0.718750	18.2559
15/64	0.234375	5.9530	47/64	0.734375	18.6527
1/4	0.250000	6.3498	**3/4**	0.750000	19.0496
17/64	0.265625	6.7467	49/64	0.765625	19.4465
9/32	0.281250	7.1436	25/32	0.781250	19.8433
19/64	0.296875	7.5404	51/64	0.796875	20.2402
5/16	0.312500	7.9373	**13/16**	0.812500	20.6371
21/64	0.328125	8.3342	53/64	0.828125	21.0339
11/32	0.343750	8.7310	27/32	0.843750	21.4308
23/64	0.359375	9.1279	55/64	0.859375	21.8277
3/8	0.375000	9.5248	**7/8**	0.875000	22.2245
25/64	0.390625	9.9216	57/64	0.890625	22.6214
13/32	0.406250	10.3185	29/32	0.906250	23.0183
27/64	0.421875	10.7154	59/64	0.921875	23.4151
7/16	0.437500	11.1122	**15/16**	0.937500	23.8120
29/64	0.453125	11.5091	61/64	0.953125	24.2089
15/32	0.468750	11.9060	31/32	0.968750	24.6057
31/64	0.484375	12.3029	63/64	0.984375	25.0026
1/2	0.500000	12.6997	**1**	1.000000	25.3995

TABLE II.—LINEAL INCHES IN DECIMAL FRACTIONS OF A LINEAL FOOT

Lineal inches	Lineal foot	Lineal inches	Lineal foot	Lineal inches	Lineal foot
1/64	0.001302083	1 7/8	0.15625	6 3/4	0.5625
1/32	0.00260416	2	0.1666	7	0.5833
1/16	0.0052083	2 1/8	0.177083	7 1/4	0.60416
1/8	0.010416	2 1/4	0.1875	7 1/2	0.625
3/16	0.015625	2 3/8	0.197916	7 3/4	0.64583
1/4	0.02083	2 1/2	0.2083	8	0.66667
5/16	0.0260416	2 5/8	0.21875	8 1/4	0.6875
3/8	0.03125	2 3/4	0.22916	8 1/2	0.7083
7/16	0.0364583	2 7/8	0.239583	8 3/4	0.72916
1/2	0.0416	3	0.25	9	0.75
9/16	0.046875	3 1/4	0.27083	9 1/4	0.77083
5/8	0.052083	3 1/2	0.2916	9 1/2	0.7916
11/16	0.0572916	3 3/4	0.3125	9 3/4	0.8125
3/4	0.0625	4	0.33333	10	0.83333
13/16	0.0677083	4 1/4	0.35416	10 1/4	0.85416
7/8	0.072916	4 1/2	0.375	10 1/2	0.875
15/16	0.078125	4 3/4	0.39583	10 3/4	0.89583
1	0.0833	5	0.4166	11	0.9166
1 1/8	0.09375	5 1/4	0.4375	11 1/4	0.9375
1 1/4	0.10416	5 1/2	0.4583	11 1/2	0.9583
1 3/8	0.114583	5 3/4	0.47916	11 3/4	0.97916
1 1/2	0.125	6	0.5	12	1.000
1 5/8	0.135416	6 1/4	0.52083		
1 3/4	0.14583	6 1/2	0.5416		

TABLE III.—WEIGHT, SPECIFIC GRAVITY AND COEFFICIENT OF EXPANSION OF METALS

	Specific gravity, range according to several authorities	Specific gravity, approximate mean value used in calculation of weight	Weight per cubic foot, pounds	Weight per cubic inch, pounds	Linear coefficient of thermal expansion, degrees Fahrenheit
Aluminum...........	2.56 to 2.71	2.67	166.5	0.0963	0.0000128
Antimony............	to 6.86	6.76	421.6	0.2439	
Bismuth.............	9.74 to 9.90	9.82	612.4	0.3544	
Brass, copper, and zinc					
80 20	7.8 to 8.6	8.60	536.3	0.3103	
70 30		8.40	523.8	0.3031	
60 40		8.36	521.3	0.3017	
50 50		8.20	511.4	0.2959	
Bronze { copper 95 to 80, tin 5 to 20 }	8.52 to 8.96	8.853	552.0	0.3195	0.0000104
Cadmium................	8.6 to 8.7	8.65	539.0	0.3121	
Calcium.................	1.58				
Chromium...............	5.0				
Cobalt..................	8.5 to 8				
Gold, pure...............	19.245 to 19 361	19.258	1,200.9	0.6949	
Copper..................	8.69 to 8 92	8.853	552.0	0.3195	0.0000093
Iridium..................	22.38 to 23 0	......	1,396.0	0.8076	
Iron, cast................	6.85 to 7 48	7.218	450.0	0.2604	0.0000060
Iron, wrought............	7.4 to 7 9	7.70	480.0	0.2779	0.0000067
Lead.....................	11.07 to 11 44	11.38	709.7	0.4106	
Manganese...............	7.0 to 8 0	8.00	499.0	0.2887	
Magnesium...	1.69 to 1 75	1.75	109.0	0.0641	
Mercury. 32°	13.60 to 13	13.62	849.3	0.4915	
60°	13.58	13.58	846.8	0.4900	
212°	13.37 to 13 38	13.38	834.4	0 4828	
Nickel.......	8.279 to 8 93	8.8	548.7	0 3175	
Platinum....	20.33 to 22 07	21.5	1,347.0	0.7758	
Potassium...	0.865				
Silver.......	10.474 to 10 511	10.505	655.1	0.3791	
Sodium......	0.97				
Steel........	7.69* to 7 932†	7.854	489.6	0.2834	0.0000065
Tin.........	7.291 to 7 409	7.350	458.3	0.2652	
Titanium....	5.3				
Tungsten....	17.0 to 17.6				
Zinc.........	6.86 to 7.20	7.00	436.5	0.2526	

* Hard and burned. † Very pure and soft. The specific gravity decreases as the carbon is increased. In the first column of figures, the lowest are usually those of cast metals, which are more or less porous; the highest are of metals finely rolled or drawn into wire.

TABLE IV.—WEIGHT OF ROUND AND SQUARE STEEL PER LINEAL FOOT IN POUNDS

(Based on 489.6 Lb. per Cubic Foot)

Size, inches	Weight of ○ 1 ft. long	Weight of □ 1 ft. long	Size, inches	Weight of ○ 1 ft. long	Weight of □ 1 ft. long	Size, inches	Weight of ○ 1 ft. long	Weight of □ 1 ft. long
1/32	0.0026	0.0033	3	24.03	30.60	6	96.14	122.4
1/16	0.0104	0.0133	3 1/16	25.04	31.89	6 1/16	98.14	125.0
1/8	0.0417	0.0531	3 1/8	26.08	33.20	6 1/8	100.2	127.6
3/16	0.0938	0.1195	3 3/16	27.13	34.55	6 3/16	102.2	130.2
1/4	0.1669	0.2123	3 1/4	28.20	35.92	6 1/4	104.3	132.8
5/16	0.2608	0.3333	3 5/16	29.30	37.31	6 5/16	106.4	135.5
3/8	0.3756	0.4782	3 3/8	30.42	38.73	6 3/8	108.5	138.2
7/16	0.5111	0.6508	3 7/16	31.56	40.18	6 7/16	110.7	140.9
1/2	0.6676	0.8500	3 1/2	32.71	41.65	6 1/2	112.8	143.6
9/16	0.8449	1.076	3 9/16	33.90	43.14	6 9/16	114.9	146.5
5/8	1.043	1.328	3 5/8	35.09	44.68	6 5/8	117.2	149.2
11/16	1.262	1.608	3 11/16	36.31	46.24	6 11/16	119.4	152.1
3/4	1.502	1.913	3 3/4	37.56	47.82	6 3/4	121.7	154.9
13/16	1.763	2.245	3 13/16	38.81	49.42	6 13/16	123.9	157.8
7/8	2.044	2.603	3 7/8	40.10	51.05	6 7/8	126.2	160.8
15/16	2.347	2.989	3 15/16	41.40	52.71	6 15/16	128.5	163.6
1	2.670	3.400	4	42.73	54.40	7	130.9	166.6
1 1/16	3.014	3.838	4 1/16	44.07	56.11	7 1/8	135.6	172.6
1 1/8	3.379	4.303	4 1/8	45.44	57.85	7 1/4	140.4	178.7
1 3/16	3.766	4.795	4 3/16	46.83	59.62	7 3/8	145.3	184.9
1 1/4	4.173	5.312	4 1/4	48.24	61.41	7 1/2	150.2	191.3
1 5/16	4.600	5.857	4 5/16	49.66	63.23	7 5/8	155.2	197.7
1 3/8	5.049	6.428	4 3/8	51.11	65.08	7 3/4	160.3	204.2
1 7/16	5.518	7.026	4 7/16	52.58	66.95	7 7/8	165.6	210.8
1 1/2	6.008	7.650	4 1/2	54.07	68.85	8	171.0	217.6
1 9/16	6.520	8.301	4 9/16	55.59	70.78	8 1/8	176.3	224.5
1 5/8	7.051	8.978	4 5/8	57.12	72.73	8 1/4	181.8	231 4
1 11/16	7.604	9.682	4 11/16	58.67	74.70	8 3/8	187.3	238.5
1 3/4	8.178	10.41	4 3/4	60.25	76.71	8 1/2	193.0	245 6
1 13/16	8.773	11.17	4 13/16	61.84	78.74	8 5/8	198.7	252.9
1 7/8	9.388	11.95	4 7/8	63.46	80.81	8 3/4	204.4	260.3
1 15/16	10.02	12.76	4 15/16	65.10	82.89	8 7/8	210.3	267.9
2	10.68	13.60	5	66.76	85.00	9	216.3	275.4
2 1/16	11.36	14.46	5 1/16	68.44	87.14	9 1/8	222.4	283.2
2 1/8	12.06	15.35	5 1/8	70.14	89.30	9 1/4	228.5	290.9
2 3/16	12.78	16.27	5 3/16	71.86	91.49	9 3/8	234.7	298.9
2 1/4	13.52	17.22	5 1/4	73.60	93.72	9 1/2	241.0	306.8
2 5/16	14.28	18.19	5 5/16	75.37	95.96	9 5/8	247.4	315.0
2 3/8	15.07	19.18	5 3/8	77.15	98.23	9 3/4	253.9	323.2
2 7/16	15.86	20.20	5 7/16	78.95	100.5	9 7/8	260.4	331.6
2 1/2	16.69	21.25	5 1/2	80.77	102.8	10	267.0	340.0
2 9/16	17.53	22.33	5 9/16	82.62	105.2	10 1/4	280.6	357.2
2 5/8	18.40	23.43	5 5/8	84.49	107.6	10 1/2	294.4	374.9
2 11/16	19.29	24.56	5 11/16	86.38	110.0	10 3/4	308.6	392.9
2 3/4	20.20	25.71	5 3/4	88.29	112.4	11	323.1	411.4
2 13/16	21.12	26.90	5 13/16	90.22	114.9	11 1/4	337.9	430.3
2 7/8	22.07	28.10	5 7/8	92.17	171.4	11 1/2	353.1	449.6
2 15/16	23.04	29.34	5 15/16	94.14	119.9	11 3/4	368.6	469 4

These figures represent the theoretical weights of steel. Iron will run about 2 per cent lighter; cast iron 9 per cent lighter.

TABLE V.—ALLOYS
(U. S. Navy Specifications for Non-ferrous Rolled Metals)

Letter	Name	Composition by percentage					
		Copper	Tin	Zinc	Lead, maximum	Iron, maximum	Miscellaneous
A	Admiralty..........	70 (min.)	1 (min.)	Remainder	0.075	0.06	
Be-r	Benedict nickel.....	84–86	....	....	......	......	Remainder nickel
B-r	Sheet brass and piping.	60–70	....	Remainder	0.5		
B-r	Commercial brass rod	60–63	....	Remainder	3.0		
Cu-r	Copper.............	99.5 (min.)					
D-r	Muntz metal.......	59–62	....	Remainder	0.6		
P-r	Phosphor-bronze[1]....	85–95	5–10	4 (max.)	0.2	0.06	Phosphorus, 0.15
Mn-r	Manganese-bronze[1]..	57–60	0.5	37–40	......	1.0	Manganese, 0.30
Mo-r	Monel metal........	Remainder	....	....	......	3.5	Nickel, 60 (min.); aluminum, 0.5 (max.)
N-r	Rolled naval brass...	59–63	0.5–1.5	Remainder	0.2	0.06	

[1] The figures given are approximate and are a guide as to the proportions of the elements, except those for lead, aluminum, and iron, which are maximum limits.

TABLE VI.—COMPOSITION OF MISCELLANEOUS ALLOYS

Alloys	Antimony	Bismuth	Copper	Iron	Lead	Nickel	Silver	Tin	Zinc
Brass, common yellow.	.....	...	61.6	.....	2.9			0.2	35.3
Brass, to be rolled.....	.....	..	32	...	..			1.5	10
Brass castings, common	....	......	20	......	...	.		2.5	1.25
Gun metal............	......	......	8	...	...			1	
Copper flanges........	......	....	9	.....	.			0.26	1
Bronze statuary......	......	......	91.4	...	1.37			1.7	5.53
German silver.........	......	......	2	6.5	......	7.9	.		6.3
Britannia metal......	50	25	......	..	......	..		25	
Chinese white copper..	......	......	20.2	......	......	15.8		1.3	12.7
Pattern letters........	15	15	......	.....	70				
Bell metal...........	......	......	4	....	......	....		1	
Chinese gongs........	......	......	40.5	......	....	......	...	9.2	
White metal, ordinary..	28.4	......	3.7	......	......	......	..	14.2	3.7
Spelter..............	......	......	1	......	...	......	.....	...	1
Type metal...........	1	......	......	......	3–7				

TABLE VII.—STRENGTH OF STANDARD AND EXTRA FLEXIBLE STEEL HOISTING ROPE. ALLOWABLE WORKING LOADS ARE 20 PER CENT OF BREAKING STRENGTH

Standard hoisting rope 6 strands, 19 wires per strand hemp center									Extra flexible steel hoisting rope 8 strands, 19 wires per strand hemp center				
Diameter, inches	Approximate circumference, inches	Approximate weight per foot, pounds	Approximate breaking strength tons (2,000 lb.)					Minimum sheave diameter	Approximate break strength tons (2,000 lb.)				Minimum sheave diameter
			Iron	Crucible steel	Extra strong crucible steel	Plow steel	Monitor plow steel	Feet Steel rope	Crucible steel	Extra strong crucible steel	Plow steel	Monitor plow steel	Feet Steel rope
1/4	3/4	0.10	1.1	2.2	2.43	2.65	3.15	1.00	1.80	2.02	2.25	....	0.75
5/16	1	0.15	1.5	3.1	3.50	3.80	4.50	1.25	2.75	3.05	3.35	....	0.83
3/8	1 1/8	0.22	2.4	4.8	5.30	5.75	6.75	1.50	4.2	4.66	5.12	....	1.00
7/16	1 1/4	0.30	2.9	6.5	7.25	8.00	9.40	1.75	5.7	6.30	6.90	....	1.16
1/2	1 1/2	0.39	3.9	8.4	9.20	10.0	12.1	2.00	7.3	8.0	8.7	9.5	1.33
9/16	1 3/4	0.50	4.7	10.0	11.2	12.3	14.5	2.25	8.7	10.1	11.6	12.0	1.50
5/8	2	0.62	6.0	12.5	14.0	15.5	19.0	2.50	10.9	12.4	14.0	15.0	1.75
3/4	2 1/4	0.89	8.5	17.5	20.2	23.0	26.3	3.00	15.3	17.6	20.0	22.0	1.83
7/8	2 3/4	1.20	11.8	23.0	26.0	29.0	35.0	3.50	20.0	23.0	26.0	28.0	2.16
1	3	1.58	14.5	30.0	34.0	38.0	45.0	4.00	26.0	29.0	33.0	36.0	2.50
1 1/8	3 1/2	2.00	18.6	38.0	43.0	47.0	56.0	4.50	34.0	38.0	43.0	46.0	2.83
1 1/4	4	2.45	22.8	47.0	53.0	58.0	69.0	5.00	42.0	47.0	52.0	56.0	3.20
1 3/8	4 1/4	3.00	28.0	56.0	64.0	72.0	84.0	5.50	51.0	57.0	64.0	68.0	3.50
1 1/2	4 3/4	3.55	33.0	64.0	73.0	82.0	98.0	6.00	58.0	66.0	74.0	80.0	3.75
Non-spinning hoisting rope, 18 strands, 7 wires per strand, hemp core													
3/8	1 1/8	0.25	2.10	3.90	4.60	5.55	6.10	1.50					
7/16	1 1/4	0.31	2.63	4.90	5.80	6.85	.....	1.75					
1/2	1 1/2	0.42	3.65	6.90	8.10	9.75	10.70	2.00					
9/16	1 3/4	0.57	4.85	9.10	10.70	12.80	.....	2.25					
5/8	2	0.70	5.90	11.10	13.10	15.75	17.30	2.50					
3/4	2 1/4	1.02	8.85	16.70	19.60	24.60	27.00	3.00					
7/8	2 3/4	1.44	11.95	22.50	26.50	31.80	35.00	3.50					
1	3	1.73	14.95	28.00	33.00	36.30	39.00	4.00					
1 1/8	3 1/2	2.34	19.60	36.80	43.20	47.50	52.20	4.50					
1 1/4	4	2.80	23.40	43.80	51.60	56.80	62.50	5.00					

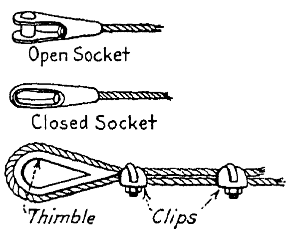


TABLE VIII.—AVERAGE STRENGTHS OF COMMON MATERIALS. COPPER, ZINC, AND TIN ALLOYS
(U. S. Government Tests)

Percentage of			Tensile strength, pounds per square inch	Percentage of			Tensile strength, pounds per square inch	Percentage of			Tensile strength, pounds per square inch
Copper	Zinc	Tin		Copper	Zinc	Tin		Copper	Zinc	Tin	
45	50	5	15,000	60	20	20	10,000	75	20	5	45,000
50	45	5	50,000	65	30	5	50,000	75	15	10	45,000
50	40	10	15,000	65	25	10	42,000	75	10	15	43,000
55	43	2	65,000	65	20	15	30,000	75	5	20	41,000
55	40	5	62,000	65	15	20	18,000	80	15	5	45,000
55	35	10	32,000	65	10	25	12,000	80	10	10	45,000
55	30	15	15,000	70	25	5	45,000	80	5	15	47,000
60	37	3	60,000	70	20	10	44,000	85	10	5	43,000
60	35	5	52,000	70	15	15	37,000	85	5	10	46,500
60	30	10	40,000	70	10	20	30,000	90	5	5	42,000

TABLE IX.—CARBON STEELS
(S. A. E. SPECIFICATIONS FOR STEELS)

Carbon, per cent		Manganese, per cent		Maximum percentage		S. A. E. Specification number
Desired	Minimum and maximum	Desired	Minimum and maximum	Phosphorus	Sulphur	
0.10	0.05 to 0.15	0.45	0.30 to 0.60	0.045	0.050	1010
0.20	0.15 to 0.25	0.45	0.30 to 0.60	0.045	0.050	1020
0.25	0.20 to 0.30	0.65	0.50 to 0.80	0.045	0.050	1025
0.35	0.30 to 0.40	0.65	0.50 to 0.80	0.045	0.050	1035
0.45	0.40 to 0.50	0.65	0.50 to 0.80	0.045	0.050	1045
0.95	0.90 to 1.05	0.35	0.25 to 0.50	0.040	0.050	1095

TABLE IX (*Continued*)

NICKEL AND NICKEL-CHROMIUM STEELS[1]

Carbon, per cent		Manganese, per cent		Nickel, per cent		Chromium, per cent		S. A. E. specification No.
Desired	Minimum and maximum	Desired	Minimum and maximum	Desired	Minimum and maximum	Desired	Minimum and maximum	
Nickel steels								
0.15	0.10 to 0.20	0.65	0.50 to 0.80	3.50	3.25 to 3.75			2315
0.20	0.15 to 0.25	0.65	0.50 to 0.80	3.50	3.25 to 3.75			2320
0.30	0.25 to 0.35	0.65	0.50 to 0.80	3.50	3.25 to 3.75			2330
0.35	0.30 to 0.40	0.65	0.50 to 0.80	3.50	3.25 to 3.75			2335
0.40	0.35 to 0.45	0.65	0.50 to 0.80	3.50	3.25 to 3.75			2340
0.45	0.40 to 0.50	0.65	0.50 to 0.80	3.50	3.25 to 3.75			2345
0.12	0.17	0.45	0.30 to 0.60	5.00	4.50 to 5.25			2512†
Nickel chromium steels[1]								
0.20	0.15 to 0.25	0.65	0.50 to 0.80	1.25	1.00 to 1.50	0.60	0.45 to 0.75	3120
0.25	0.20 to 0.30	0.65	0.50 to 0.80	1.25	1.00 to 1.50	0.60	0.45 to 0.75	3125
0.30	0.25 to 0.35	0.65	0.50 to 0.80	1.25	1.00 to 1.50	0.60	0.45 to 0.75	3130
0.35	0.30 to 0.40	0.65	0.50 to 0.80	1.25	1.00 to 1.50	0.60	0.45 to 0.75	3135
0.40	0.35 to 0.45	0.65	0.50 to 0.80	1.25	1.00 to 1.50	0.60	0.45 to 0.75	3140
0.20	0.15 to 0.25	0.45	0.30 to 0.60	1.75	1.50 to 2.00	1.10	0.90 to 1.25	3220
0.30	0.25 to 0.35	0.45	0.30 to 0.60	1.75	1.50 to 2.00	1.10	0.90 to 1.25	3230
0.40	0.35 to 0.45	0.45	0.30 to 0.60	1.75	1.50 to 2.00	1.10	0.90 to 1.25	3240
0.50	0.45 to 0.55	0.45	0.30 to 0.60	1.75	1.50 to 2.00	1.10	0.90 to 1.25	3250
0.15	0.10 to 0.20	0.60	0.45 to 0.75	3.00	2.75 to 3.25	0.80	0.60 to 0.95	3415
0.35	0.30 to 0.40	0.60	0.45 to 0.75	3.00	2.75 to 3.25	0.80	0.60 to 0.95	3435
0.50	0.45 to 0.55	0.60	0.45 to 0.75	3.00	2.75 to 3.25	0.80	0.60 to 0.95	3450
0.20	0.15 to 0.25	0.45	0.30 to 0.60	3.50	3.25 to 3.75	1.50	1.25 to 1.75	3320
0.30	0.25 to 0.35	0.45	0.30 to 0.60	3.50	3.25 to 3.75	1.50	1.25 to 1.75	3330
0.40	0.35 to 0.45	0.45	0.30 to 0.60	3.50	3.25 to 3.75	1.50	1.25 to 1.75	3340

[1] The phosphorus in the nickel and nickel-chromium steels must not exceed 0.040 per cent. The sulphur in nickel steels must not exceed 0.045 per cent. The sulphur in nickel-chromium steels, up to and including specification No. 3140, must not exceed 0.045 per cent, and for all specification numbers above 3140, the sulphur must not exceed 0.040 per cent.

† When it is necessary to machine, after carburizing, nickel steel parts made according to specification No. 2512, the nickel content should be maintained as close to the lower limit as possible.

TABLE IX (*Continued*)

CHROMIUM AND CHROMIUM-VANADIUM STEELS*

Carbon, per cent		Manganese, per cent		Chromium, per cent		Vanadium, per cent		S. A. E. specification No.
Desired	Minimum and maximum	Desired	Minimum and maximum	Desired	Minimum and maximum	Desired	Minimum	
Chromium steels								
0.20	0.15 to 0.25	†	†	0.75	0.60 to 0.90	......	....	5,120
0.40	0.35 to 0.45	†	†	0.75	0.60 to 0.90	......	....	5,140
0.65	0.60 to 0.70	†	†	0.75	0.60 to 0.90	......	....	5,165
1.00	0.95 to 1.10	0.35	0.20 to 0.50	1.35	1.20 to 1.50	......	....	52,100
Chromium-vanadium steels								
0.20	0.15 to 0.25	0.65	0.50 to 0.80	0.95	0.80 to 1.10	0.18	0.15	6,120
0.25	0.20 to 0.30	0.65	0.50 to 0.80	0.95	0.80 to 1.10	0.18	0.15	6,125
0.30	0.25 to 0.35	0.65	0.50 to 0.80	0.95	0.80 to 1.10	0.18	0.15	6,130
0.35	0.30 to 0.40	0.65	0.50 to 0.80	0.95	0.80 to 1.10	0.18	0.15	6,135
0.40	0.35 to 0.45	0.65	0.50 to 0.80	0.95	0.80 to 1.10	0.18	0.15	6,140
0.45	0.40 to 0.50	0.65	0.50 to 0.80	0.95	0.80 to 1.10	0.18	0.15	6,145
0.50	0.45 to 0.55	0.65	0.50 to 0.80	0.95	0.80 to 1.10	0.18	0.15	6,150
0.95	0.90 to 1.05	0.35	0.20 to 0.45	0.95	0.80 to 1.10	0.18	0.15	6,195

* The phosphorus in chromium steels up to specification No. 5,165 inclusive must not exceed 0.040 per cent; the maximum amount for No. 52,100 is 0.030 per cent. The maximum sulphur content is 0.045 per cent, except for steel No. 52,100, which must not have over 0.030 per cent sulphur. The maximum amount for both phosphorus and sulphur for all chromium vanadium steels is 0.040 per cent, except No. 6,195 which must not have over 0.03 per cent.

† Two types of steel are available in this class, one with manganese from 0.25 to 0.50 per cent (0.35 per cent desired) and silicon not over 0.20 per cent; the other with manganese from 0.60 to 0.80 per cent (0.70 per cent desired) and silicon from 0.15 to 0.50 per cent.

TUNGSTEN STEELS

Carbon, per cent		Manganese, maximum, per cent	Phosphorus, maximum, per cent	Sulphur, maximum, per cent	Chromium, per cent	Tungsten, per cent	S. A. E. specification No.
Desired	Minimum and maximum						
0.60	0.50 to 0.70	0.30	0.035	0.035	3 00 to 4.00	12 0 to 15 0	71,360
0.60	0.50 to 0.70	0.30	0.035	0.035	3.00 to 4.00	15.0 to 18.0	71,660
0.60	0.50 to 0.70	0.30	0.035	0.035	0.50 to 1.00	1.5 to 2.00	7,260

SILICO-MANGANESE STEELS

Carbon, per cent		Manganese, per cent		Silicon, per cent		Phosphorus and sulphur, maximum	S. A. E. specification No.
Desired	Minimum and maximum	Desired	Minimum and maximum	Desired	Minimum and maximum		
0.50	0.45 to 0.55	0.70	0.60 to 0.80	1.95	1.80 to 2.10	0.045	9,250
0.60	0.55 to 0.65	0.60	0.50 to 0.70	1.65	1.50 to 1.80	0.045	9,260

TABLE X.—AVERAGE ULTIMATE STRENGTHS OF COMMON MATERIALS

WIRE

Material in wire	Tension	Compression	Shear	Modulus of elasticity
	Pounds per square inch			
Copper, annealed	36,000	....	....	15,000,000
Copper, unannealed	60.000	....	....	18,000,000
Iron, annealed	60,000	....	....	15,000,000
Iron, unannealed	80,000	....	....	25,000,000
Steel, annealed	80,000	....	....	30,000,000
Steel, unannealed	120,000	....	....	30,000,000
Steel, crucible	180,000	....	....	30,000,000
Steel, susp. bridge	200,000	....	....	30,000,000
Steel, piano	300,000			
Steel, plow	270,000			

BRONZES
(U. S. Government Tests)

Per cent of		Tensile	Yield point	Compressive	Elongation per cent
Copper	Tin	Pounds per square inch			
100	..	27,000	14,000	41,000	8.0
95	5	31,000	17,000	46,000	10.0
90	10	29,000	21,000	54,000	4.0
85	15	33,000	26,000	74,000	1.6
80	20	32,000	28,000	124,000	0.5
75	25	18,000	18,000	150,000	
70	30	6,500	6,500	143,000	
65	35	2,800	2,800	75,000	

BRASSES

Per cent of		Tensile	Yield point	Compressive	Elongation per cent
Copper	Zinc	Pounds per square inch			
100	..	27,000	14,000	41,000	7
95	5	28,000	12,000	28,000	12
90	10	30,000	10,000	29,000	18
85	15	32,000	9,000	33,000	25
80	20	34,000	8,000	39,000	33
75	25	37,000	9,000	46,000	38
70	30	41,000	10,000	54,000	38
65	35	46,000	13,000	63,000	33
60	40	49,000	17,000	74,000	19
55	45	44,000	20,000	90,000	10
50	50	30,000	24,000	116,000	4
45	55	14,000	14,000	126,000	

TABLE XI.—PHYSICAL PROPERTIES OF HEAT-TREATED ALLOY STEELS (S. A. E. Reports of Iron and Steel Division)

Carbon steel

Range of carbon content	Range of manganese content	Physical properties (average minimum values given)[1] Heating temperature	Reheating temperature	Tensile strength	Elastic limit	Reduction of area	Elongation in 2 in.
Per cent		Degrees Fahrenheit		Pounds per square inch		Per cent	
0.15 to 0.25	0.30 to 0.60	1560 to 1580	400	80,000	50,000	60.0	20.0
			900	75,000	42,500	65.0	26.5
			1400	70,000	35,000	70.0	32.5
0.20 to 0.30	0.50 to 0.80	1540 to 1560	400	90,000	60,000	55.0	17.0
			900	82,500	50,000	61.0	23.5
			1400	75,000	40,000	67.5	30.0
0.30 to 0.40	0.50 to 0.80	1510 to 1530	400	105,000	75,000	42.5	15.0
			900	94,000	63,000	52.5	21.5
			1400	82,000	50,000	62.5	28.0
0.40 to 0.50	0.50 to 0.80	1490 to 1510	400	125,000	90,000	35.0	12.5
			900	110,000	75,000	45.0	17.5
			1400	95,000	60,000	55.0	22.5
Nickel steels							
	Range of nickel						
0.15 to 0.25	3.25 to 3.75	1510 to 1540	400	170,000	140,000	45.0	11.0
			900	130,000	99,000	60.5	21.5
			1400	70,000	40,000	75.0	30.0
0.25 to 0.35	3.25 to 3.75	1485 to 1515	400	220,000	190,000	35.0	10.0
			900	140,000	115,000	54.0	16.0
			1400	80,000	50,000	70.0	25.0
0.35 to 0.45	3.25 to 3.75	1435 to 1465	400	240,000	215,000	32.5	10.0
			900	155,000	130,000	51.0	16.0
			1400	90,000	60,000	62.5	22.5
Nickel-chromium steels[2]							
0.15 to 0.25	1.00 to 1.50	1585 to 1615	400	160,000	120,000	52.5	15.0
			900	111,000	84,000	69.0	21.0
			1400	75,000	50,000	72.5	35.0
0.25 to 0.35	1.00 to 1.50	1535 to 1565	400	190,000	155,000	37.5	10.0
			900	134,000	102,000	63.0	17.5
			1400	80,000	70,000	70.0	30.0
0.35 to 0.45	1.00 to 1.50	1485 to 1515	400	230,000	200,000	27.0	7.5
			900	157,000	126,000	46.5	14.0
			1400	90,000	75,000	62.0	20.0

[1] Round specimens varying from ½ to 1½ in diameter. Heated from 15 to 30 min., quenched in oil, reheated for 30 min. to the temperature given in the table and cooled in air.

[2] The chromium content for all steels listed, ranges from 0.45 to 0.75 per cent in the standard specifications. 0.60 being desired.

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